DEFORMATION BEHAVIOR OF MATERIALS

B.TECH, 4th SEMESTER

LECTURES NOTE

BY

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Disclaimer

5Th Semester

Deformation Behaviour of Materials

Module I: **(8 hours)**

Introduction: Elastic, plastic and visco-elastic deformation.
Continuum mechanics: Concepts of stress and strain in 3D stress and strain tensor, principal stresses and strains and principal axes, mean stress, stress deviator, maximum shear, equilibrium of stresses, equations of compatibility.
Plastic response of materials: a continuum approach: classification of stress-strain curves, yield criteria

Module II: **(8 hours)**

Plastic deformation of single crystals: Concepts of crystal geometry, lattice defects, deformation by slip, slip in a perfect lattice, slip by dislocation movement, critical resolved shear stress, deformation by twinning, stacking faults, deformation band and kink band, strain hardening of single crystal; stress-strain curves of fcc, bcc and hcp materials

Module III: **(6 hours)**

Dislocation Theory: Elements of dislocation theory, movement of dislocation, elastic properties of dislocation, intersection of dislocation, dislocation reactions in different crystal structures, origin and multiplication of dislocations, dislocation pile-ups.

Module IV: **(6 hours)**

Plastic deformation of polycrystalline materials: Role of grain boundaries in deformation, strengthening by grain boundaries, yield point phenomenon, strain ageing, strengthening by solutes, precipitates, dispersoids and fibres.

Module V: **(12 hours)**

Fracture: Types of fracture in metals, theoretical cohesive strength of metals, Griffth theory of brittle fracture, fracture of single crystals, metallographic aspects of fracture, dislocation theories of brittle fracture, ductile fracture.
Tension test: Engineering & true stress-strain curves, evaluation of tensile properties, Tensile instability, Effect of strain-rate & temperature on flow properties.
Deformation in non-metallic materials: structure and deformation of polymers, concept Super-lattice dislocations in intermetallics, concept of charge associated with dislocations in ceramics.

Introduction to mechanical metallurgy

Subjects of interest

- *Introduction to mechanical metallurgy*
- *Strength of materials – Basic assumptions*
- *Elastic and plastic behaviour*
- *Average stress and strain*
- *Tensile deformation of ductile metals*
- *Ductile vs brittle behaviour*
- *What constitute failure?*
- *Concept of stress and the types of stresses*
- *Units of stress and other quantities*

Objectives

• This chapter provides a background of continuum description of stress and strain and extends it to the defect mechanisms of flow and fracture of metals.

• Elastic and plastic behaviours of metals are highlighted and factors influencing failure in metals are finally addressed.

Introduction

Mechanical metallurgy : Response of metals to forces or loads.

Mechanical assessment of Materials
- Structural materials
- Machine, aircraft, ship, car etc

We need to know limiting values of which materials in service can withstand without failure.

Forming of metals into useful shapes
- Forging, rolling, extrusion.
- drawing, machining, etc

We need to know conditions of load and temperature to minimise the forces that are needed to deform metal without failure.

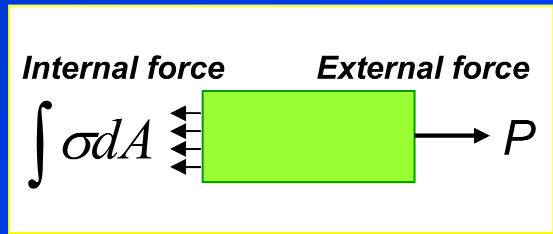
Strength of materials

Strength of materials deals with relationships between;

- *internal resisting forces*
- *deformation*
- *external loads*

, which act on some part of a body (member) in equilibrium.

- In equilibrium condition, if there are the ***external forces*** acting on the member, there will be the ***internal forces*** resisting the action of the external loads.



- The ***internal resisting forces*** are usually expressed by the ***stress*** acting over a certain ***area***, so that the internal force is the integral of the stress times the differential area over which it acts.

$$P = \int \sigma dA \qquad \text{…Eq.1}$$

Assumptions in strength of materials

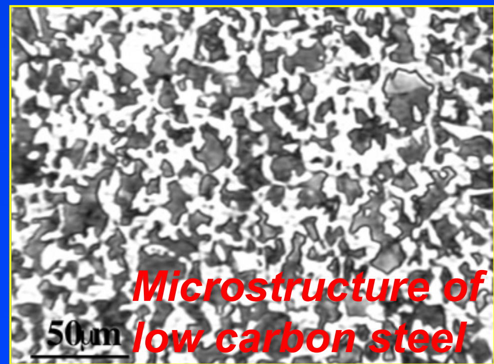
The body (member) is

- ***Continuous:*** No voids or empty spaces.
- ***Homogeneous:*** Has identical properties at all points.
- ***Isotropic:*** Has similar properties in all directions or orientation.

Remark:
Anisotropic *is when the body has property that varies with orientation.*

Macroscopic scale, engineering materials such as steel, cast iron, aluminium seems to be continuous, homogeneous and isotropic.

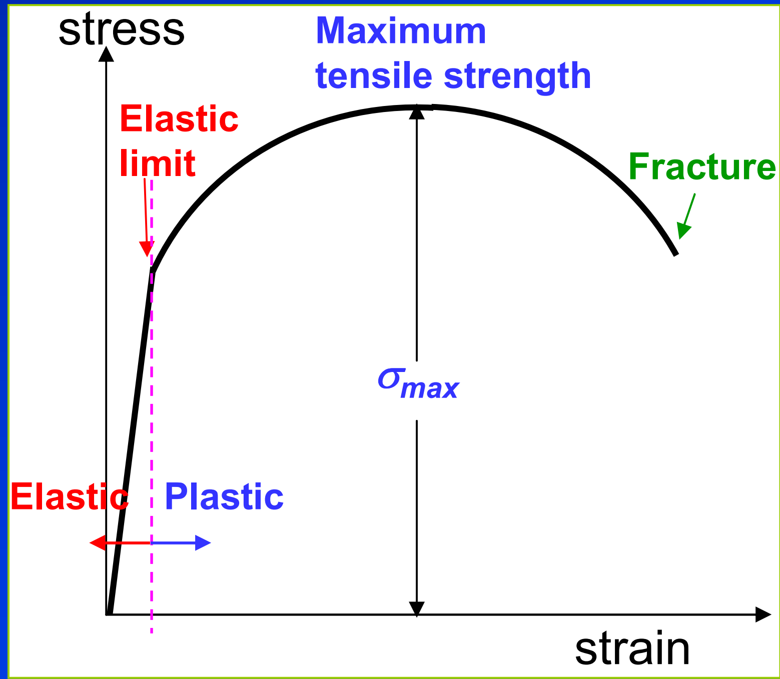
Microscopic scale, metals are made up of an aggregate of crystal grains having different properties in different crystallographic directions. However, these crystal grains are very small, and therefore the properties are homogenous in the macroscopic scale.



Microstructure of low carbon steel

Elastic and plastic behaviour

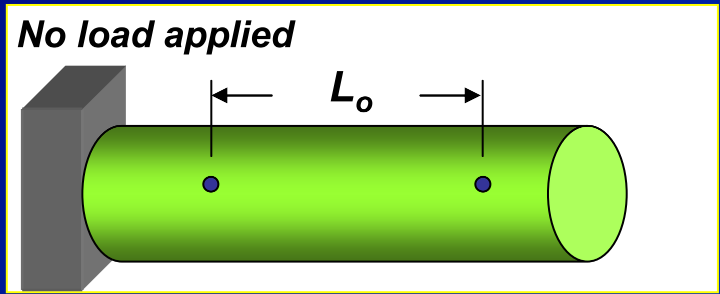
- All solid materials can be ***deformed*** when subjected to external load.
- In elastic region, stress is proportional to strain. This follows ***Hook's law*** up to the elastic limit . The material now has ***elastic behaviour***.
- At the ***elastic limit***, when the load is removed, the material will change back to its original shape.
- Beyond the ***elastic limit***, material permanently deformed or the material has undergone ***plastic deformation***.

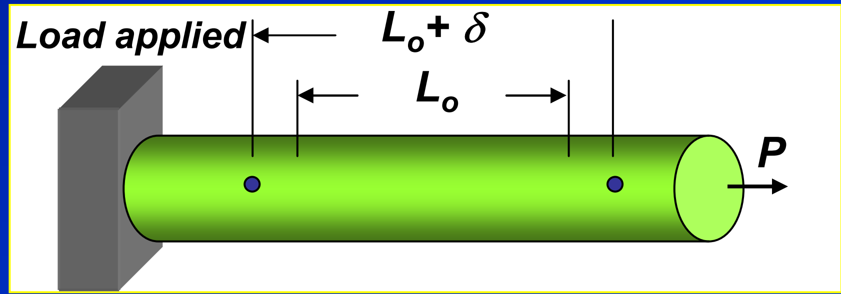


Load and extension curve of under uniaxial tensile loading.

Note: elastic deformations in metals are relatively small in comparison to plastic deformations.

Average stress and strain



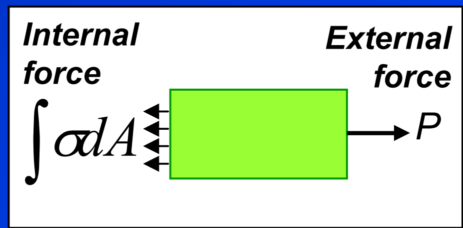


• A uniform cylindrical bar which is subjected to an axial tensile load ***P***. The original gauge length L_o has undergone a slight increase in length to $L_o + \delta$ with a slight decrease in diameter.

• The ***average elastic strain e*** is the ratio of change in length to the original length.

$$e = \frac{\delta}{L_o} = \frac{\Delta L}{L_o} = \frac{L - L_o}{L_o} \qquad \text{...Eq.2}$$

The ***external load P*** is balanced by the ***internal resisting force***, giving the equilibrium equation;



$$P = \int \sigma dA$$

If the stress σ is uniform over the area ***A***, σ = constant, then

$$P = \sigma \int dA = \sigma A$$

The average stress

$$\sigma = \frac{P}{A} \qquad \text{...Eq.3}$$

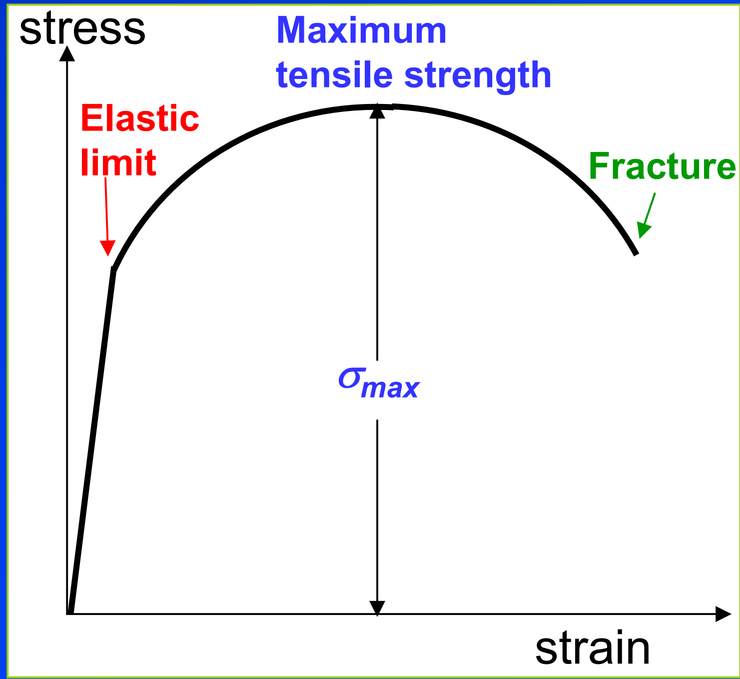
Average stress and strain

• In general, the stress is not uniform over the area ***A***
→ ***average stress***.
• ***On a microscopic scale***, metals consist of more than one phase and therefore give rise to nonuniformity of stress.

• Below the ***elastic limit***, ***Hook's law*** can be applied, so that the average stress is proportional to the average strain,

$$\frac{\sigma}{\varepsilon} = E \qquad \text{...Eq.4}$$

The constant ***E*** is the ***modulus of elasticity*** or ***Young's modulus***.



Load and extention curve of under uniaxial tensile loading

Tensile deformation of ductile metal

• In ***tension test,*** specimen is subjected to axial tensile load until fracture. Load and extension are measured and expressed as *stress* and *strain*, see *Fig*.

• The *initial linear portion* of the curve (*OA*) is the *elastic region*, following *Hook's law*.

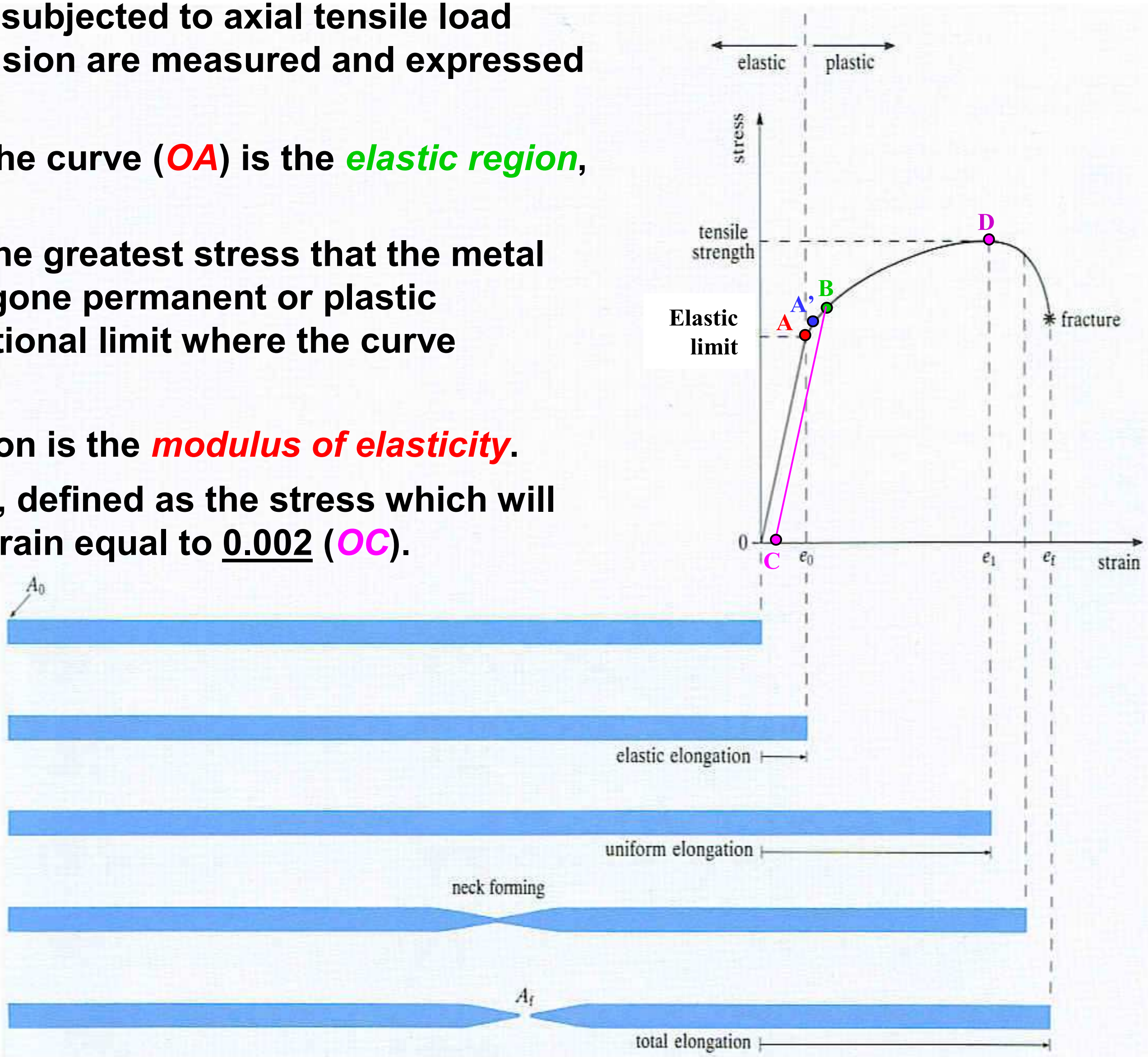
• Point *A* is the *elastic limit* (the greatest stress that the metal can withstand without undergone permanent or plastic deformation. *A'* is the proportional limit where the curve deviates from linearity.

• The slope of the linear portion is the *modulus of elasticity*.

• Point *B* is the ***yield strength***, defined as the stress which will produce a small amount of strain equal to 0.002 (*OC*).

• As the plastic deformation increases, the metal becomes stronger (strain hardening) until reaching the maximum load, giving ***ultimate tensile strength*** *D.*

• Beyond *D*, metal necks (reduce in x-section). Load needed to continue deformation drops off till failure.

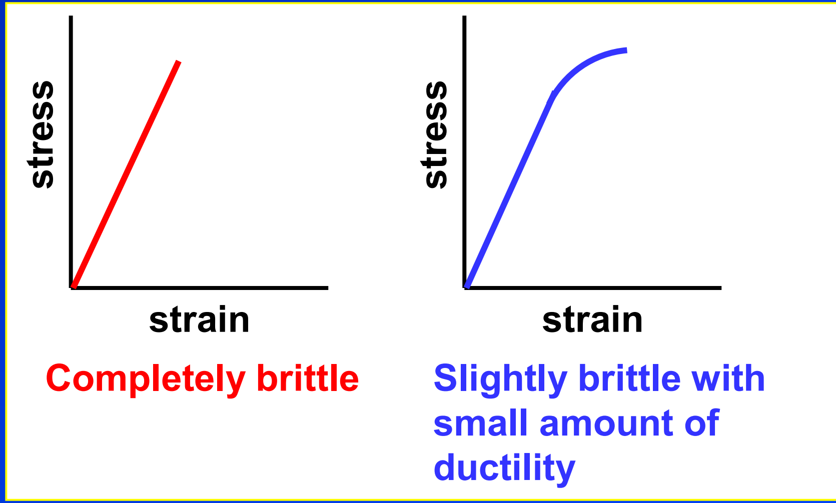


Ductile and brittle behaviour

• Completely brittle materials i.e., ceramics would ***fracture almost at the elastic limit***.

• Brittle metals such as cast iron show ***small amounts of plasticity before failure***.

• For engineering materials, ***adequate ductility*** is important because it allows the materials to redistribute localised stresses.

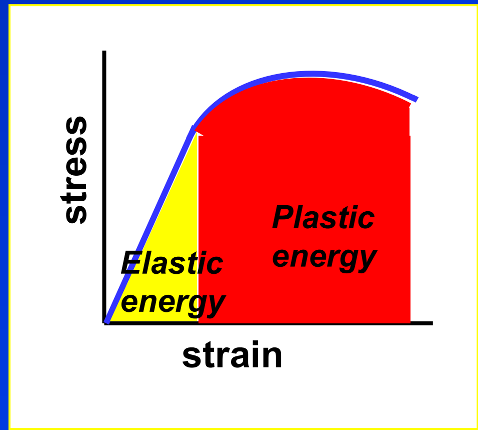


• Fracture behaviour of metal (ductile or brittle) also depends on some conditions, i.e., temperature, tension or compression, state of stresses, strain rate and embrittling agent.

What constitute fracture?

Three general ways that cause failures in structural members and machine elements:

1) Excessive elastic deformation
2) Yielding or excessive plastic deformation
3) Fracture



Types of Load/forces

Nature of Materials

Structural design

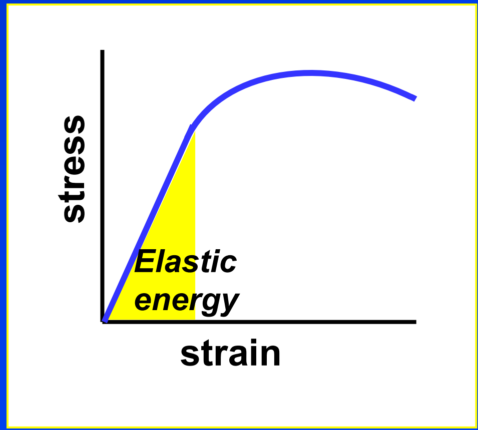
→ ***Good structural components in service without failure***

Excessive elastic deformation

• Failure due to excessive elastic deformation are controlled by the ***modulus of elasticity*** not the strength of the materials.

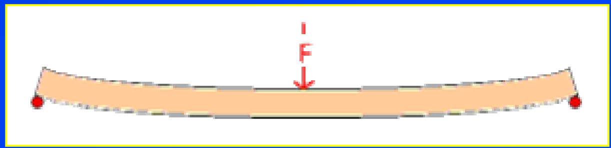
Two general types of excessive elastic deformation:

1) Excessive elastic deformation under condition of stable equilibrium.



EX: too much deflection in a shaft can cause wear in bearing and damage other parts.

2) Excessive elastic deformation under condition of unstable equilibrium.



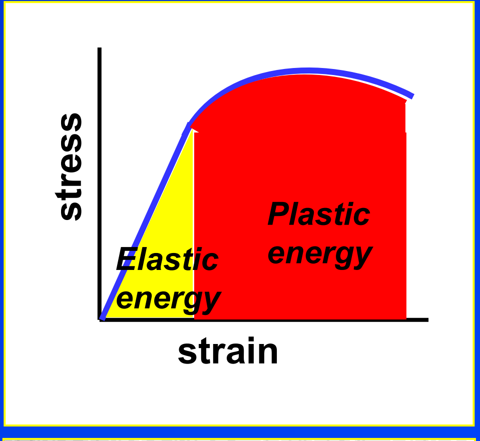
EX: A sudden deflection or ***buckling*** of a slender column.

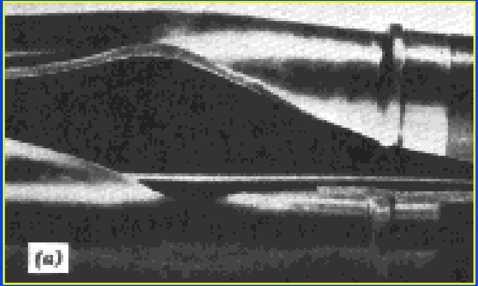
Excessive plastic deformation

- Excessive plastic deformation occurs when the ***elastic limit is exceeded*** → ***yielding***.
- Yielding produce permanent change of shape and can cause fracture.

EX: When component changed in shape, it cannot function properly any longer.

- Failure by ***excessive plastic deformation*** is controlled by the ***yield strength*** of the materials. Even in more complex loading conditions, yield strength is still a significant parameter.





Large deformation that cause failure in a pipe

Fracture

Metals fail by fracture in three general ways:

1) ***Sudden brittle fracture***

Ex : some ductile metal such as ***plain carbon steel*** will undergo ductile to brittle transition with decreasing temperature, increasing rate of loading and triaxial state of stresses.

2) ***Fatigue, or progressive fracture***

Most fracture in machine parts are due to fatigue (subjected to alternating stresses). Failure is due to localised tensile stress at spot or notch or stress concentration.

3) ***Delayed fracture***

Ex: Stress-rupture failure, which occurs when a metal has been statistically loaded at an elevated temperature.

Working stress

To prevent structural members or machine elements from failure, such members should be used under a stress level that is lower than its ***yield stress*** σ_o. This stress level is called the ***working stress*** σ_w.

According to ***American Society of Mechanical Engineering*** (ASME), the ***working stress*** σ_w may be considered as either the yield strength σ_o or the tensile strength σ_u divided by a number called ***safety factor***.

$$\sigma_w = \frac{\sigma_o}{N_o} \quad \text{...Eq.5} \qquad \text{or} \qquad \sigma_w = \frac{\sigma_u}{N_u} \quad \text{...Eq.6}$$

Where
σ_w = working stress
σ_o = yield strength
σ_u = tensile strength
N_o = safety factor based on yield strength
N_u = safety factor based on tensile strength

Safety factor depends on loading/service conditions, consequences, etc.

Concept of stress and the type of stresses

Definition: stress is ***force per unit area***.

There are two kinds of external forces which may act on a body;

1) ***Surface forces :*** forces distributed over the surface of the body, i.e., hydrostatic pressure

2) ***Body forces :*** forces distributed over the volume of the body, i.e., gravitational force, magnetic force, centrifugal force, thermal stress.

Stress at a point

• Consider a **body** having forces $F_1 \ldots F_4$ acting on it, see *Fig (a).* The body is cut by a plane passing through point **O**. If one half is removed and replaced by an equivalent force **F**, acting on the x-sectional area **A** to remain the static equilibrium, see *Fig (b).*

• We can resolve **F** into component normal to the plane F_n and tangential to the plane F_s.

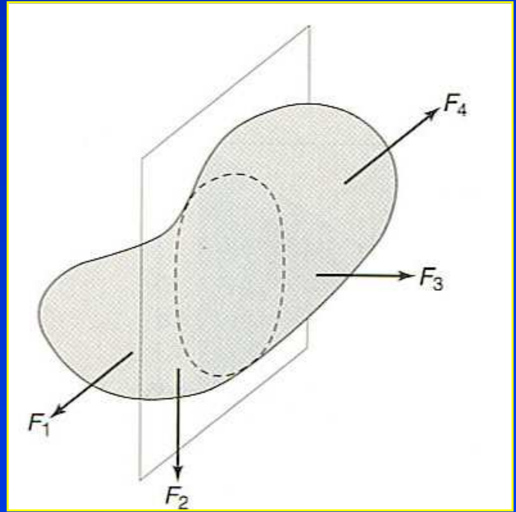
• The concept of stress at a point is shrinking the area **A** into infinitesimal dimensions.

$$\sigma = \lim_{\Delta A \to 0} \frac{|F_n|}{A} \quad \ldots Eq.7$$

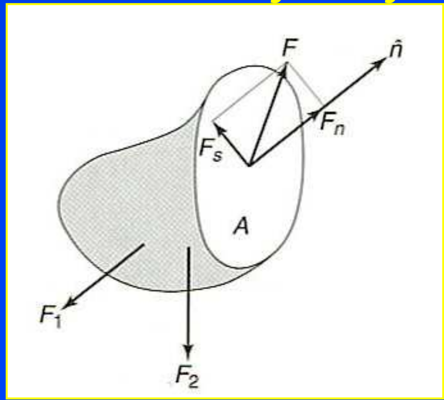
and

$$\tau = \lim_{\Delta A \to 0} \frac{|F_s|}{A} \quad \ldots Eq.8$$

Note: *σ and τ depend on the orientation of the plane passing through **P** and will vary from point to point.*



(a) Equilibrium of an arbitrary body.



(b) Force acting on parts.

The total stress can be resolved into two components;

1) Normal stress σ perpendicular to ***A***.

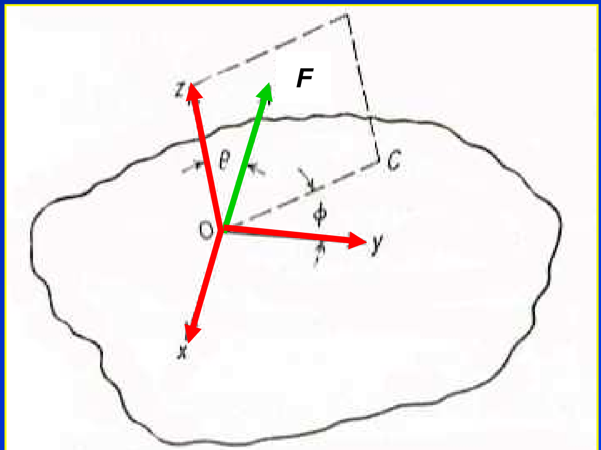
2) Shear stress τ lying in the plane of the area.

- The force ***F*** makes an angle θ with the normal ***z*** to the plane ***x-y*** of the area ***A***.

- The plane containing the normal ***z*** and ***F*** intersects the plane ***A*** along a dash line that makes an angle ϕ with the ***y*** axis.

The ***normal stress*** is given by

$$\sigma = \frac{F}{A}\cos\theta \quad \text{...Eq.9}$$



Resolutions of total stress into its components.

The ***shear stress*** in the plane acting along ***OC*** has the magnitude.

$$\tau = \frac{F}{A}\sin\theta \quad \text{...Eq.10}$$

x direction

$$\tau = \frac{F}{A}\sin\theta\sin\phi \quad \text{...Eq.11}$$

y direction

$$\tau = \frac{F}{A}\sin\theta\cos\phi \quad \text{...Eq.12}$$

Concept of strain and types of strain

The ***average linear strain*** was defined as the ratio of the change in length to the original length.

$$e = \frac{\delta}{L_o} = \frac{\Delta L}{L_o} = \frac{L - L_o}{L_o}$$

…*Eq.2*

Where **e** = average linear strain
δ = deformation.

Strain at a point is the ratio of the deformation to the gauge length as the gauge length → 0.

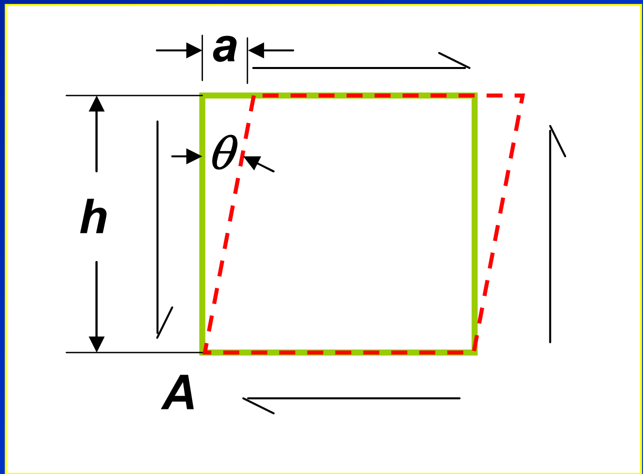
True strain or natural strain is the strain as the change in linear dimension divided by the instantaneous value of the dimension.

$$\varepsilon = \int_{L_o}^{L_f} \frac{dL}{L} = \ln \frac{L_f}{L_o}$$

…*Eq.13*

Shear strain

- ***Shear strain*** is the angular change in a right angle.
- The angle at ***A***, which is originally 90°, is decreased by a small amount θ when the shear stress is applied.
- The ***shear strain*** γ will be given by



Shear strain

$$\gamma = \frac{a}{h} = \tan\theta = \theta$$

…Eq.14

Units of stress and other quantities

- ***Following SI unit***

Length	metre (m)
Mass	kilogram (kg)
Time	second (s)
Electric current	ampere (A)
Temperature	Kelvin (K)
Amount of substance	mole (mol)
Luminous intensity	candela (cd)

Frequency	(s^{-1}, Hz)
Force	Newton (N)
Stress	($N.m^{-2}$, Pa)
Strain	dimensionless

Example: The shear stress required to nucleate a grain boundary crack in high-temperature deformation has been estimated to be

$$\tau = \left(\frac{3\pi\gamma_b G}{8(1-\nu)L} \right)^{\frac{1}{2}}$$

Where γ_b is the grain boundary surface energy ~ 2 J.m^{-2}, ***G*** is shear modulus, 75 GPa, ***L*** is the grain boundary sliding distance, assume = grain diameter 0.01 mm, and the Poisson's ratio ν = 0.3.

Checking the unit

$$\tau = \left(\frac{\frac{Nm}{m^2} \times \frac{N}{m^2}}{m} \right)^{\frac{1}{2}} = \left(\frac{N^2}{m^4} \right)^{\frac{1}{2}} = \frac{N}{m^2}$$

$$\tau = \left(\frac{3\pi \times 2 \times 75 \times 10^9}{8(1-0.3) \times 10^{-2} \times 10^{-3}} \right)^{\frac{1}{2}} = 15.89 \times 10^7 \ N.m^{-2}$$

$$\tau = 158.9 \ \ MN.m^{-2} = 158.9 \ \ MPa$$

References

- Dieter, G.E., ***Mechanical metallurgy***, 1988, SI metric edition, McGraw-Hill, ISBN 0-07-100406-8.
- Sanford, R.J., ***Principles of fracture mechanics***, 2003, Prentice Hall, ISBN 0-13-192992-1.
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Stress and strain relationships for elastic behaviour

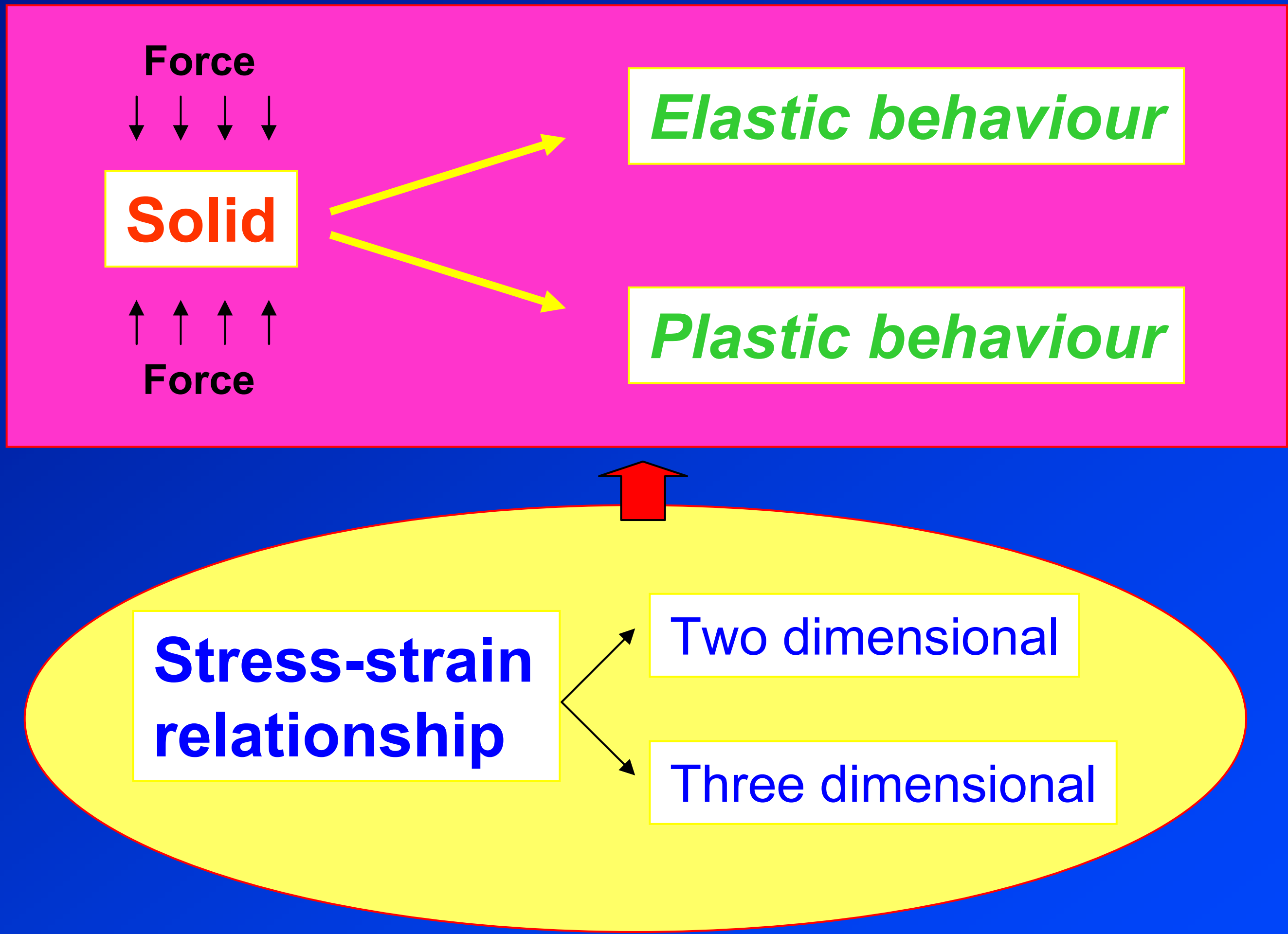
Subjects of interest

- *Introduction/Objectives*
- *Description of stress at a point*
- *State of stresses in two dimensions (Plane stress)*
- *Mohr's circle of stress – two dimensions*
- *State of stress in three dimensions*
- *Strain at a point*
- *Hydrostatic and deviator components of stress*
- *Elastic stress-strain relations*
- *Strain energy*
- *Stress concentration*
- *Finite element method*

Objectives

- This chapter provides mathematical relationships to understand relationships between stress and strain in a solid which obeys Hook's law.
- Stress and strain at a point and in three dimensions will be understood.

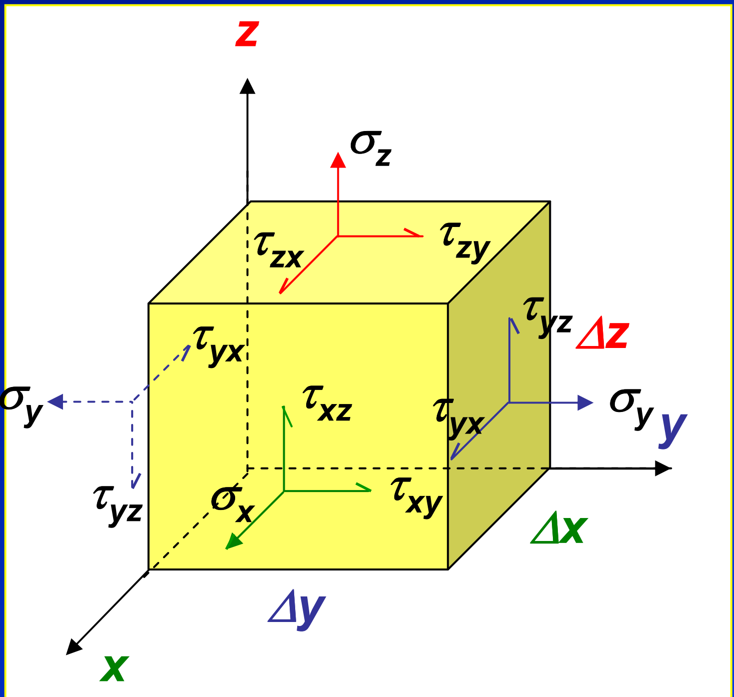
Introduction



Note: if solid is metal, we need to include metallurgical factors.

Description of stress at a point

- ***Stress*** at a point is resolved into normal and shear components.
- ***Shear components*** are at arbitrary angles to the coordinate axes.



Stress acting on an element cube.

- The ***normal stress*** σ_x acting on the plane perpendicular to the ***x*** direction. (this also applies to σ_y and σ_z.)

- The ***shearing stress*** has two components and need two subscripts;

  - The ***first subscript*** is the plane in which the stress acts.

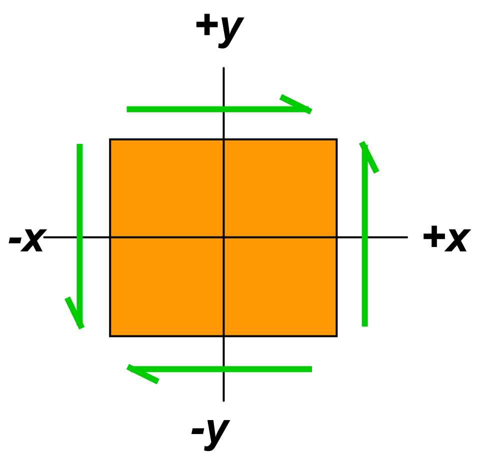
  - The ***second subscript*** is the direction in which the stress acts.

Ex: τ_{yz} is the ***shear stress*** in the plane perpendicular to the ***y*** axis in the ***z*** direction.

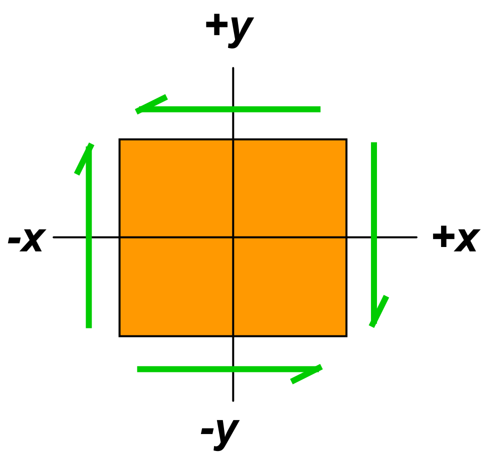
Sign convention for shear stress

• A ***shear stress*** is ***positive*** if it points in the ***positive*** direction on the ***positive*** face of a unit cube. (and negative direction on the negative face).

• A ***shear stress*** is ***negative*** if it points in the ***negative*** direction of a ***positive*** face of a unit cube. (and positive direction on the negative face).



(a) Positive shear stresses.



(b) Negative shear stresses.

Stress components

- In order to establish state of stress at a point, nine quantities must be defined; σ_x , σ_y , σ_z , τ_{xy} , τ_{xz} , τ_{yx} , τ_{yx} , τ_{zx} and τ_{zy} .
- If stress are slowly varying across the infinitesimal cube, moment equilibrium about the centroid of the cube requires that

$$\tau_{xy} = \tau_{yx} \;,\; \tau_{xz} = \tau_{zx} \;,\; \tau_{yz} = \tau_{zy} \qquad \text{...Eq. 1}$$

- Nine stress components can now reduce to six independent quantities σ_x , σ_y , σ_z , τ_{xy} , τ_{xz} , and τ_{zy} , which can be written as

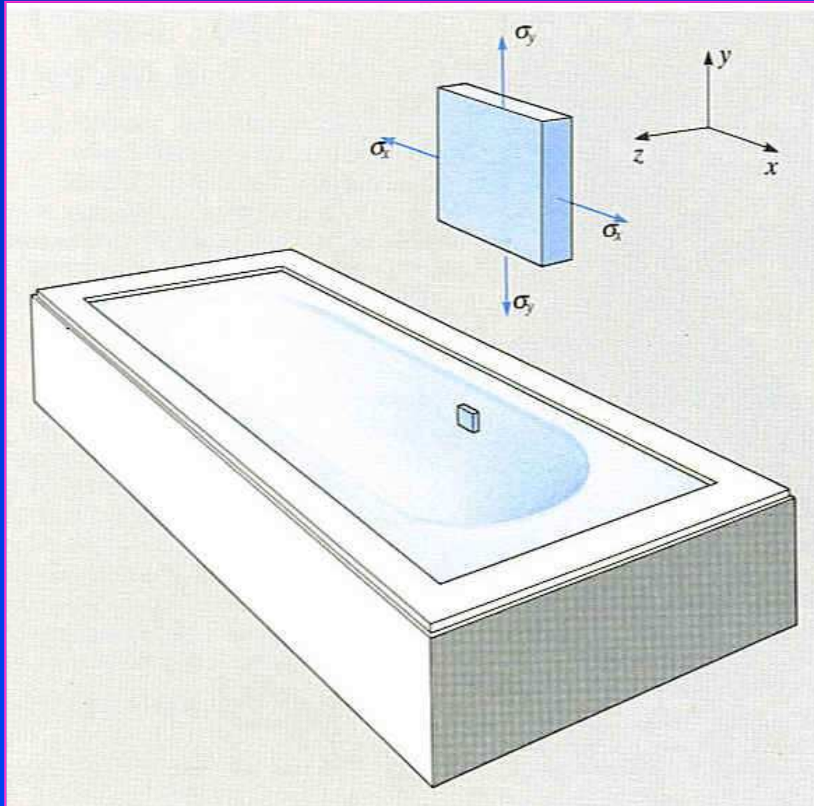
$$\sigma_{ij} = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{pmatrix} \qquad \text{...Eq. 2}$$

State of stress in two dimensions (Plane stress)

Definition: Plane stress is a stress condition in which the stresses are zero in one of the primary directions.

• In a ***thin plate*** where load will be on the plane of the plate and there will be no stress acting perpendicular to the surface of the plate.

• The stress system consists of two normal stresses σ_x and σ_y and a shear stress τ_{xy}.



(a)

Stress on oblique plane

- Consider an ***oblique plane*** normal to the plane of the paper crossing ***x*** and ***y*** axis.
- The direction ***x'*** is normal to the ***oblique plane*** and ***y'*** direction is lying in the ***oblique plane***.
- The normal stress σ and shear stress τ are acting on this plane and ***A*** is the area on the ***oblique plane***.
- S_x and S_y denote the ***x*** and ***y*** components of the total stress acting on the inclined face. The direction cosines between ***x'*** and the ***x*** and ***y*** axes are ***l*** and ***m***, hence $l = \cos\theta$ and $m = \sin\theta$.

Summation of the forces

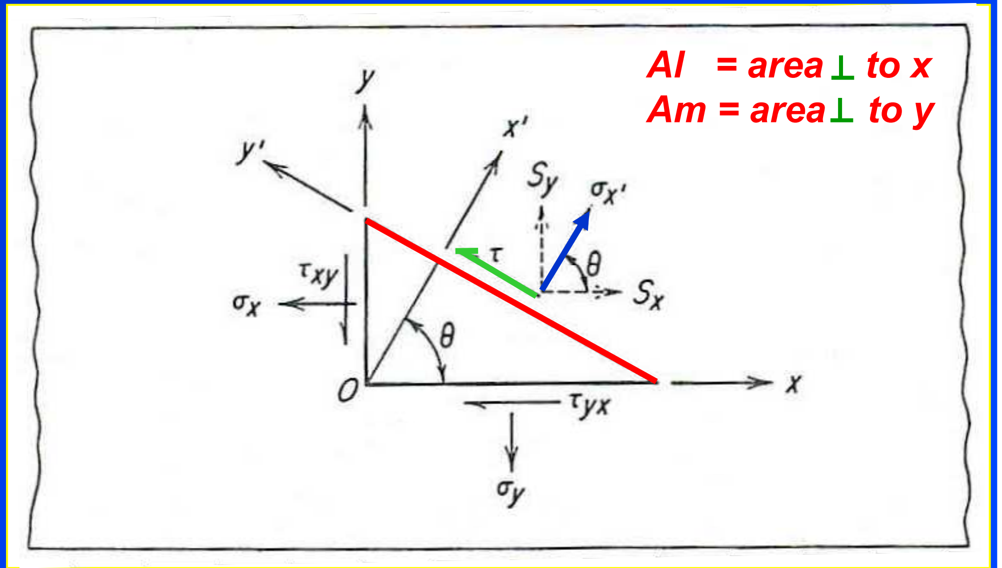
x direction $$S_x A = \sigma_x Al + \tau_{xy} Am$$

y direction $$S_y A = \sigma_y Am + \tau_{xy} Al$$

or

$$S_x = \sigma_x \cos\theta + \tau_{xy} \sin\theta$$

$$S_y = \sigma_y \sin\theta + \tau_{xy} \cos\theta$$



Stress on oblique plane (two dimensions)

Stress on oblique plane

- The components of S_x and S_y in the direction of the normal stress σ are

$$S_{xN} = S_x \cos\theta \quad and \quad S_{yn} = S_y \sin\theta$$

- The ***normal stress*** acting on the oblique plane is given by

$$\sigma_{x'} = S_x \cos\theta + S_y \sin\theta$$

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta\cos\theta$$

…Eq. 3

- The ***shearing stress*** on the oblique plane is given by

$$\tau_{x'y'} = S_y \cos\theta - S_x \sin\theta$$

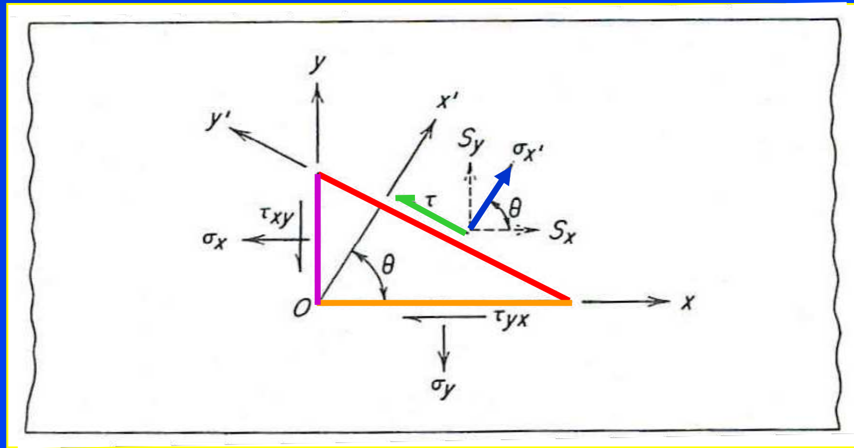
$$\tau_{x'y'} = \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right) + \left(\sigma_y - \sigma_x\right)\sin\theta\cos\theta$$

…Eq. 4

- The stress $\sigma_{y'}$ can be found by substituting $\theta + \pi/2$ for θ, we then have

…Eq. 5

$$\sigma_{y'} = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy} \sin\theta\cos\theta$$



If stresses in an *xy* coordinate system and the angle θ are known, we will get the stresses in any *x'y'* coordinate.

Stress on oblique plane

- ***Equations 3-5*** can be expressed in terms of ***double angle 2θ***.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \text{...Eq. 6}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \qquad \text{...Eq. 7}$$

$$\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \text{...Eq. 8}$$

Note: $\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y$ → Thus ***the sum of the normal stresses*** on two perpendicular plan is an ***invariant quantity*** and independent of orientation or angle θ.

Eq. 3-8 describe the ***normal stress*** and ***shear stress*** on any plane through a point in a body subjected to a ***plane-stress situation***.

Variation of normal stress and shear stress with θ for the biaxial-plane stress situation.

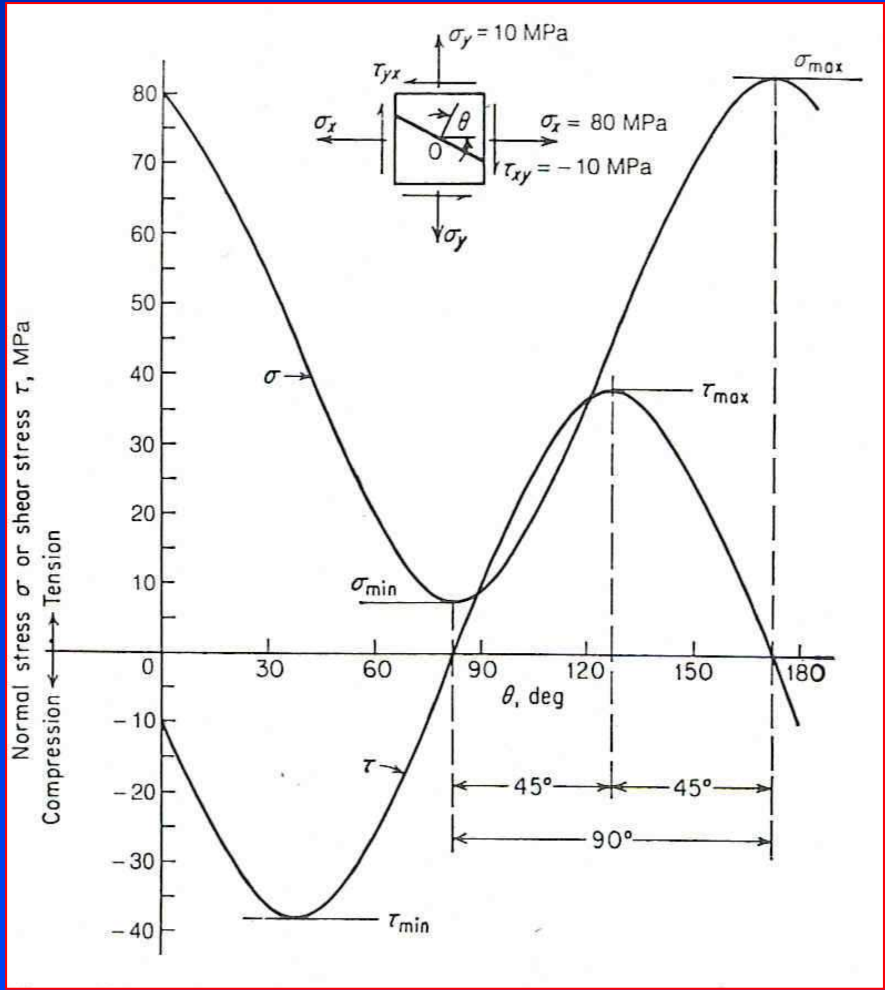
1) When τ is zero →give ***max*** and ***min*** values of σ.

2) The ***max*** and ***min*** values of σ and τ occur at angles which are 90^o apart.

3) The ***max*** τ occurs at an angle halfway between the ***max*** and ***min*** σ.

4) The variation of σ and τ occurs in the form of a ***sine wave***, with a period of θ = 180^o.

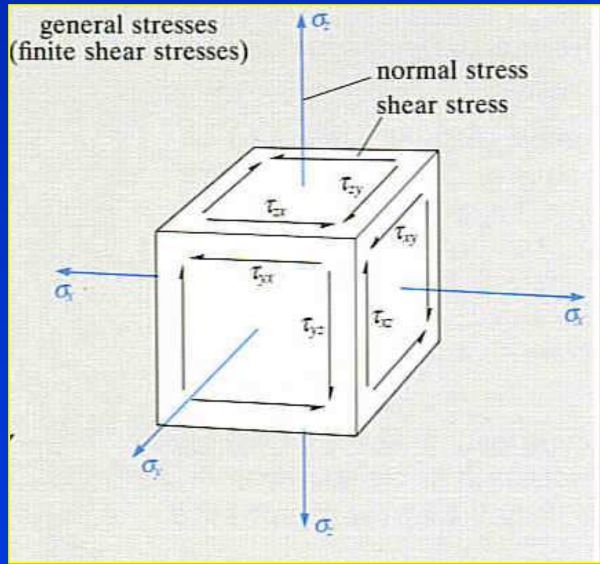
The relationships are valid for any state of stress.



Principal stresses

• *When there is no shear stresses acting on the planes → giving the maximum normal stress acting on the planes.*

• *These planes are called the* ***principal planes****, and stresses normal to these planes are the* ***principal stresses*** σ_1*,* σ_2 *and* σ_3 *which in general do not coincide with the* ***cartesian-coordinate axes x, y, z****. Directions of principal stresses are 1, 2 and 3.*

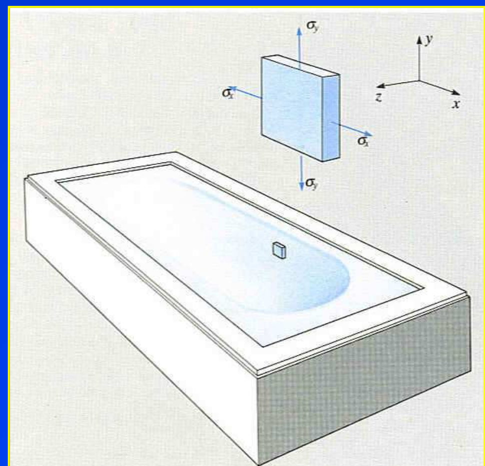


Biaxial-plane stress condition

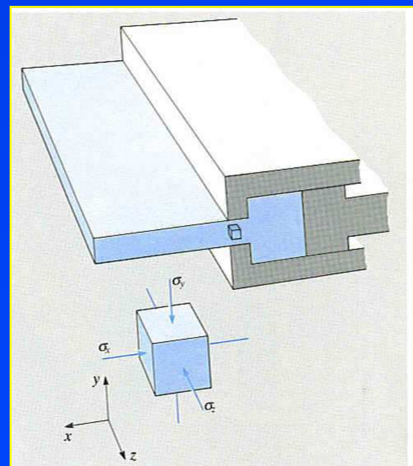
• Two principal stresses, σ_1 and σ_2.

Triaxial-plane strain condition

• Three principal stresses, σ_1 , σ_2 and σ_3 , where $\sigma_1 > \sigma_2 > \sigma_3$.



(a) ***Biaxial stresses***



(b) ***Triaxial stresses***

Maximum and minimum principal stresses in biaxial state of stress

- On a principal plane there is no shear stress; thereby, $\tau_{x'y'} = 0$

From *Eq. 5*

$$\tau_{xy}\left(\cos^2\theta - \sin^2\theta\right) + \left(\sigma_y - \sigma_x\right)\sin\theta\cos\theta = 0$$

$$\frac{\tau_{xy}}{\sigma_x - \sigma_y} = \frac{\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{\frac{1}{2}(\sin 2\theta)}{\cos 2\theta} = \frac{1}{2}\tan 2\theta$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

…Eq. 9

- Since ***$\tan 2\theta = \tan(\pi + 2\theta)$***, Eq.9 has two roots, θ_1 and $\theta_2 = \theta_1 + n\pi/2$. These roots define ***two mutually perpendicular planes*** which are ***free from shear***.

- The ***maximum*** and ***minimum principal stresses*** for biaxial state of stress are given by

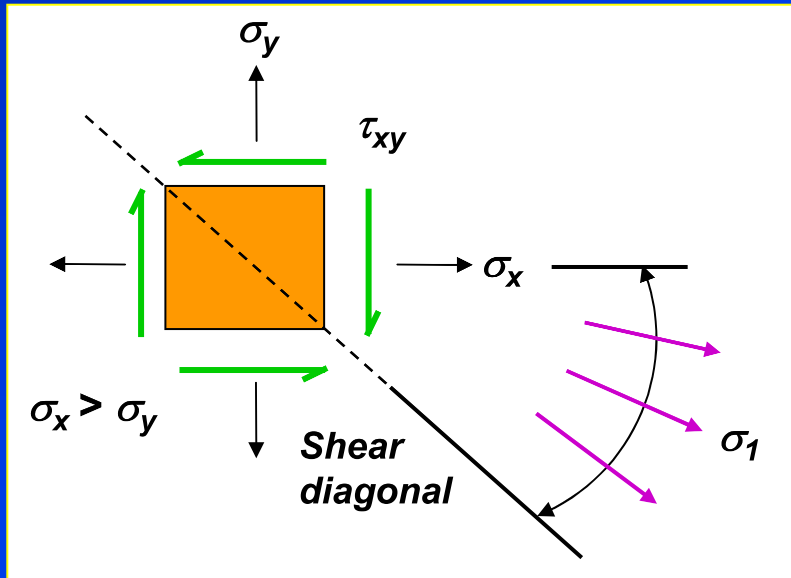
$$\left.\begin{aligned}\sigma_{max} &= \sigma_1 \\ \sigma_{min} &= \sigma_2\end{aligned}\right\} = \frac{\sigma_x + \sigma_y}{2} \pm \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

…Eq. 10

Directions of the maximum principal stress

- The largest principal stress σ_1 will lie between the largest ***normal stress*** and the ***shear diagonal***.

- (If there is no shear, $\sigma_x = \sigma_1$, and if there is only shear the principal stress σ_1 would exist along the shear diagonal. If both normal and shear stresses act on the element, then σ_1 lies between the influence of these two effects.



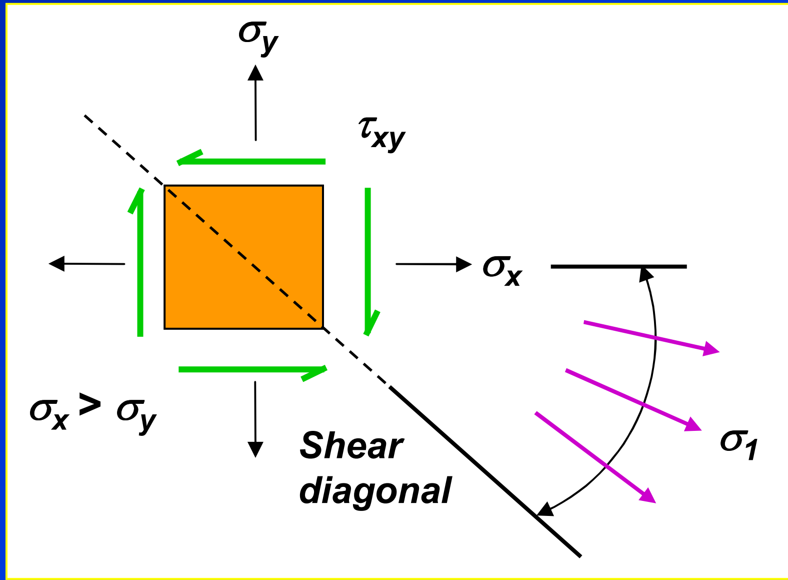
Method of establishing direction of σ_1

The maximum shear stress in biaxial stress

- The ***maximum shear stress*** can be found by differentiating *Eq. 8* and set to zero.

$$\frac{d\tau_{x'y'}}{d\theta} = (\sigma_y - \sigma_x)\cos 2\theta - 2\tau_{xy}\sin 2\theta = 0$$

$$\tan 2\theta_s = \frac{\sigma_y - \sigma_x}{2\tau_{xy}} = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} \quad \text{...Eq. 11}$$



Method of establishing direction of σ_1.

***Note:** $\tan 2\theta_s$ is the negative reciprocal of $\tan 2\theta_n$.*

This means that $2\theta_s$ and $2\theta_n$ are orthogonal and that θ_s and θ_n are separated in space by 45°.

- The magnitude of the maximum shear stress is given by

$$\tau_{max} = \pm\left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

Example: *The state of stress is given by σ_x = 25p and σ_y = 5p plus shearing stresses τ_{xy}. On a plane at 45^o counterclockwise to the plane on which σ_x acts on the state of stress is 50 MPa tension and 5 MPa shear. Determine the values of σ_x, σ_y, τ_{xy}.*

From *Eq.6*

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$50\times 10^6 = \frac{25p+5p}{2} + \frac{25p-5p}{2}\cos 90^o + \tau_{xy}\sin 90^o$$

$$50\times 10^6 = 15p + \tau_{xy}$$

From *Eq.8*

$$\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$5\times 10^6 = \left(\frac{5p-25p}{2}\right)\sin 90^o + \tau_{xy}\cos 90^o$$

$$p = -5\times 10^6 \ \ Pa$$

$$\sigma_x = 25(-5\times 10^6) = -12.5 \ \ MPa$$

$$\sigma_y = 5(p) = 2.5 \ \ MPa$$

$$\tau_{xy} = 50\times 10^6 - 15(-5\times 10^5)$$

$$\tau_{xy} = 57.5 \ \ MPa$$

Since $\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'}$

$$\sigma_{y'} = \sigma_x + \sigma_y - \sigma_{x'}$$

$$\sigma_{y'} = -12.5 - 2.5 - 50 = -65 \ \ MPa$$

Mohr's circle of stress - two dimensions

***O. Mohr** used a graphical method to represent the state of stress at a point on an oblique plane through the point.*

From *Eq. 6* and *Eq. 8*

$$\sigma_{x'} - \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad \text{...Eq. 6}$$

$$\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \quad \text{...Eq. 8}$$

By squaring each of these equations and adding, we have

$$\left(\sigma_{x'} - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \quad \text{...Eq. 12}$$

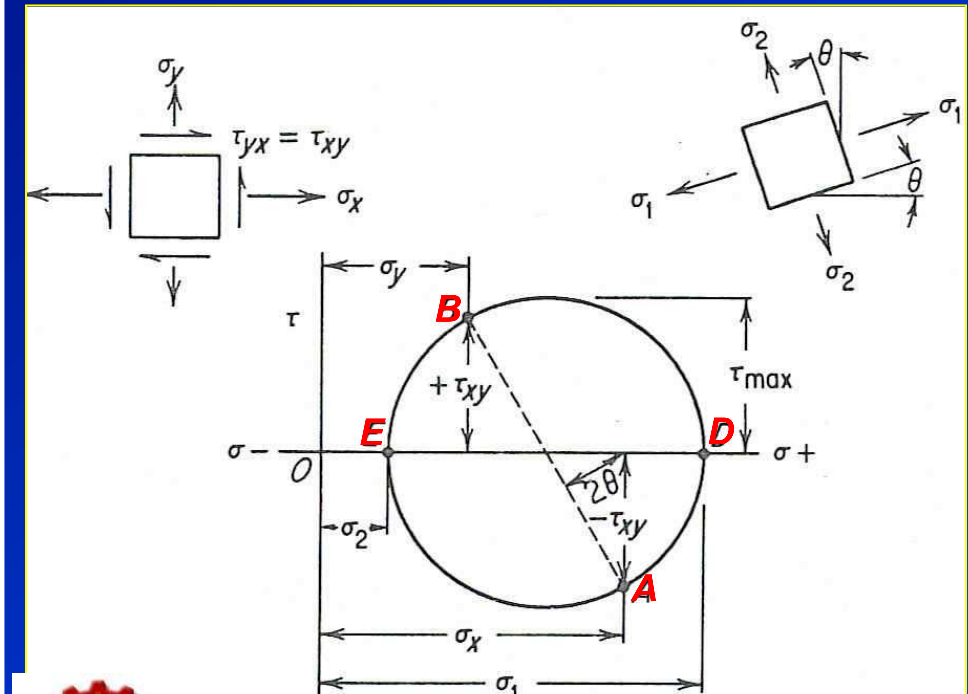
Eq. 12 is in the equation of a circle of the form ***$(x-h)^2 + y^2 = r^2$***. Therefore ***Mohr's circle*** is a circle in $\sigma_{x'}$ and $\tau_{x'y'}$ coordinates which ***$r = \tau_{max}$*** and the centre is displaced ***$(\sigma_x+\sigma_y)/2$*** to the right of the origin.

Mohr's circle

Conventions

• A shear stress causing a clockwise rotation about any point in the physical element is plotted above the horizontal axis of the ***Mohr's circle***.

• A point on Mohr's circle gives the magnitude and direction of the ***normal and shear stresses*** on any plane in the physical element.



Mohr's circle for two-dimensional state of stress

• ***Normal stresses*** are plotted along the ***x*** axis, ***shear stresses*** along the ***y*** axis.

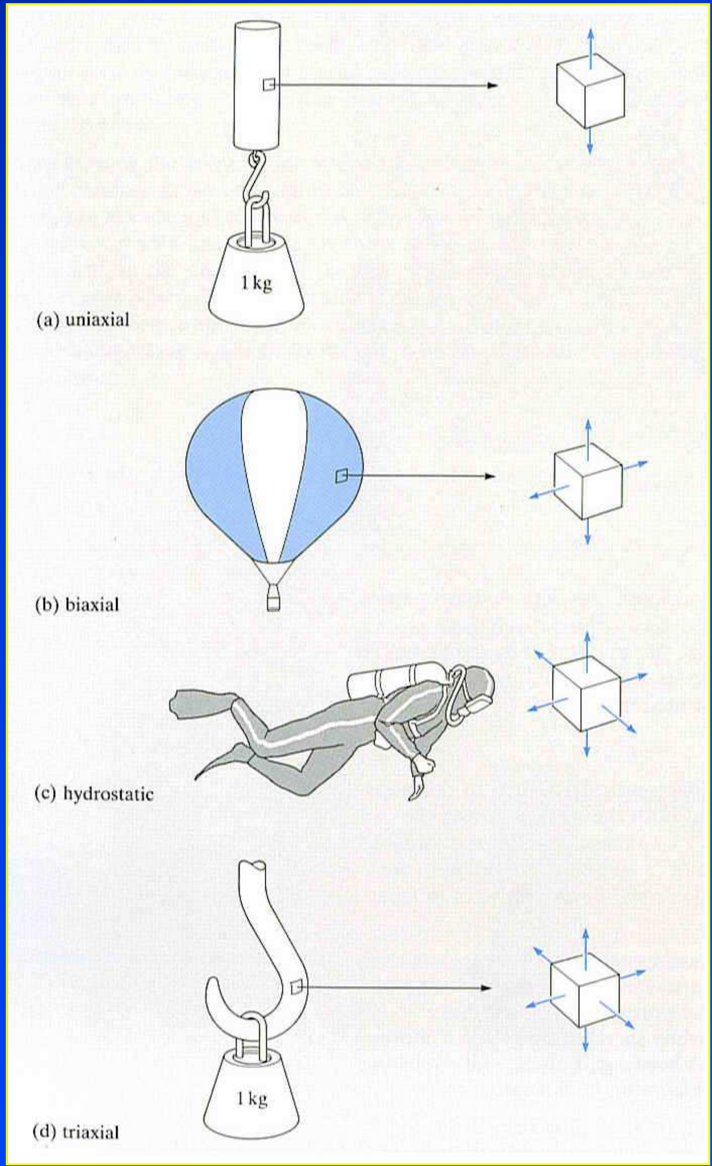
• The stresses on the planes normal to the ***x*** and ***y*** axes are plotted as points ***A*** and ***B***.

• The ***shear stress*** is zero at points ***D*** and ***E***, representing the values of the principal stresses σ_1 and σ_2 respectively.

• The Angle between σ_x and σ_1 on ***Mohr's circle*** is 2θ.

State of stress in three dimensions

- In general three dimensional state of stress consists of ***three unequal principal stresses*** acting at a point, which is called a ***triaxial state of stress***.
- If two of the three principal stresses are equal → ***cylindrical***.
- If $\sigma_1 = \sigma_2 = \sigma_3$ → ***hydrostatic or spherical***.



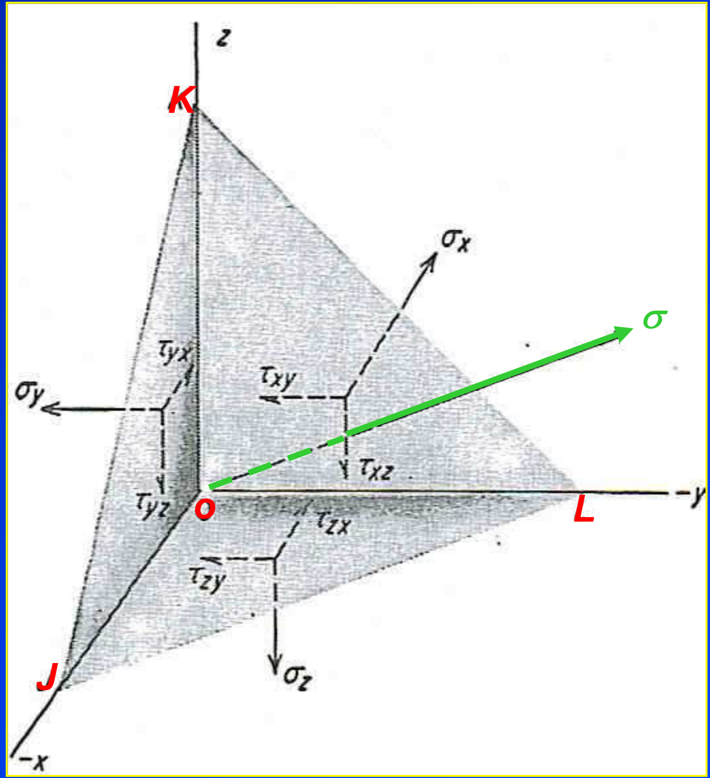
Stress in three dimensions

- Considering an elemental free body with diagonal plane ***JKL*** of area ***A***, which is assumed to be a ***principal plane*** cutting through the unit cube.

- The ***principal stress*** σ is acting normal to the plane ***JKL***. The direction cosines of σ and ***x***, ***y*** and ***z*** axes is ***l***, ***m*** and ***n*** respectively.

In equilibrium, the forces acting on each of its face must balance. S_x, S_y and S_z are the components of σ along the axes.

$S_x = \sigma l$	$S_y = \sigma m$	$S_z = \sigma n$
Area KOL =Al	**Area JOK = Am**	**Area JOL = An**



Stresses acting on elemental free body

Taking summation of the forces in the x direction results in

$$\sigma Al - \sigma_x Al - \tau_{yx} Am - \tau_{zx} An = 0$$

Which reduces to

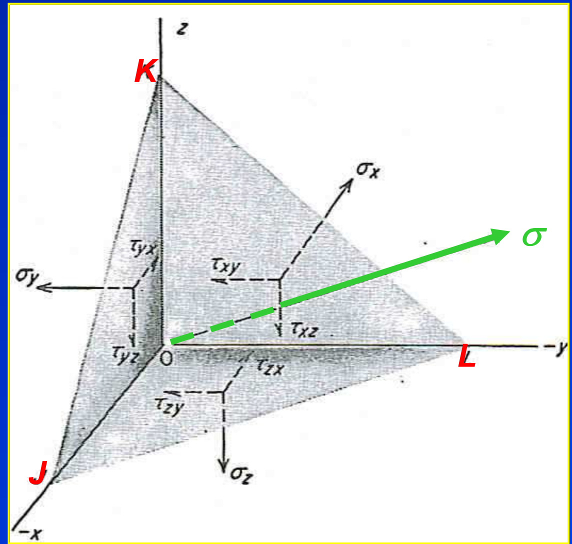
$$(\sigma - \sigma_x)l - \tau_{yx} m - \tau_{zx} n = 0 \quad \text{...Eq. 13}$$

$$(\sigma - \sigma_x)l - \tau_{yx}m - \tau_{zx}n = 0 \qquad \text{…Eq. 13(a)}$$

Summation of the forces in the other two directions results in

$$-\tau_{xy}l + (\sigma - \sigma_y)m - \tau_{zy}n = 0 \qquad \text{…Eq. 13(b)}$$

$$-\tau_{xz}l - \tau_{yz}m + (\sigma - \sigma_z)n = 0 \qquad \text{…Eq. 13(c)}$$



Stresses acting on elemental free body

By setting the ***determinant of the coefficients*** of ***l***, ***m*** and ***n*** = 0

$$\begin{vmatrix} \sigma - \sigma_x & -\tau_{yx} & -\tau_{zx} \\ -\tau_{xy} & \sigma - \sigma_y & -\tau_{zy} \\ -\tau_{xz} & -\tau_{yx} & \sigma - \sigma_z \end{vmatrix} = 0$$

Will give the solution of the determinant which results in a cubic equation in σ

$$\sigma^3 - (\sigma_x + \sigma_y + \sigma_z)\sigma^2 + (\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_x\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{xz}^2)\sigma$$
$$-(\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{xz} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\tau_{xy}^2) = 0$$

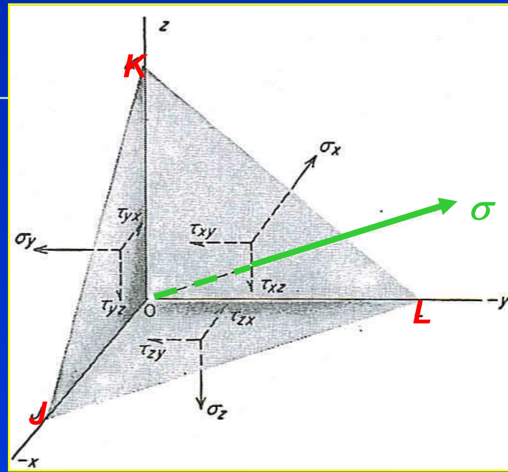
And invariant coefficients

$$I_1 = \sigma_x + \sigma_y + \sigma_z$$
$$I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_x\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{xz}^2$$
$$I_3 = \sigma_x\sigma_y\sigma_y + 2\tau_{xy}\tau_{yz}\tau_{xz} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\tau_{xy}^2$$

Note:

$$\sigma_x + \sigma_y + \sigma_z = \sigma_{x'} + \sigma_{y'} + \sigma_{z'} = \sigma_1 + \sigma_2 + \sigma_3$$

Determination of the normal stress on any oblique plane



- On any oblique plane whose normal has the direction cosines ***l, m, n*** with the ***x, y, z*** axes, The total stress on the plane ***S*** will not now be coaxial with the normal stress, and that $S^2 = \sigma^2 + \tau_2$.

- The ***total stress*** can be resolved into components S_x, S_y, S_z, so that

$$S^2 = S_x^2 + S_y^2 + S_z^2 \quad \text{…Eq. 15}$$

- Taking the summation of the forces in the ***x, y, z*** directions, the expressions for the orthogonal components of the ***total stress*** are given by;

$$S_x = \sigma_x l + \tau_{yx} m + \tau_{zx} n$$
$$S_y = \tau_{xy} l + \sigma_y m + \tau_{zy} n \quad \text{…Eq. 16}$$
$$S_z = \tau_{xz} l + \tau_{yz} m + \sigma_z n$$

- The normal stress σ on the oblique plane;

$$\sigma = S_x l + S_y m + S_z n$$

Substituting Eq and simplifying $\tau_{xy} = \tau_{yx}$ Etc.

$$\sigma = \sigma_x l^2 + \sigma_y m^2 + \sigma_z n^2 + 2\tau_{xy} lm + 2\tau_{yz} mn + 2\tau_{zx} nl \quad \text{…Eq. 17}$$

The maximum or principal shear stress

- Since plastic flow involves shearing stresses, it is important to identify the planes on which the maximum or principal shear stresses occur.

- The principal shear planes can be defined in terms of the ***three principal axes 1, 2, 3.***

$$\tau^2 = (\sigma_1 - \sigma_2)^2 l^2 m^2 + (\sigma_1 - \sigma_3)^2 l^2 n^2 + (\sigma_2 - \sigma_3)^2 m^2 n^2$$

…Eq. 14

Where ***l***, ***m*** and ***n*** are the direction cosines between the normal to the oblique plane and the principal axes.

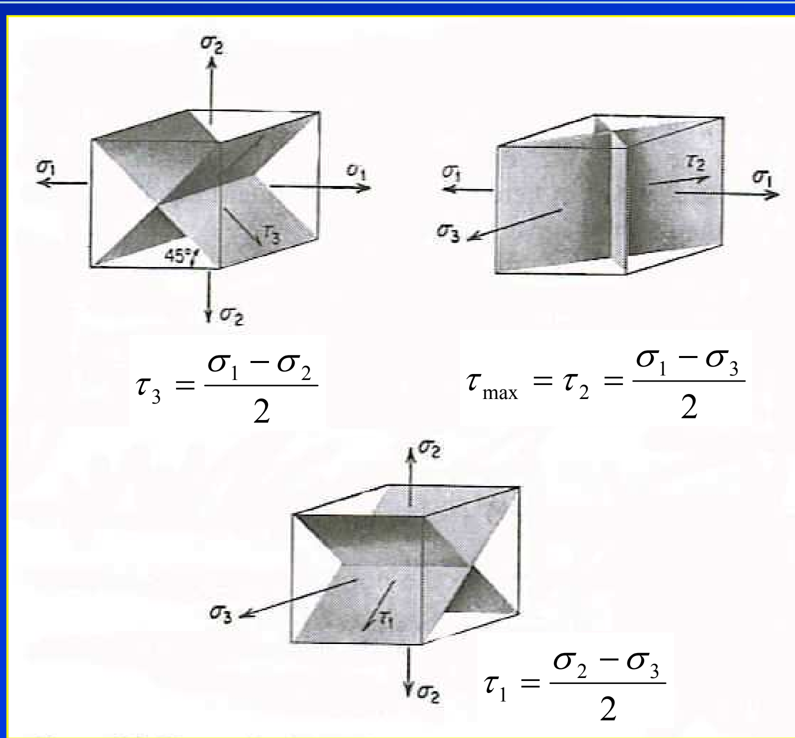
*The **principal shear stresses** occur for the following combination of direction cosines that bisect the angle between two of the three principal axes:*

l	m	n	τ
0	$\pm\sqrt{\frac{1}{2}}$	$\pm\sqrt{\frac{1}{2}}$	$\tau_1 = \frac{\sigma_2 - \sigma_3}{2}$
$\pm\sqrt{\frac{1}{2}}$	0	$\pm\sqrt{\frac{1}{2}}$	$\tau_2 = \frac{\sigma_1 - \sigma_3}{2}$
$\pm\sqrt{\frac{1}{2}}$	$\pm\sqrt{\frac{1}{2}}$	0	$\tau_3 = \frac{\sigma_1 - \sigma_2}{2}$

The maximum or principal shear stress

- ***Principal shear stresses*** for a cube whose faces are the principal planes.
- For each pair of ***principal stresses***, there are two planes of ***principal shear stress***, which bisect the directions of the principal stresses.

According to convention σ_1 is the greatest principal normal stress and σ_3 is the smallest principal stress, τ_2 therefore has the largest value



Planes of principal shear stresses.

- The ***maximum principal shear stress*** τ_{max} is given by

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \quad \text{...Eq. 18}$$

Stress tensor

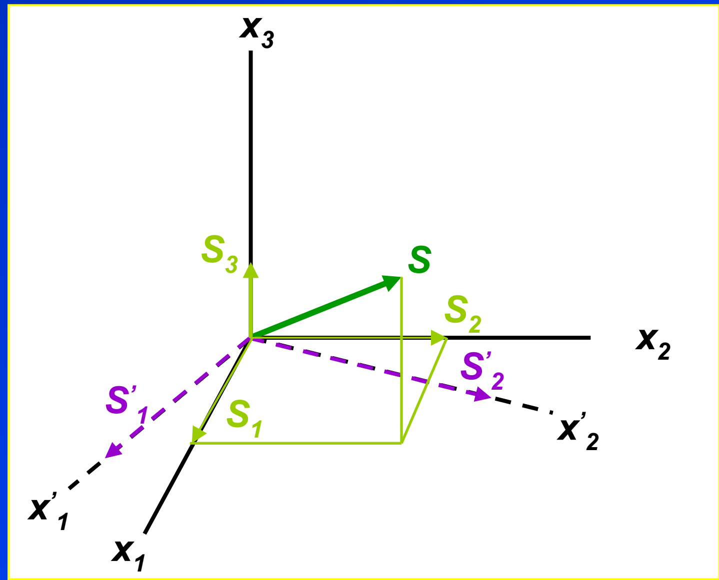
Stress tensor *is used to simplify the equations for the transformation of the stress components from one set of coordinate axes to another coordinate system.*

- First, we consider the transformation of a vector (first-rank tensor) from one coordinate system to another.

$\mathbf{S} = S_1 i_1 + S_2 i_2 + S_3 i_3$ when the unit vectors i_1, i_2, i_3 are in the direction x_1, x_2, x_3.

Where S_1, S_2, S_3 are the components of $\mathbf{S}$ referred to the axes x_1, x_2, x_3.

- The components of $\mathbf{S}$ referred to the x'_1, x'_2, x'_3 is obtained by resolving S_1, S_2, S_3 along the new direction x'_1.



Transformation of axes for a vector.

$$S'_1 = S_1\cos(x_1x'_1) + S_2\cos(x_2x'_1) + S_3\cos(x_3x'_1)$$

or

$$S'_1 = a_{11}S_1 + a_{12}S_2 + a_{13}S_3$$
$$S'_2 = a_{21}S_1 + a_{22}S_2 + a_{23}S_3$$
$$S'_3 = a_{31}S_1 + a_{32}S_2 + a_{33}S_3$$

...Eq. 19

Where a_{11} is the direction cosine between x'_1 and x_1
a_{12} is the direction cosine between x'_1 and x_2, etc..

• We could write the equations (from last slide) as

$$S_1' = \sum_{j=1}^{3} a_{1j} S_j \qquad S_2' = \sum_{j=1}^{3} a_{2j} S_j \qquad S_3' \sum_{j=1}^{3} a_{3j} S_j$$

• These three equations could be combined by writing;

$$S_i' = \sum_{j=1}^{3} a_{ij} S_j \,(i = 1,2,3) = a_{i1} S_1 + a_{i2} S_2 + a_{i3} S_3$$

…Eq. 20

• In greater brevity, the equation is obtained by writing in the Einstein suffix notation.

$$S_i' = a_{ij} S_j$$

…Eq. 21

• The suffix notation ***(j)*** indicates ***summation*** when a suffix occurs twice in the same term. The summation will take place over ***j***.

• The transformation of the stress tensor σ_{ij} from the x_1, x_2, x_3 system of axes to the x'_1, x'_2, x'_3 axes is given by

$$\sigma_{kl} = a_{ki} a_{lj} \sigma_{ij}$$

Rank of tensors

- ***Scalars*** are ***tensors of rank zero***, in which their quantity remains unchanged with the transformation of axes.

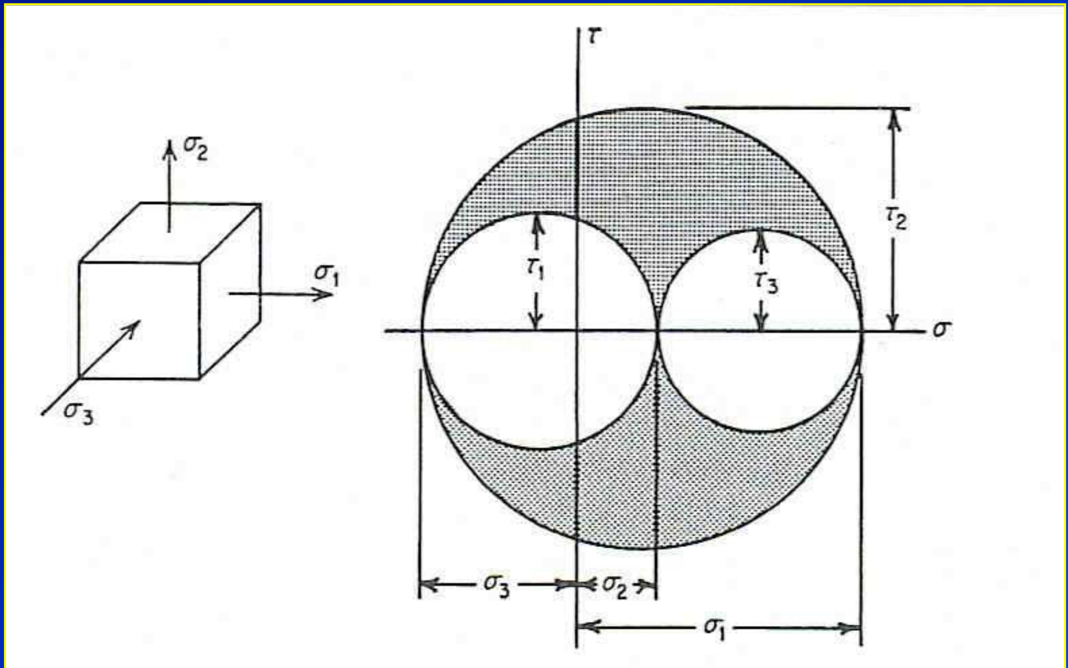
- ***Vectors*** are ***tensors of first rank***.

- ***Physical quantities*** such as stress, strain and many other quantities are ***second-rank tensors*** which ***transform with coordinate axes***.

The number of components required to specify a quantity is 3^n, when n is the rank of the tensor.

Mohr's circle – three dimensions

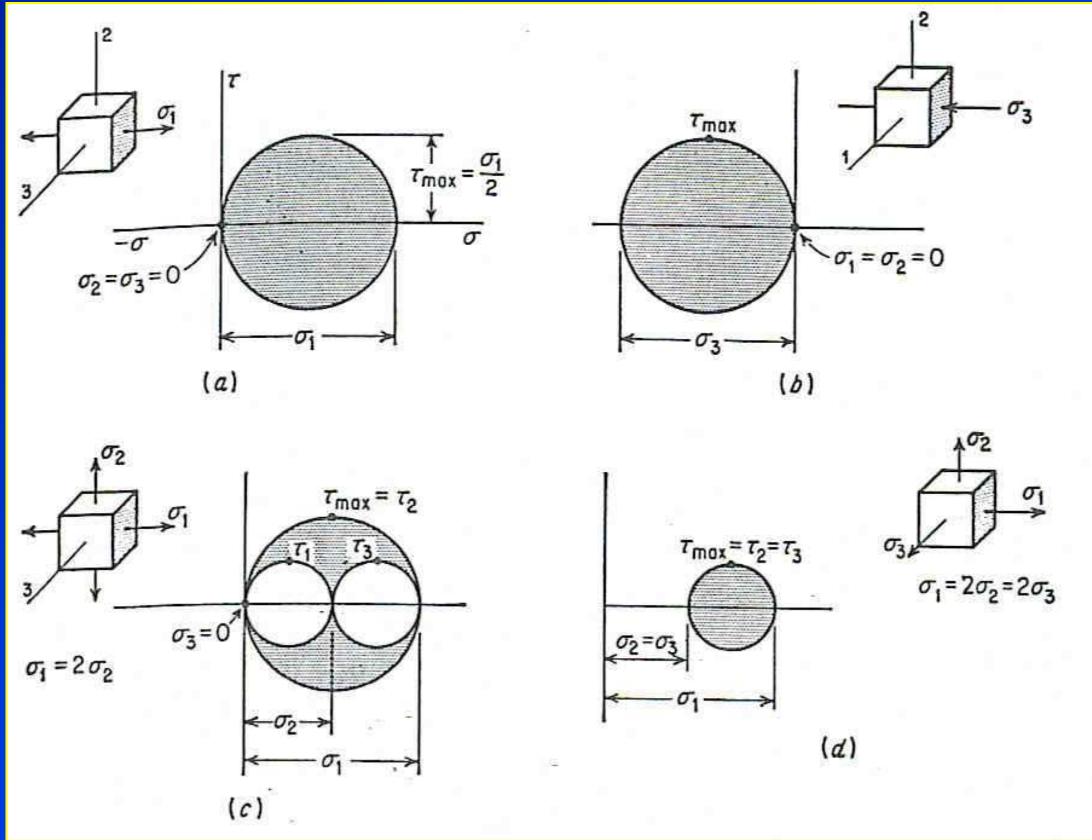
Mohr's circle in three dimensions show how a triaxial state of stress is presented.



- All the possible stress conditions within the body fall within the ***shaded area*** between the circles.
- The ***y*** axis is the ***shear stress*** τ and the ***x*** axis is the ***normal stress*** σ.
- σ_1, σ_2 , σ_3 are shown on the ***x*** axis and τ_1, τ_2 , τ_3 are shown on the ***y*** axis.

Mohr's circle for various states of stress

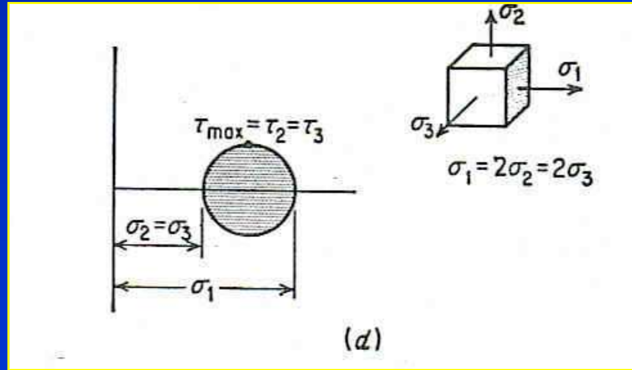
- ***Mohr's circle*** give a geometrical representation of the equations that express the transformation of stress components to different sets of axes.
- The introduction of σ_2 at a right angle to σ_1 results in a reduction in the ***principal shear stress***, but τ_2 remains similar see *fig (c).*
- The ***maximum shear stress*** is reduced appreciably when the ***third principal stress*** is introduced, see *fig (d).*



- Hydrostatic tension $\sigma_1 = \sigma_2 = \sigma_3$, Mohr's circle reduces to a point with no shear stress.

Mohr's circle for various states of stress

Biaxial and triaxial tension stresses effectively reduce the shear stresses and this results in a ***considerable decrease in ductility*** of the material, because plastic deformation is produced by shear stresses.



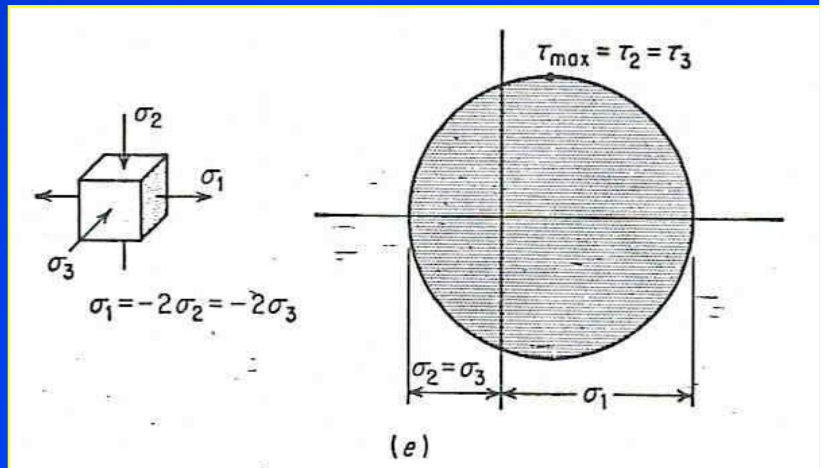
Triaxial state of stress → Reduction in ductility

Brittle fracture

Notch → Triaxial state of stress

Stress raiser → Triaxial state of stress

Uniaxial tension plus biaxial compression stresses produce high value of shear stress and contribute to an excellent opportunity to deform plastically without fracturing.



***Ex:* forming by metal extrusion gives better ductility than simple uniaxial tension.**

Description of strain at a point

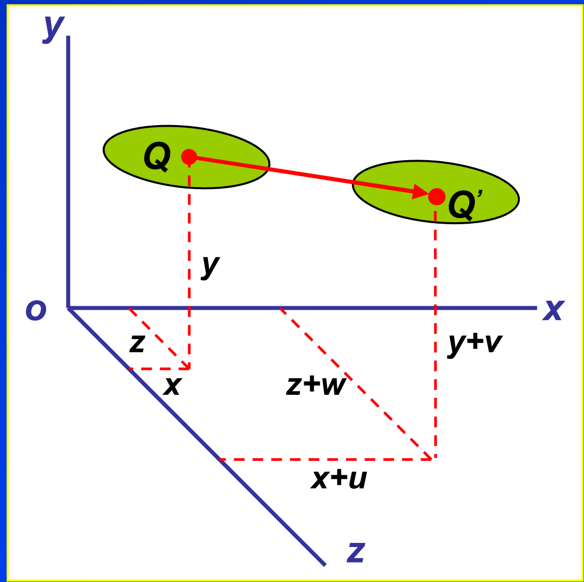
• Deformation of a solid may be made up of dilatation (change in volume), or distortion (change in shape). This results in ***displacement of points*** in a continuum body.

• Consider a solid body in fixed coordinates ***x, y, z*** with a displacement from point ***Q*** to ***Q'***.

• The components of displacement are ***u, v, w***.

• The displacement of ***Q*** is the vector $u_Q = f(u,v,w)$

• Displacement is a function of distance, $u_i = f(x_i)$ and for elastic solid and small displacement, u_i is a linear function of x_i.



Displacement of point Q.

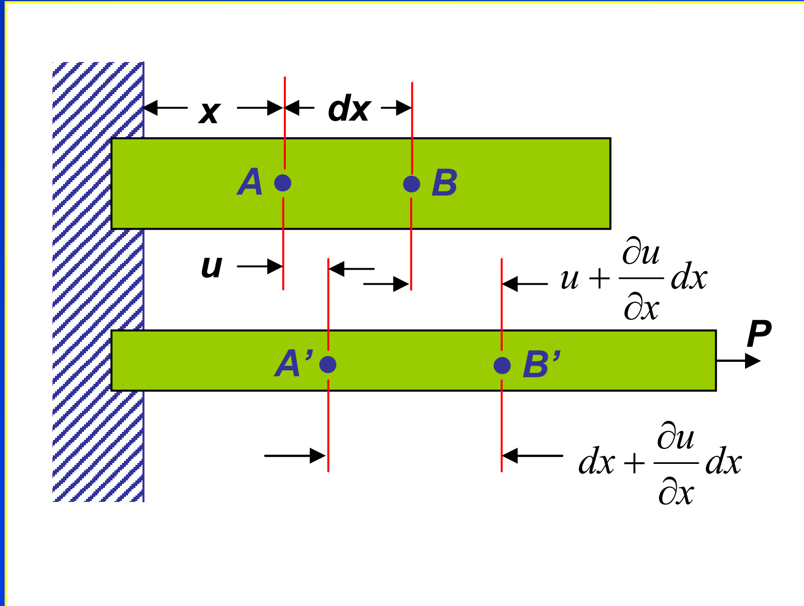
Note: in other materials the displacement may not be linear with distance, which leads to cumbersome mathematical relationships.

One-dimensional strain

- Consider a simple one-dimensional strain, which has been deform from the original distance ***AB*** to ***A'B'.***
- Displacement ***u*** is in one dimension (as a function of ***x***).
- The normal strain is given by

$$e_x = \frac{\Delta L}{L} = \frac{A'B' - AB}{AB} = \frac{dx + \frac{\partial u}{\partial x}dx - dx}{dx} = \frac{\partial u}{\partial x}$$

…Eq. 22



One-dimensional strain

The displacement in one dimensional case is given by

$$u = e_x x$$

…Eq. 23

Three - dimensional strain

- In three dimensional strain, each of the component of the displacement will be linearly related to each of the three initial coordinates of the point.

$$u = e_{xx}x + e_{xy}y + e_{xz}z$$
$$v = e_{yx}x + e_{yy}y + e_{yz}z$$
$$w = e_{zx}x + e_{zy}y + e_{zz}z$$
$$\text{or} \quad u_i = e_{ij}x_j$$

…Eq. 24

- Three coefficients for the normal strains,

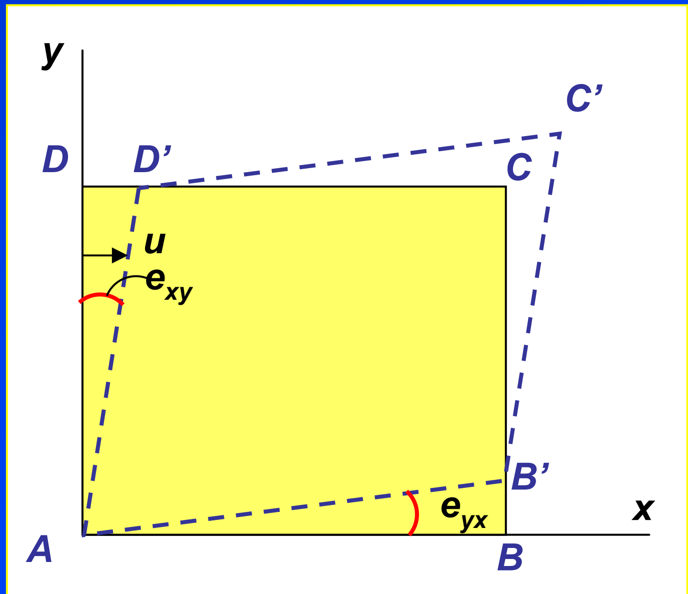
$$e_{xx} = \frac{\partial u}{\partial x} \quad e_{yy} = \frac{\partial v}{\partial y} \quad e_{zz} = \frac{\partial w}{\partial z}$$

- Consider an ***angular distortion*** of an element in the ***xy*** plane by shearing stresses, we have

$$e_{xy} = \frac{DD'}{DA} = \frac{\partial u}{\partial y} \qquad e_{yx} = \frac{BB'}{AB} = \frac{\partial v}{\partial x}$$

$$e_{ij} = \begin{vmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{vmatrix} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$

…Eq. 25



Angular distortion of an element

Strain tensor and rotation tensor

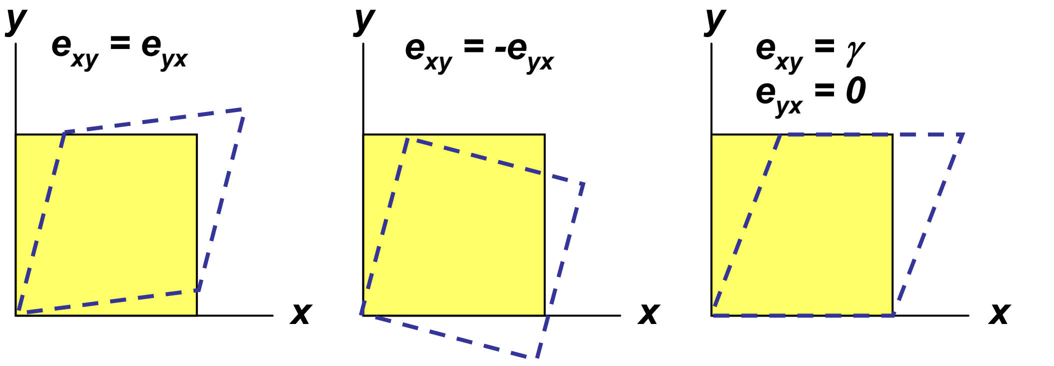
• In general, displacement components such as e_{xy} , e_{yx}.. produce both ***shear strain*** and ***rigid-body rotation***.

• From tensor theory, any second-rank tensor can be decomposed into a symmetric tensor and an anisymmetric tensor.

$$e_{ij} = \tfrac{1}{2}(e_{ij} + e_{ji}) + \tfrac{1}{2}(e_{ij} - e_{ji})$$

$$e_{ij} = \varepsilon_{ij} + \omega_{ij}$$

...Eq. 26



(a) Pure shear without rotation. ***(b) Pure rotation without shear.*** ***(c) Simple shear.***

where

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_i}\right)$$ *Strain tensor*

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_i}{\partial x_i}\right)$$ *Rotation tensor*

The general displacement equations

$$u_i = \varepsilon_{ij} x_j + \omega_{ij} x_j$$

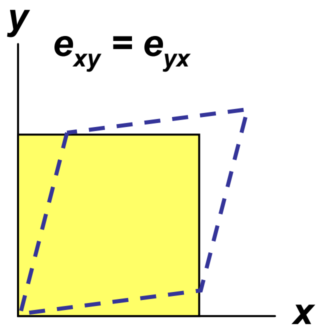
...Eq. 27

Shear strain

The ***shear strain*** γ was defined as the total angular change from a right angle.

$$\gamma = e_{xy} + e_{yx} = \varepsilon_{xy} + \varepsilon_{yx} = 2\varepsilon_{xy}$$

…Eq. 28



(a) Pure shear without rotation.

And the definition of shear strain, $\gamma_{ij} = \varepsilon_{ij}$ is called the ***engineering shear strain***.

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$
$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$
$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$$

…Eq. 29

Since strain is a ***second-rank tensor***, following the transformation of stress previously done, the strain tensor may be transformed one set of coordinate axes of coordinate axes to a new system of axes by

$$\varepsilon_{kl} = a_{ki} a_{lj} \varepsilon_{ij}$$

…Eq. 30

Principal shear strain

Following *Eq.17*, substituting ε for σ and $\gamma/2$ for τ. The **normal strain** on an oblique plane is given by

$$\varepsilon = \varepsilon_x l^2 + \varepsilon_y m^2 + \varepsilon_z n^2 + \gamma_{xy} lm + \gamma_{yz} mn + \gamma_{xz} nl$$

…Eq. 31

Similar to stress, ***the direction of the principal strains*** coincide with ***the principal stress directions***. The three principal strains are the roots of the cubic equation.

$$\varepsilon^3 - I_1\varepsilon^2 + I_2\varepsilon - I_3 = 0$$

…Eq. 32

Where

$$I_1 = \varepsilon_x + \varepsilon_y + \varepsilon_z$$
$$I_2 = \varepsilon_x\varepsilon_y + \varepsilon_y\varepsilon_z + \varepsilon_z\varepsilon_x - \tfrac{1}{4}\left(\gamma_{xy}^2 + \gamma_{zx}^2 + \gamma_{yz}^2\right)$$
$$I_3 = \varepsilon_x\varepsilon_y\varepsilon_z + \tfrac{1}{4}\left(\varepsilon_x\gamma_{yz}^2 + \varepsilon_y\gamma_{zx}^2 + \varepsilon_z\gamma_{xy}^2\right)$$

…Eq. 33

The ***maximum shearing strains*** can be obtained from

$$\gamma_1 = \varepsilon_2 - \varepsilon_3$$
$$\gamma_{max} = \gamma_2 = \varepsilon_1 - \varepsilon_3$$
$$\gamma_3 = \varepsilon_1 - \varepsilon_2$$

…Eq. 34

Strain tensor

Strain tensor can be divided into a ***hydrostatic or mean strain*** and a ***strain deviator***.

1) Hydrostatic or mean strain (volume change)

…Eq. 35

$$\varepsilon_m = \frac{\varepsilon_x + \varepsilon_y + \varepsilon_z}{3} = \frac{\varepsilon_{kk}}{3} = \frac{\Delta}{3}$$

Where Δ is the volume strain (change in volume).

2) Strain deviator (shape change)

We can do by subtracting ε_m from the normal strain components, thus

…Eq. 36

$$\varepsilon'_{ij} = \begin{vmatrix} \varepsilon_x - \varepsilon_m & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_y - \varepsilon_m & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_z - \varepsilon_m \end{vmatrix}$$

These strains represent elongations or contractions along the principal axes that change the shape of the body at constant volume.

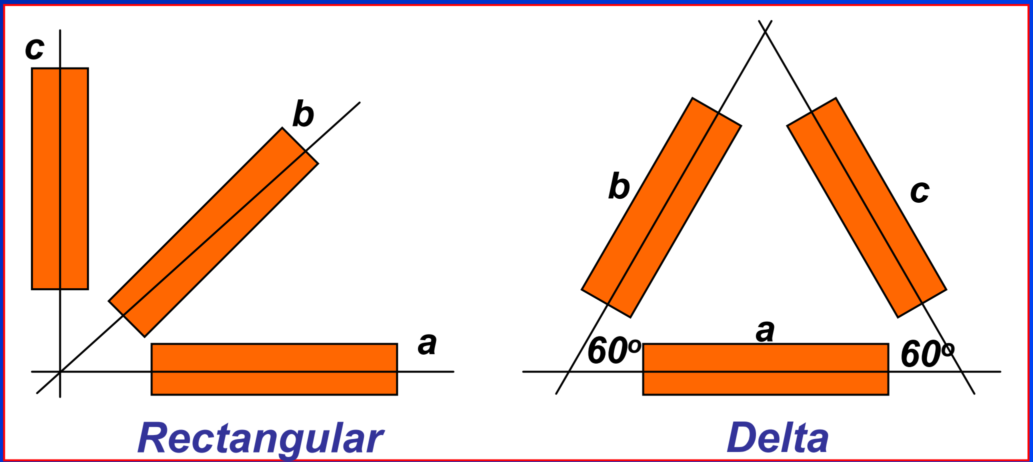
Strain measurement

• Strain can be measured by using a bonded-wire resistance gauge or ***strain gauge***.

• When the body is deformed, the wires in the strain gauge are strained and their electrical resistance is altered.

• The change in resistance, which is proportional to strain can therefore be determined.

• Strain gauges can make only direct readings of linear strain, while shear strains must be determined indirectly.



Typical strain gauge rosettes.

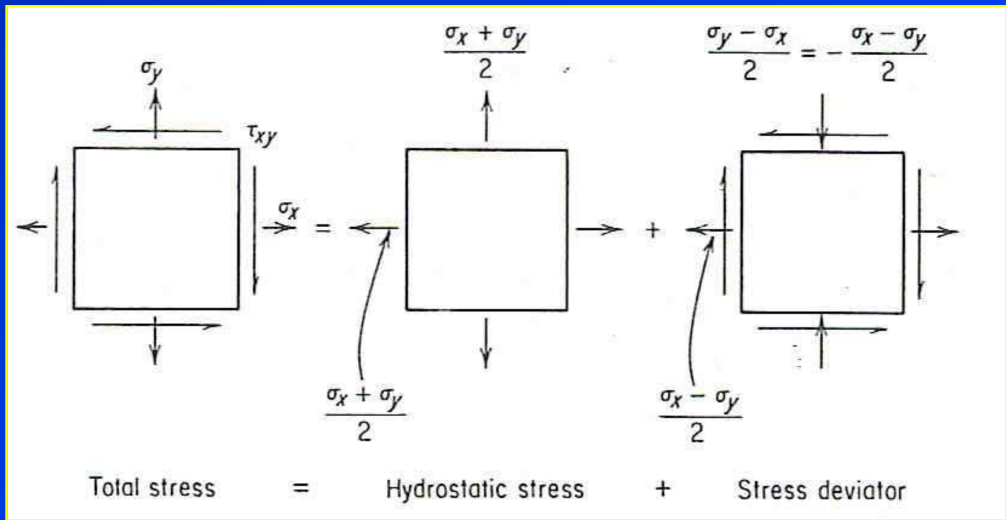
Hydrostatic and deviator components of stress

Similar to strain tensor, the total stress tensor can be divided into

1) ***Hydrostatic or mean stress tensor***, σ_m, which involves only pure tension or compression. → Produce ***elastic volume changes***.

$$\sigma_m = \frac{\sigma_{kk}}{3} = \frac{\sigma_x + \sigma_y + \sigma_z}{3} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \qquad \text{...Eq. 37}$$

2) ***Deviator stress tensor*** σ'_{ij}. Which represents the shear stress in the total state of stress. → important in causing ***plastic deformation***.



Deviator of stress tensor

Since the decomposition of the stress tensor is given by

$$\sigma_{ij} = \sigma'_{ij} + \tfrac{1}{3}\delta_{ij}\sigma_{kk} \qquad \text{...Eq. 38}$$

Stress deviator involves shear stress. For example, referring σ'_{ij} to a system of principal axes.

$$\sigma'_1 = \frac{2\sigma_1 - \sigma_2 - \sigma_3}{3} = \frac{(\sigma_1 - \sigma_2) + (\sigma_1 - \sigma_3)}{3}$$

$$\sigma'_1 = \frac{2}{3}\left(\frac{\sigma_1 - \sigma_2}{2} + \frac{\sigma_1 - \sigma_3}{2}\right) = \frac{2}{3}(\tau_3 + \tau_2) \qquad \text{...Eq. 39}$$

Where τ_3 and τ_2 are principal shearing stresses.

Deviator of stress tensor

- The principal values of the stress deviator are the roots of the cubic equation:

$$(\sigma')^3 - J_1(\sigma')^2 - J_2\sigma' - J_3 = 0$$

Where J_1 , J_2 , J_3 are the invariants of the deviator stress tensor.

$$J_1 = (\sigma_x - \sigma_m) + (\sigma_y - \sigma_m) + (\sigma_z - \sigma_m) = 0$$

$$J_2 = \tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2 - \sigma_x'\sigma_y' - \sigma_y'\sigma_z' - \sigma_x'\sigma_z'$$
$$J_2 = \tfrac{1}{6}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_x - \sigma_z)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)\right]$$

Elastic stress – strain relations

The ***elastic stress*** is linearly related to ***elastic strain*** following ***Hooke's law.***

Where ***E*** is the modulus of elasticity in tension or compression.

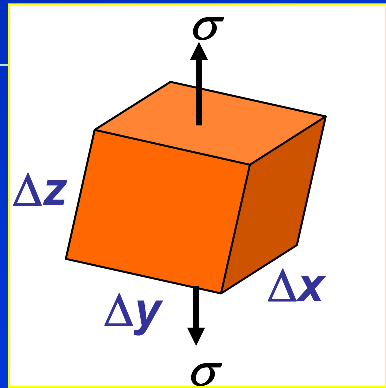
$$\sigma_x = E\varepsilon_x$$ *…Eq. 40*

However, during linear extension, i.e., in ***x*** axis, the contraction in the transverse ***y*** and ***z*** direction causes a constant fraction of the strain in the longitudinal direction known as ***Poisson's ratio v***.

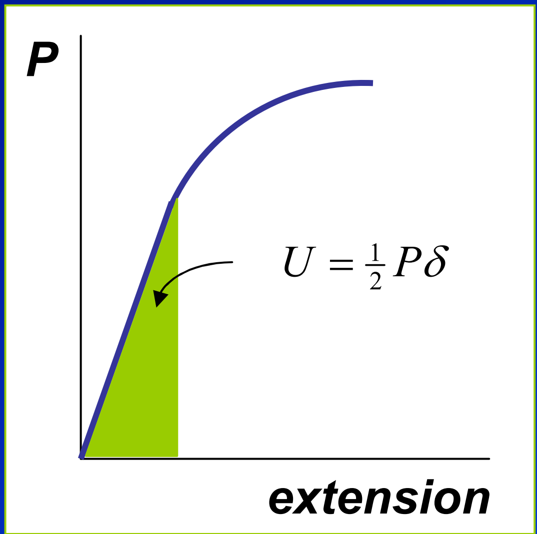
$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -\frac{\nu\sigma_x}{E}$$ *…Eq. 41*

Note: for most metals ***v*** ~ 0.33

Strain energy



*When a material is deformed by an external loading, **work done during elastic deformation** is stored as elastic energy and will be recovered when the load is released. → **Strain energy**.*



Work is force x distance over it acts, therefore

$$U = \tfrac{1}{2} P\delta \qquad \text{...Eq. 42}$$

Where **P/2** is the average force from zero.
δ is the extension.

For linear elastic (i.e., **x** axis) then ***Hooke's law*** is applied ***(σ = Eε)***

$$U_o = \frac{1}{2}\sigma_x \varepsilon_x = \frac{1}{2}\frac{\sigma_x^2}{E} = \frac{1}{2}\varepsilon_x^2 E \qquad \text{...Eq. 43}$$

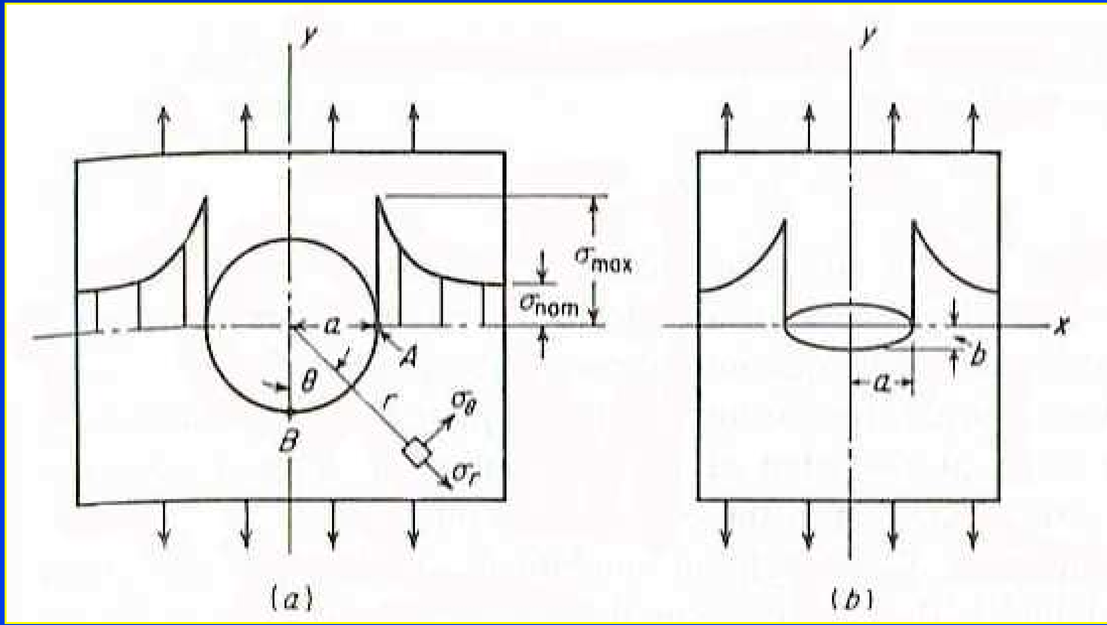
Stress concentration

• Discontinuity such as a ***hole*** or a ***notch*** results in ***non-uniform stress distribution*** at the vicinity of the discontinuity.→ ***stress concentration or stress raiser***.

• The distribution of the axial stress reaches a high value at the edges of the hole and drops of rapidly with distance away from the hole.

• The stress concentration is expressed by a theoretical stress-concentration factor K_t.

$$K_t = \frac{\sigma_{max}}{\sigma_{nom}} \quad \text{...Eq. 44}$$



Stress distributions due to (a) circular hole and (b) elliptical hole.

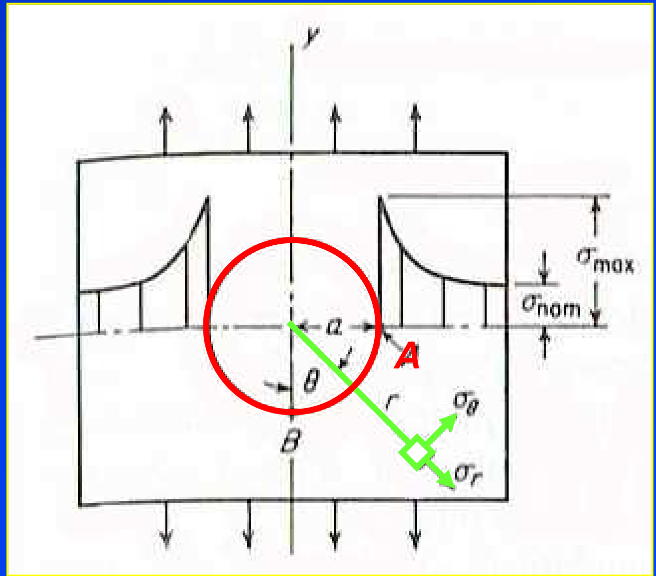
Stress concentration at a circular hole in a plate

- For the circular hole in a plate subjected to an axial load, a radial stress is produced as well as a longitudinal stress.
- From elastic analysis, the stresses can be expressed as:

$$\sigma_r = \frac{\sigma}{2}\left(1-\frac{a^2}{r^2}\right)+\frac{\sigma}{2}\left(1+3\frac{a^4}{r^4}-4\frac{a^2}{r^2}\right)\cos 2\theta$$

$$\sigma_\theta = \frac{\sigma}{2}\left(1+\frac{a^2}{r^2}\right)-\frac{\sigma}{2}\left(1+3\frac{a^4}{r^4}\right)\cos 2\theta \quad \text{...Eq. 45}$$

$$\tau = -\frac{\sigma}{2}\left(1-3\frac{a^4}{r^4}+2\frac{a^2}{r^2}\right)\sin 2\theta$$



Stress distribution at a circular hole

The maximum stress occurs at point **A** when $\theta = \pi/2$ and $r = a$

$$\sigma_\theta = 3\sigma = \sigma_{max} \quad \text{...Eq. 46}$$

The theoretical stress-concentration factor = 3

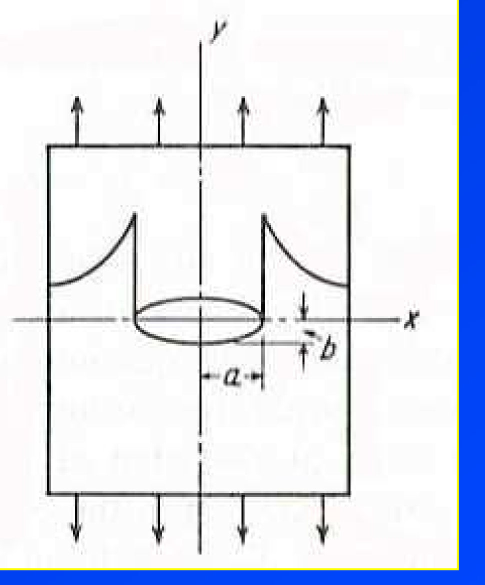
Stress concentration at an elliptical hole in a plate

In the case of an elliptical hole in a plate, the maximum stress at the ends of the hole is given by the equation

$$\sigma_{max} = \sigma\left(1 + 2\frac{a}{b}\right) \quad \text{...Eq. 47}$$

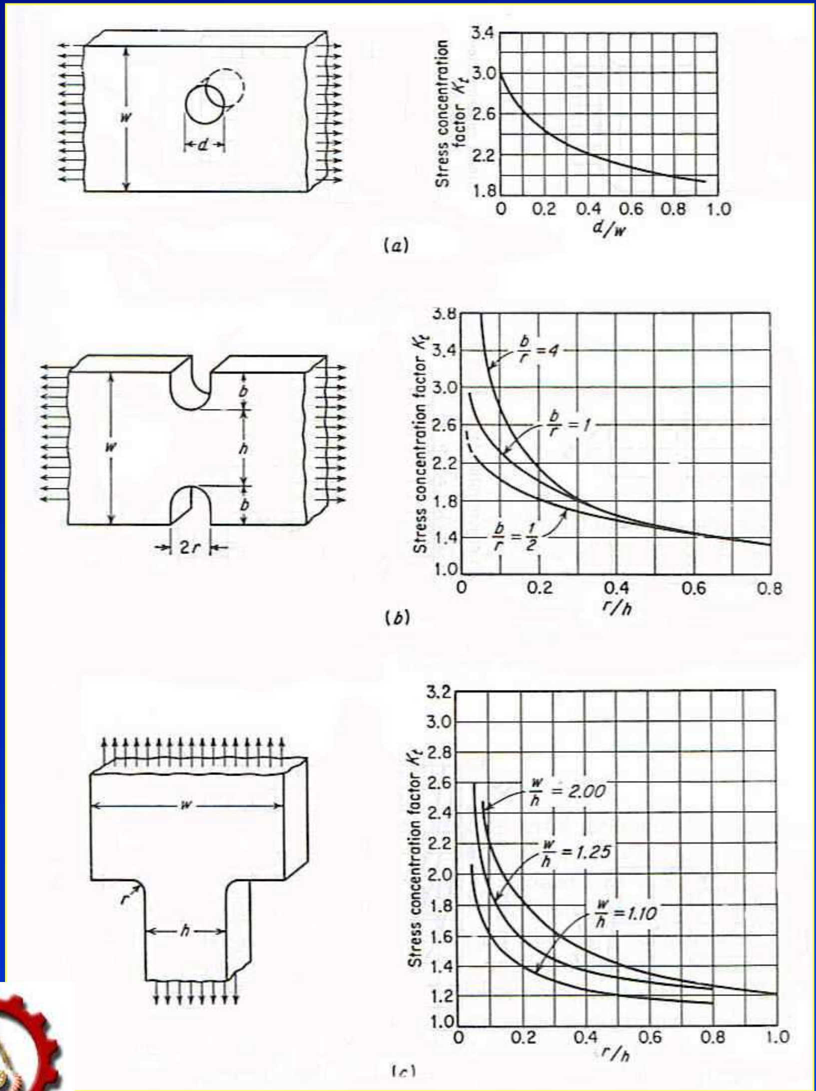
a/b ⬆ **stress** ⬆

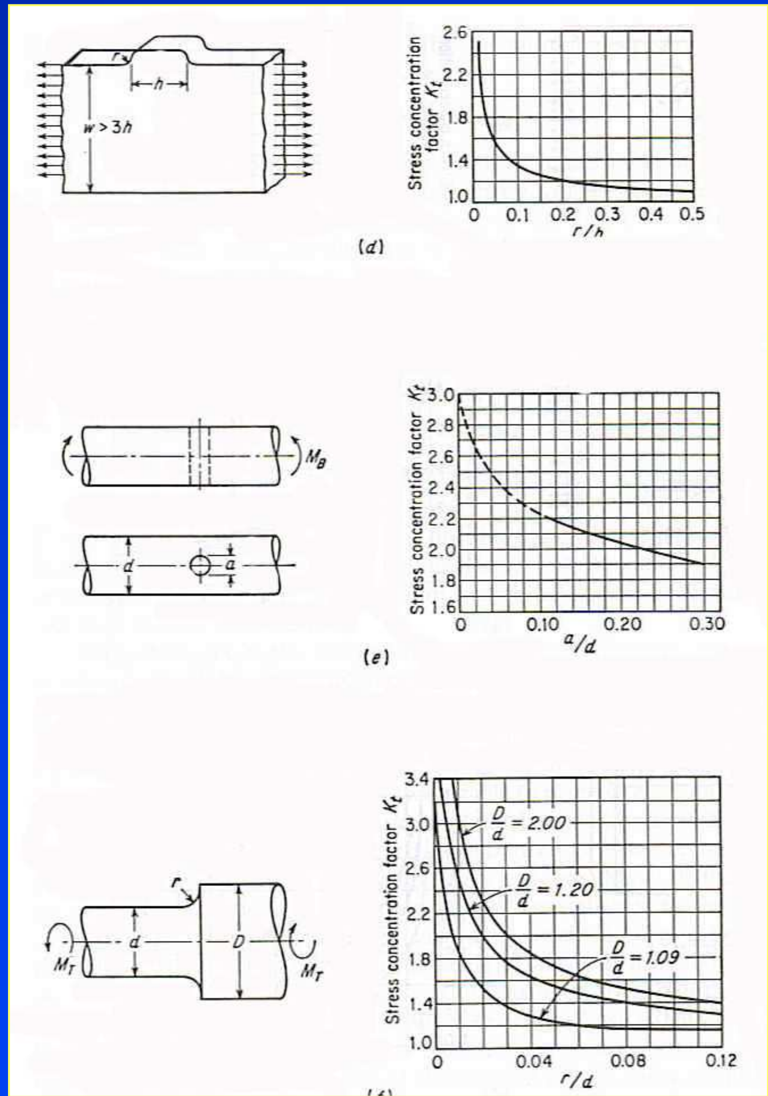
There for a very sharp crack normal to the tensile direction will result in a very high stress concentration.



Stress distribution at an elliptical hole

Stress concentrations for different geometrical shapes





Stress raiser in brittle and ductile materials

• The effect of ***stress raiser*** is much more pronounced in brittle materials than in ductile materials.

Ductile materials

• Plastic deformation occurs when the yield stress is exceeded at the point of maximum stress.

• In ductile materials, further increase in load produce strain hardening (work hardening). → ***redistribution of stress*** → the materials will not develop the full theoretically stress-concentration factor.

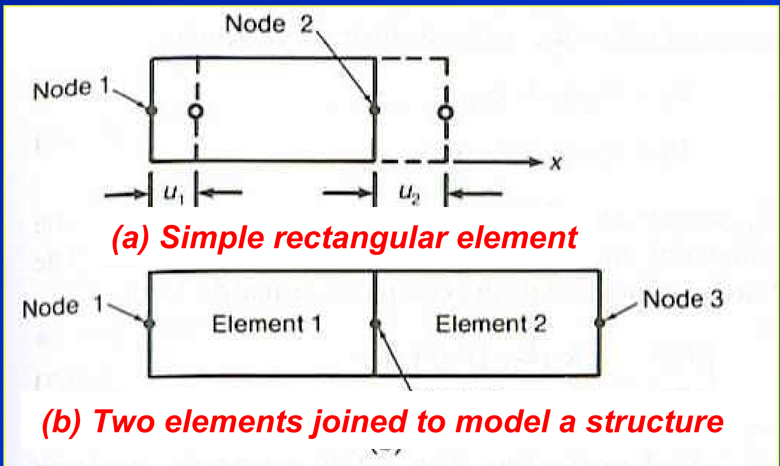
Brittle materials

• ***Stress redistribution*** will not occur in to any extent in ***brittle materials***. → stress concentration of close to the theoretical value will result.

Finite element method

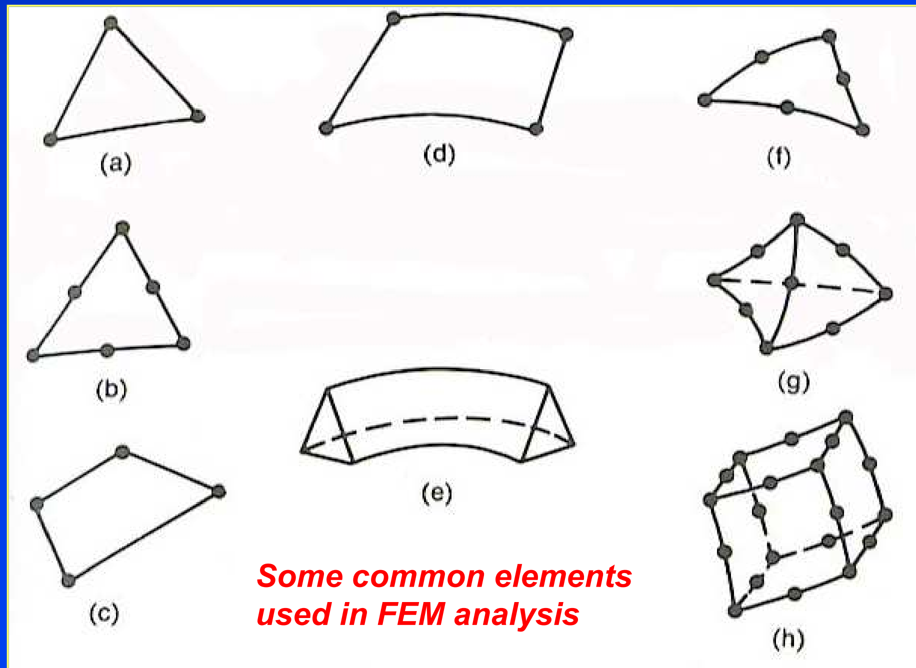
• ***Finite element method (FEM)*** is a very powerful technique for determining stresses and deflections in structures too complex to analyse by strictly analytical methods.

• The structure is divided into ***a network of small elements connected to each other at node points***. One node has one degree of freedom (*see fig* →showing two and three dimensional element)



(a) Simple rectangular element

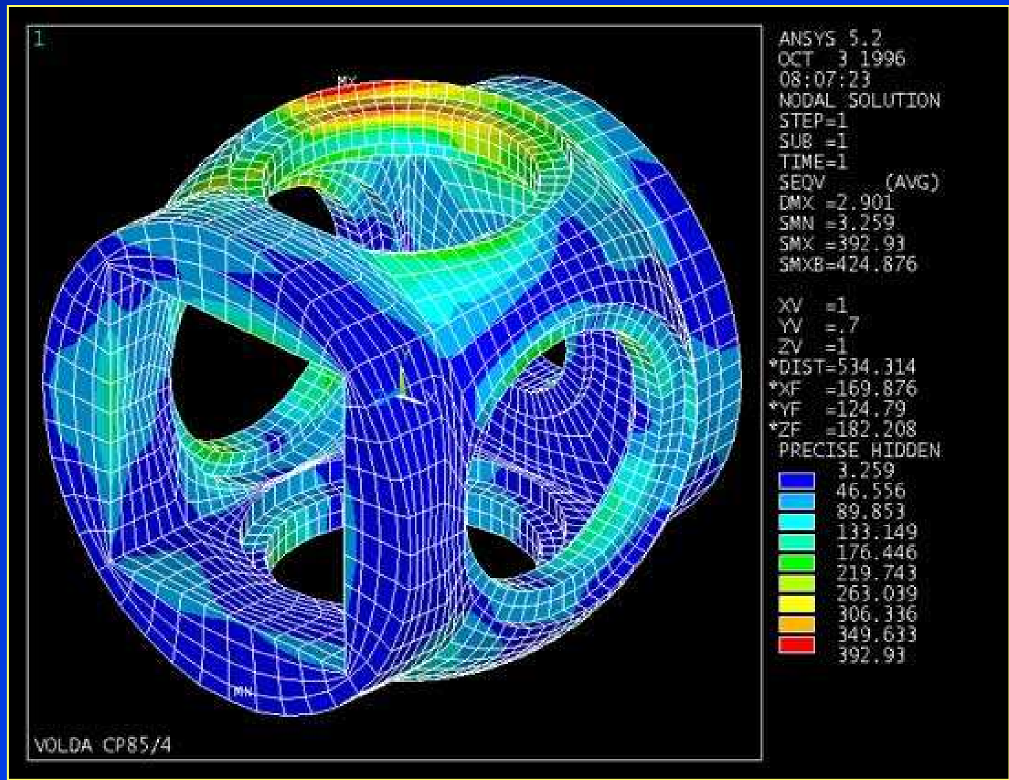
(b) Two elements joined to model a structure



Some common elements used in FEM analysis

Finite element method

- A finite element solution involves ***calculating*** the stiffness matrices for every element in the structure.
- A cumbersome part of the finite element solution is the ***preparation of the input data***. Required data such as topology of the element mesh, node numbers, coordinates of the node points.



Propeller designed by means of FEM analysis

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Elements of the theory of plasticity

Subjects of interest

- *Introduction/objectives*
- *The flow curve*
- *True stress and true strain*
- *Yielding criteria for ductile materials*
- *Combined stress tests*
- *The yield locus*
- *Anisotropy in yielding*

Objective

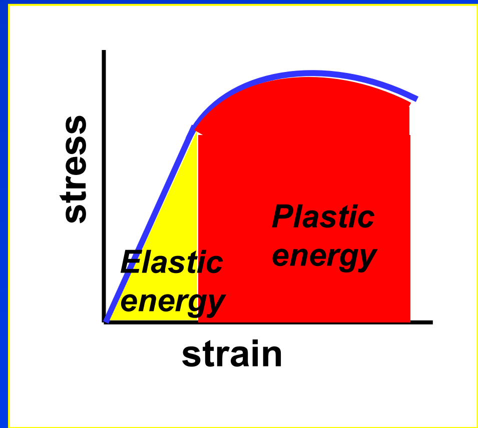
• This chapter provides a basic theory of plasticity for the understanding of the flow curve.

• Differences between the true stress – true strain curve and the engineering stress – engineering strain curves will be highlighted.

• Finally the understanding of the yielding criteria for ductile materials will be made.

Introduction

- Plastic deformation is a ***non reversible*** process where ***Hooke's law*** is no longer valid.
- One aspect of plasticity in the viewpoint of structural design is that it is concerned with ***predicting the maximum load***, which can be applied to a body without causing excessive yielding.
- Another aspect of plasticity is about the ***plastic forming of metals*** where large plastic deformation is required to change metals into desired shapes.



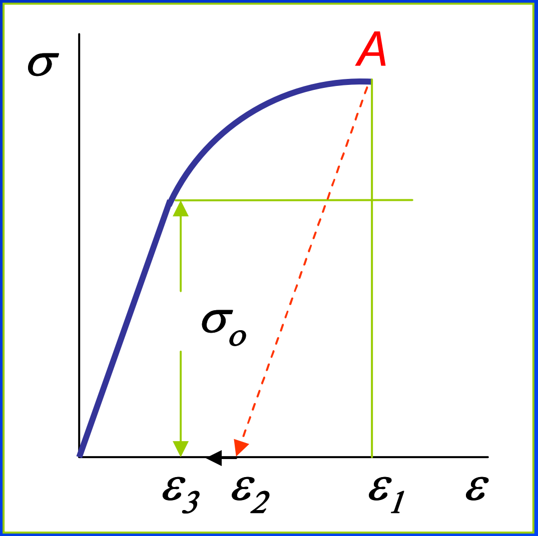
Plastic and elastic deformation in uniaxial tension

The flow curve

- True stress-strain curve for typical ductile materials, i.e., aluminium, show that the stress - strain relationship follows up the ***Hooke's law*** up to the ***yield point***, σ_o.

- Beyond σ_o, the metal ***deforms plastically*** with ***strain-hardening***. This cannot be related by any simple constant of proportionality.

- If the load is released from straining up to point ***A***, the ***total strain*** will immediately decrease from ε_1 to ε_2. by an amount of σ/E.



Typical true stress-strain curves for a ductile metal.

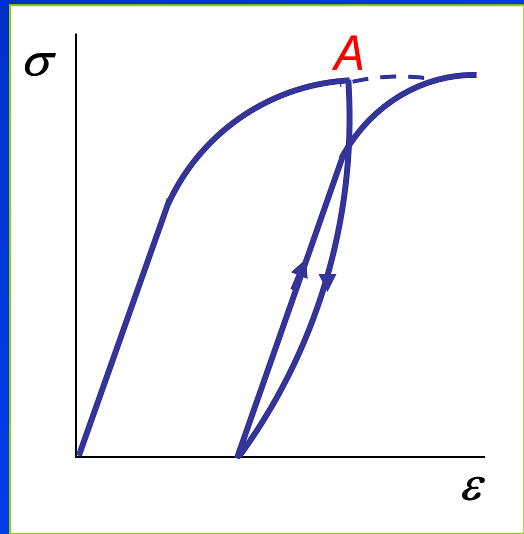
- The strain ε_1-ε_2 is the ***recoverable elastic strain***. Also there will be a small amount of the plastic strain ε_2-ε_3 known as ***anelastic behaviour*** which will disappear by time.→ (neglected in plasticity theories.)

The flow curve

• Usually the stress-strain curve on ***unloading*** from a ***plastic strain will not be exactly linear*** and parallel to the elastic portion of the curve.

• On reloading the curve will generally ***bend over*** as the stress pass through the original value from which it was unloaded.

• With this little effect of ***unloading and loading from a plastic strain***, the stress-strain curve becomes a continuation of the ***hysteresis behaviour.*** (but generally neglected in plasticity theories.)

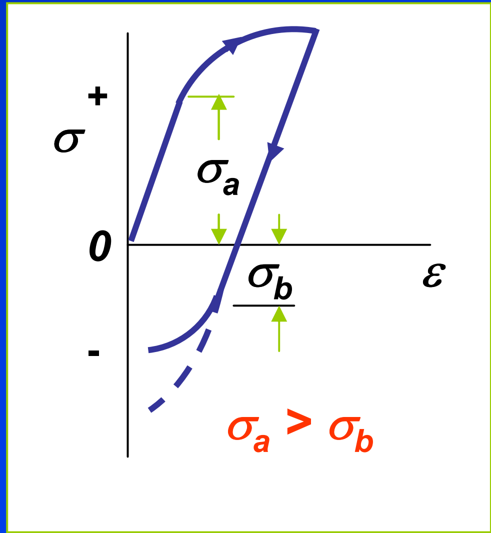


Stress-strain curve on unloading from a plastic strain.

The flow curve

• If specimen is deformed plastically beyond the yield stress in ***tension*** (+), and then in ***compression*** (-), it is found that the yield stress on reloading in compression is less than the original yield stress.

• The dependence of the yield stress on loading path and direction is called the ***Bauschinger effect***. → (however it is neglected in plasticity theories and it is assumed that the yield stress in tension and compression are the same).



Bauschinger effect

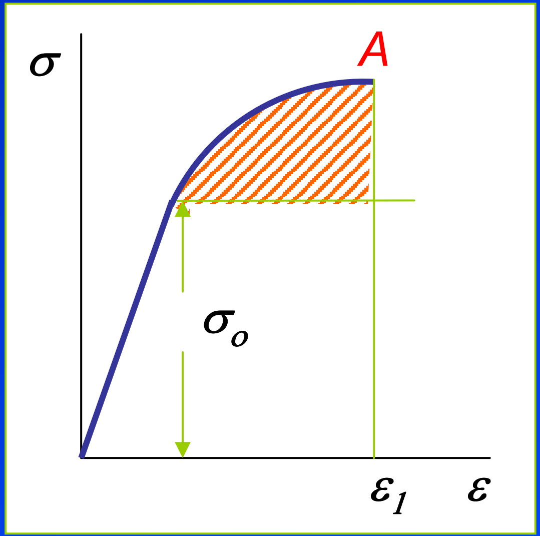
The flow curve

• A true stress – strain curve provides the stress required to cause the metal to flow plastically at any strain → it is often called a '***flow curve***'.

• A mathematical equation that fit to this curve from the beginning of the plastic flow to the maximum load before necking is a ***power expression*** of the type

$$\sigma = K\varepsilon^n \qquad \ldots Eq.1$$

Where ***K*** is the stress at ε **= 1.0**
n is the strain – hardening exponent
(slope of a log-log plot of *Eq.1*)



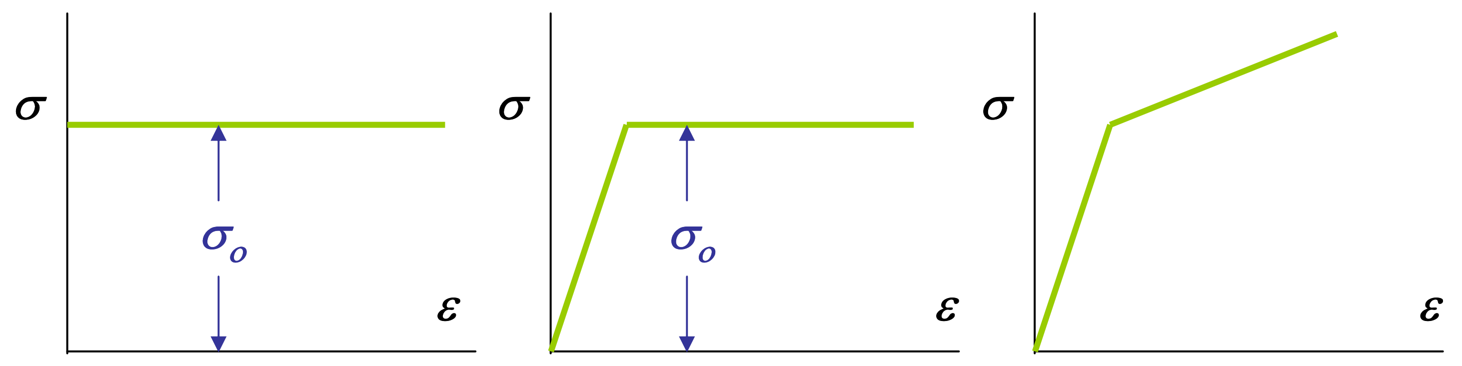
Typical true stress-strain curves for a ductile metal.

Note: higher σ_o → greater elastic region, but ***less ductility*** (less plastic region).

Idealised flow curves

Due to ***considerable mathematical complexity*** concerning the theory of plasticity, the ***idealised flow curves*** are therefore utilised to simplify the mathematics.

1) ***Rigid ideal plastic material*** : no elastic strain, no strain hardening.
2) ***Perfectly plastic material with an elastic region***, i.e., plain carbon steel.
3) ***Piecewise linear (strain-hardening material)*** : with elastic region and strain hardening region → more realistic approach but complicated mathematics.

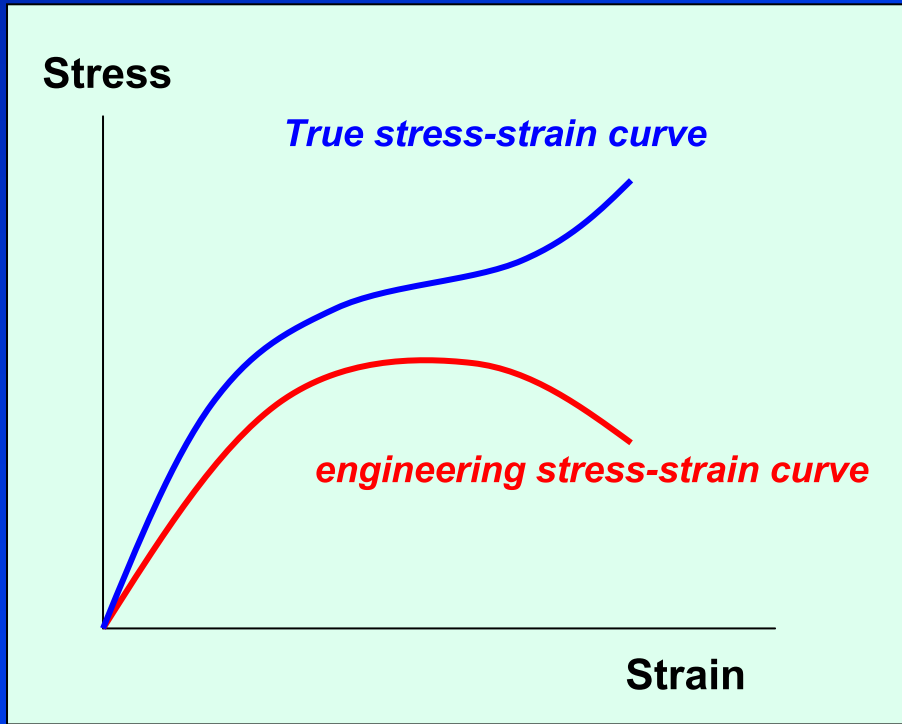


(a) Rigid ideal plastic material. ***(b) Ideal plastic material with elastic region.*** ***(c) Piecewise linear (stain-hardening) material.***

True stress and true strain

• The ***engineering stress – strain curve*** is based entirely on the ***original dimensions*** of the specimen → This cannot represent true deformation characteristic of the material.

• The ***true stress – strain curve*** is based on the ***instantaneous specimen dimensions***.



Engineering stress-strain and true stress-strain curves.

The true strain

According to the concept of unit linear strain,

$$e = \frac{\Delta L}{L_o} = \frac{1}{L_o}\int_{L_o}^{L} dL$$

…Eq.2

This satisfies for elastic strain where ΔL is very small, but not for plastic strain.

Definition: ***true strain*** or ***natural strain*** (first proposed by ***Ludwik***) is the change in length referred to the instantaneous gauge length.

$$\varepsilon = \sum \frac{L_1 - L_o}{L_o} + \frac{L_2 - L_1}{L_1} + \frac{L_3 - L_2}{L_2} + \ldots$$

$$\varepsilon = \int_{L_o}^{L} \frac{dL}{L} = \ln \frac{L}{L_o}$$

…Eq.3

Hence the relationship between the ***true strain*** and the ***conventional linear strain*** becomes

$$e = \frac{\Delta L}{L_o} = \frac{L - L_o}{L_o} = \frac{L}{L_o} - 1$$

$$e + 1 = \frac{L}{L_o}$$

$$\varepsilon = \ln \frac{L}{L_o} = \ln(e + 1)$$

…Eq.4

Comparison of true strain and conventional linear strain

True stain ε	0.01	0.10	0.20	0.50	1.0	4.0
Conventional strain e	0.01	0.105	0.22	0.65	1.72	53.6

• In true strain, the same amount of strain (but the opposite sign) is produced in tension and compression respectively.

Ex: *Expanding the cylinder to twice its length.*

Tension

$$\varepsilon = \ln(2L_o / L_o) = \ln 2$$

…Eq.5

Ex: *Compression to half the original length.*

Compression

$$\varepsilon = \ln[(L_o / 2)L_o] = -\ln 2$$

…Eq.6

Total true strain and conventional strain

Increment	Length of rod	
0	50	
1	55	e_{o-1} = 5/50=0.1
2	60.5	e_{1-2} = 5.5/55=0.1
3	66.5	e_{2-3} = 6.05/60.5= 0.1

The total conventional strain e

• The total conventional strain e_{0-3} is not equal to $e_{0-1} + e_{1-2} + e_{2-3}$.

$$e_{0-1} + e_{1-2} + e_{2-3} = 0.3 \neq e_{0-3} = 16.55/50 = 0.331$$ ...Eq.7

The total true strain ε

• The total true strain = the summation of the incremental true strains.

$$\varepsilon_{0-1} + \varepsilon_{1-2} + \varepsilon_{2-3} = \ln\frac{55}{50} + \ln\frac{60.5}{55} + \ln\frac{66.55}{60.5} = \ln\frac{66.55}{50} = \varepsilon_{0-3} = 0.286$$ ...Eq.8

The volume strain

According to the volume strain Δ

$$\Delta = \frac{\Delta V}{V} = \frac{(1+e_x)(1+e_y)(1+e_z)d_x d_y d_z - d_x d_y d_z}{d_x d_y d_z}$$

$$\Delta = (1+e_x)(1+e_y)(1+e_z) - 1$$

…Eq.9

During plastic deformation, it is considered that the ***volume of a solid remain constant*** *→ (Δ = 0)*

$$\Delta + 1 = (1+e_x)(1+e_y)(1+e_z)$$

$$\ln 1 = 0 = \ln(1+e_x) + \ln(1+e_y) + \ln(1+e_z)$$

But $\varepsilon_x = \ln(1+e_x)$, hence

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0$$

…Eq.10

Due to the constant volume $A_oL_o = AL$, therefore

$$\varepsilon = \ln\frac{L}{L_o} = \ln\frac{A_o}{A}$$

…Eq.11

The true stress

Definition: ***the true stress*** is the load divided by the instantaneous area.

True stress $\sigma = \frac{P}{A}$

Engineering stress $s = \frac{P}{A_o}$

Relationship between the true stress and the engineering stress

Since $\sigma = \frac{P}{A} = \frac{P}{A_o}\frac{A_o}{A}$

But $\frac{A_o}{A} = \frac{L}{L_o} = e + 1$

Hence, $\sigma = \frac{P}{A_o}(e+1) = s(e+1)$ *…Eq.12*

Example: *A tensile specimen with a 12 mm initial diameter and 50 mm gauge length reaches maximum load at 90 kN and fractures at 70 kN. The maximum diameter at fracture is 10 mm. Determine engineering stress at maximum load (the ultimate tensile strength), true fracture stress, true strain at fracture and engineering strain at fracture*

Engineering stress at maximum load

$$\frac{P_{max}}{A_{max}} = \frac{90\times10^3}{\pi(12\times10^{-3})^2/4} = 796 \quad MPa$$

True fracture stress

$$\frac{P_f}{A_f} = \frac{70\times10^3}{\pi(10\times10^{-3})/4} = 891 \quad MPa$$

True strain at fracture

$$\varepsilon_f = \ln\frac{A_o}{A_f} = \ln\left(\frac{12}{10}\right)^2 = 2\ln 1.2 = 0.365$$

Engineering strain at fracture

$$e_f = \exp(\varepsilon) - 1 = \exp(0.365) - 1 = 0.44$$

Yielding criteria for ductile metals

• ***Plastic yielding*** of the material subjected to any external forces is of considerable importance in the field of plasticity.

• For predicting the ***onset of yielding*** in ductile material, there are at present two generally accepted criteria,

1) ***Von Mises'*** or Distortion-energy criterion

2) ***Tresca*** or Maximum shear stress criterion

Von Mises' criterion

Von Mises proposed that yielding occur when the second invariant of the stress deviator J_2 > critical value k^2.

$$\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]=6k^2$$ ...Eq.13

For yielding in uniaxial tension,
$\sigma_1 = \sigma_o$, $\sigma_2 = \sigma_3 = 0$

$$2\sigma_o^2 = 6k^2, then\ k=\frac{\sigma_o}{\sqrt{3}}$$ ...Eq.14

Substituting ***k*** from *Eq.14* in *Eq.13*, we then have the ***von Mises' yield criterion***

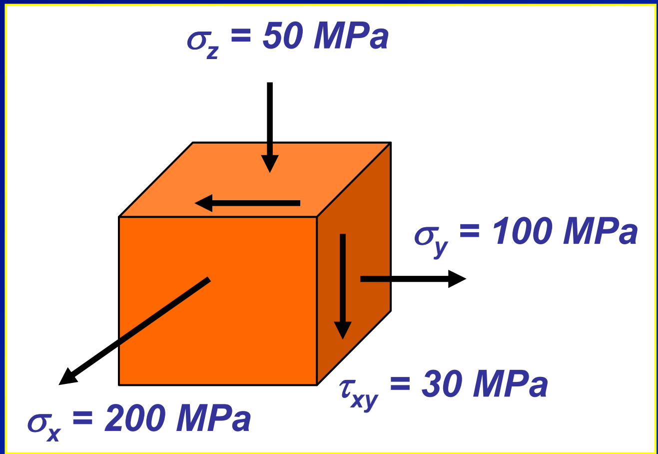
$$\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{\frac{1}{2}}=\sqrt{2}\sigma_o$$ ...Eq.15

In ***pure shear***, to evaluate the constant ***k***, note $\sigma_1 = \sigma_3 = \tau_y$, $\sigma_2 = 0$, where σ_o is the yield stress; when yields: $\tau_y^2+\tau_y^2+4\tau_y^2 = 6k^2$ then $k = \tau_y$

By comparing with *Eq 14,*
we then have

$$\tau_y = 0.577\sigma_o$$ *** ...Eq.16

Example: Stress analysis of a spacecraft structural member gives the state of stress shown below. If the part is made from 7075-T6 aluminium alloy with σ_o = 500 MPa, will it exhibit yielding? If not, what is the safety factor?



From *Eq.16*

$$\sigma_o = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)\right]^{1/2}$$

$$\sigma_o = \frac{1}{\sqrt{2}}\left[(200 - 100)^2 + (100 - (-50))^2 + (-50 - 200)^2 + 6(30)^2\right]^{1/2}$$

$$\sigma_o = 224 \quad MPa$$

The calculated σ_o = 224 MPa < the yield stress (500 MPa), therefore yielding will not occur.

Safety factor = 500/224 = 2.2.

Tresca yield criterion

Yielding occurs when the ***maximum shear stress*** τ_{max} reaches the value of the shear stress in the uniaxial-tension test, τ_o .

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} \qquad \text{...Eq.17}$$

Where σ_1 is the algebraically largest and σ_3 is the algebraically smallest principal stress.

For uniaxial tension, $\sigma_1 = \sigma_o$, $\sigma_2 = \sigma_3 = 0$, and the shearing yield stress $\tau_o = \sigma_o/2$.

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \tau_o = \frac{\sigma_o}{2} \qquad \text{...Eq.18}$$

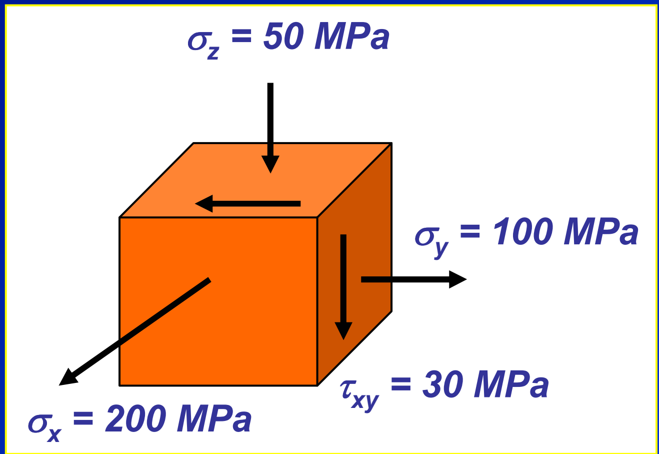
Therefore the maximum - shear stress criterion is given by

$$\sigma_1 - \sigma_3 = \sigma_o \qquad \text{...Eq.19}$$

In pure shear, $\sigma_1 = -\sigma_3 = k$, $\sigma_2 = 0$, → $\tau_{max} = \tau_y$

$$\tau_y = 0.5\sigma_o \qquad *** \qquad \text{...Eq.20}$$

Example: Use the maximum-shear-stress criterion to establish whether yielding will occur for the stress state shown in the previous example.



$$\tau_{\max} = \frac{\sigma_x - \sigma_z}{2} = \frac{\sigma_o}{2}$$

$$200 - (-50) = \sigma_o$$

$$\sigma_o = 250 \quad MPa$$

The calculated value of σ_o is less then the yield stress (500 MPa), therefore yielding will not occur.

Summation

1) Von Mises' yield criterion

- Yielding is based on differences of normal stress, but independent of hydrostatic stress.
- Complicated mathematical equations.
- Used in most theoretical work.

$$\tau_y = 0.577\sigma_o$$ ***

2) Tresca yield criterion

- Less complicated mathematical equation → used in engineering design.

$$\tau_y = 0.5\sigma_o$$ ***

Note: the difference between the two criteria are approximately 1-15%.

Combined stress tests

In a ***thin-wall tube***, states of stress are various combinations of ***uniaxial*** and ***torsion*** with maybe a hydrostatic pressure being introduced to produce a circumferential ***hoop stress*** in the tube.

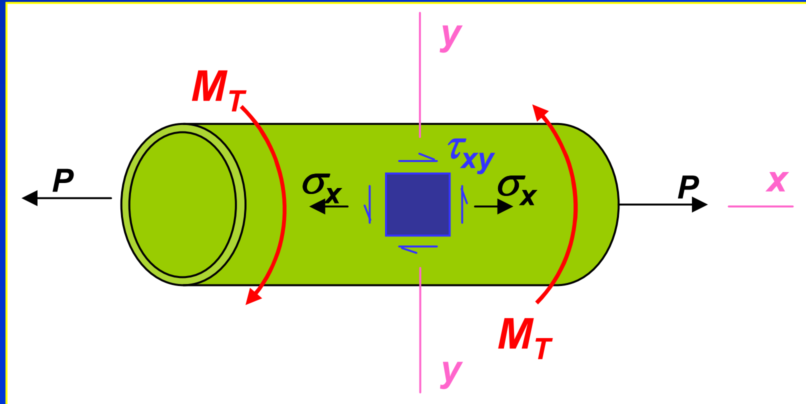
In a ***thin*** wall, $\sigma_1 = -\sigma_3$, $\sigma_2 = 0$

The ***maximum shear-stress criterion*** of yielding in the thin wall tube is given by

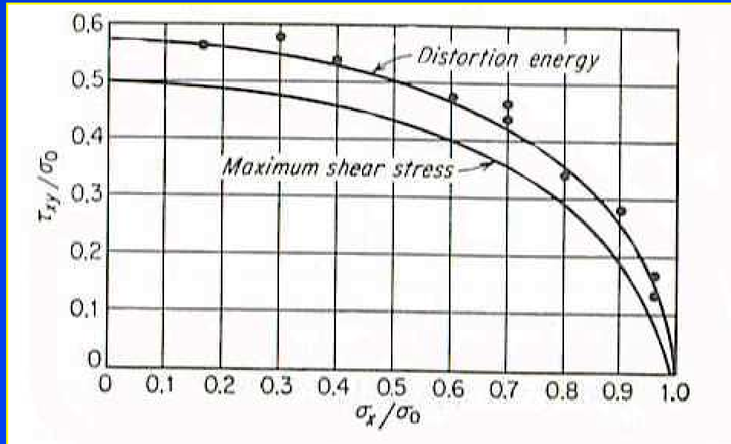
$$\left(\frac{\sigma_x}{\sigma_o}\right)^2 + 4\left(\frac{\tau_{xy}}{\sigma_o}\right)^2 = 1 \qquad \text{…Eq.21}$$

The ***distortion-energy theory*** of yielding is expressed by

$$\left(\frac{\sigma_x}{\sigma_o}\right)^2 + 3\left(\frac{\tau_{xy}}{\sigma_o}\right)^2 = 1 \qquad \text{…Eq.22}$$



Combined tension and torsion in a thin-wall tube.



Comparison between maximum-shear-stress theory and distortion-energy (von Mise's) theory.

The yield locus

For a ***biaxial plane-stress*** condition ***(σ_2 = 0)*** the ***von-Mise's yield criterion*** can be expressed as

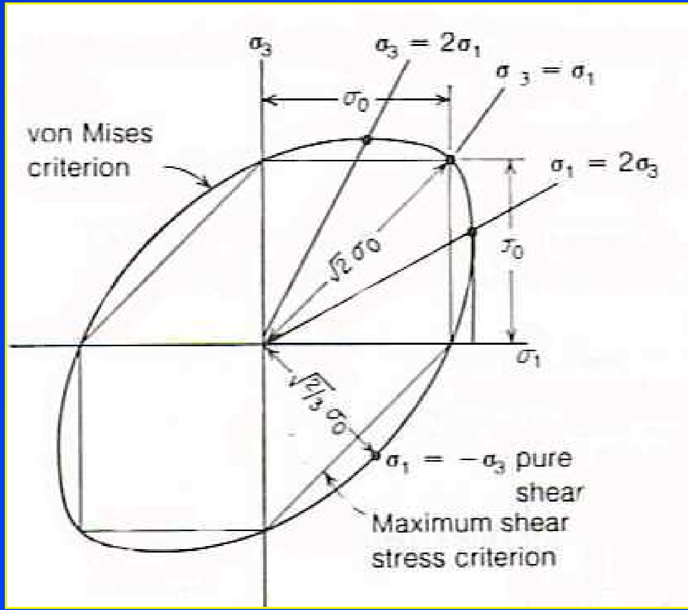
$$\sigma_1^2 + \sigma_3^2 - \sigma_1\sigma_3 = \sigma_o^2 \quad \text{...Eq.23}$$

The equation is an ellipse type with

-major semiaxis $\sqrt{2}\sigma_o$ - minor semiaxis $\sqrt{\tfrac{2}{3}}\sigma_o$ → ***Yield locus***

• The ***yield locus*** for the ***maximum shear stress criterion*** falls inside the ***von Mise's yield ellipse***.

• The yield stress predicted by the ***von Mise's criterion*** is 15.5% > than the yield stress predicted by the ***maximum-shear-stress criterion***.



Comparison of yield criteria for plane stress

References

- Dieter, G.E., ***Mechanical metallurgy***, 1988, SI metric edition, McGraw-Hill, ISBN 0-07-100406-8.
- Hibbeler, R.C. ***Mechanics of materials***, 2005, SI second edition, Person Prentice Hall, ISBN 0-13-186-638-9.

Plastic deformation of single crystals

Subjects of interest

- *Introduction/Objectives*
- *Concepts of crystal geometry*
- *Lattice defects*
- *Deformation by slip*
- *Slip by dislocation motion*
- *Crystal resolved shear stress by slip*

Plastic deformation of single crystals

Subjects of interest (continued)

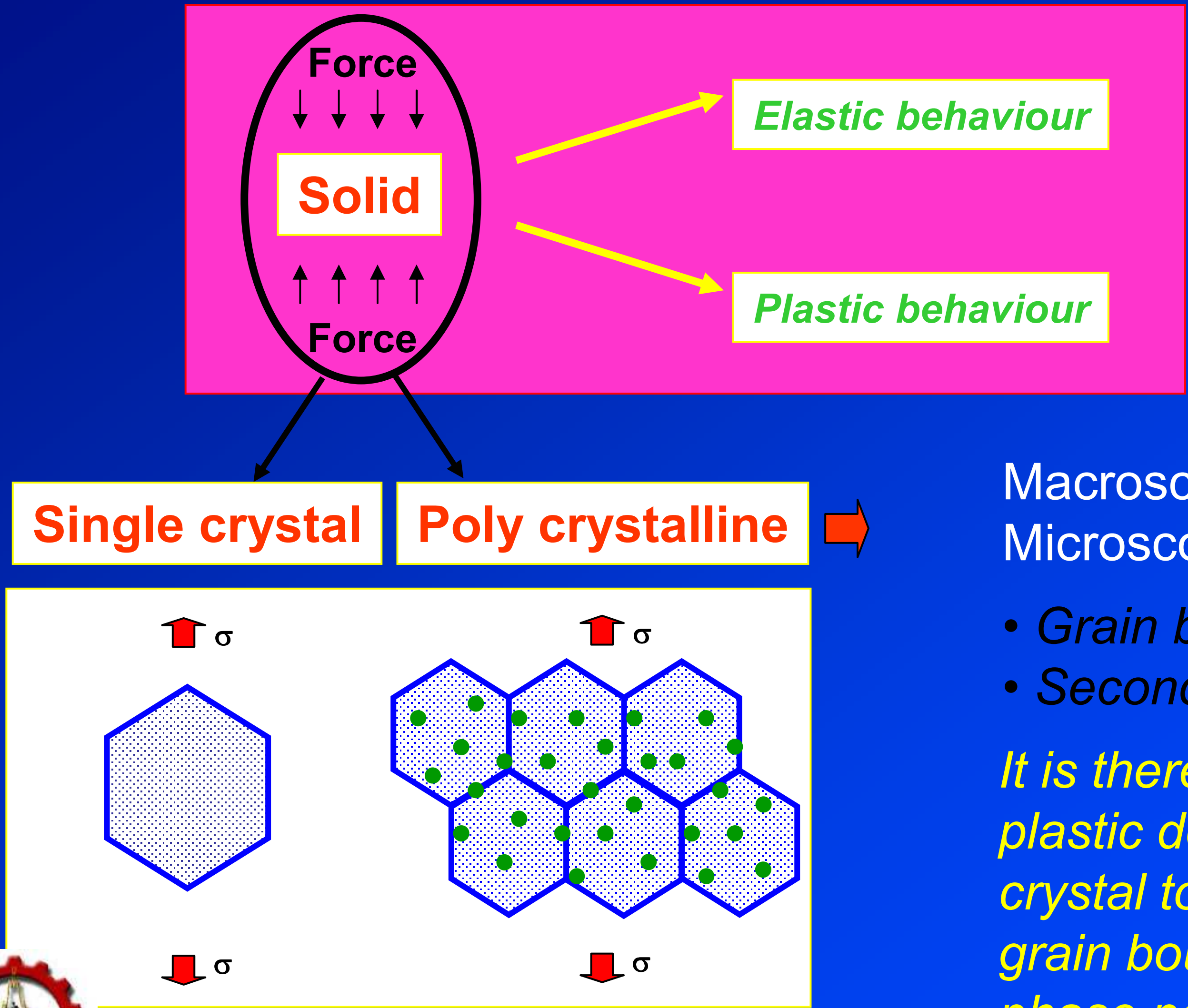
- *Deformation of single crystals*
- *Deformation of face-centred cubic crystals*
- *Deformation by twinning*
- *Stacking faults*
- *Deformation bands and kink bands*
- *Microstrain behaviour*
- *Strain hardening of single crystals*

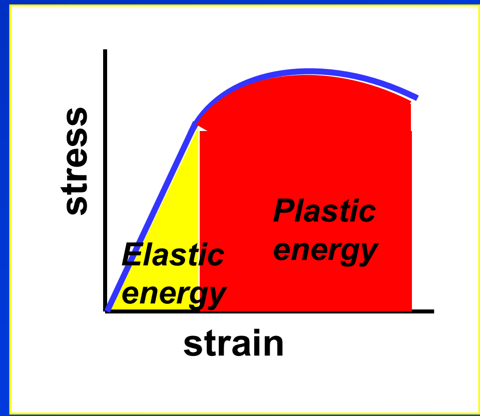
Objectives

• Metallurgical fundamentals on the plastic deformation of single crystal are provided in this chapter. This is, for example, the response of single crystal when subjected to external load.

• Different types of crystal defects and their influences on deformation behaviour of materials will also be discussed.

Introduction





Macroscopically homogeneous
Microscopically heterogeneous

- *Grain boundaries*
- *Second phase particles*

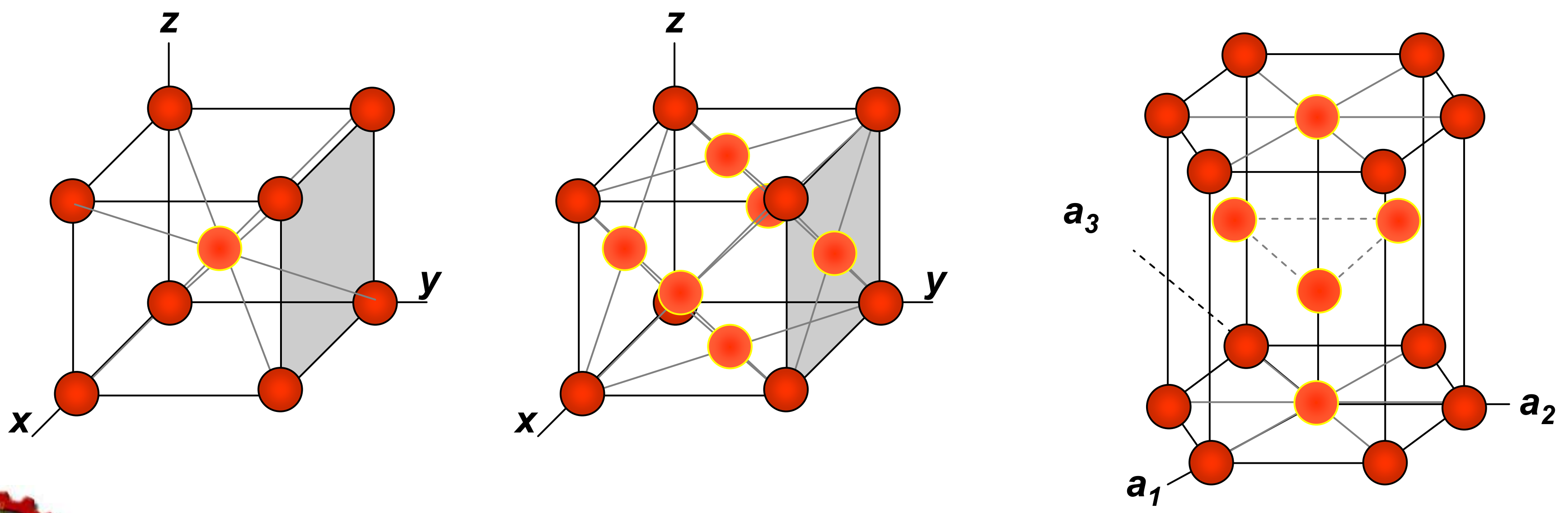
It is therefore easier to study plastic deformation in a single crystal to eliminate the effects of grain boundaries and second phase particles.

Concept of crystal geometry

Metal crystal consists of atoms arranged in a regular repeated three dimensional pattern.

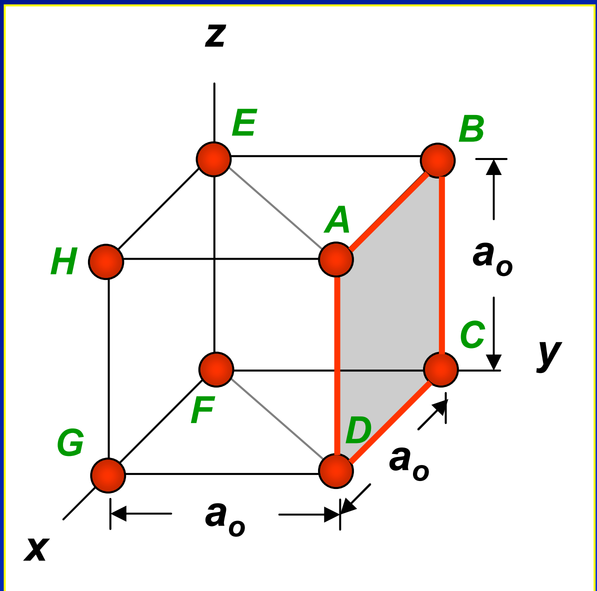
- There are three basic types of crystal structures in metals;

Body centre cubic (bcc)	Face centre cubic (fcc)	Hexagonal close packed (hcp)



Miller indices

***Miller indices** give the crystallographic information in terms of crystallographic planes and directions with respect to three principal axes.*



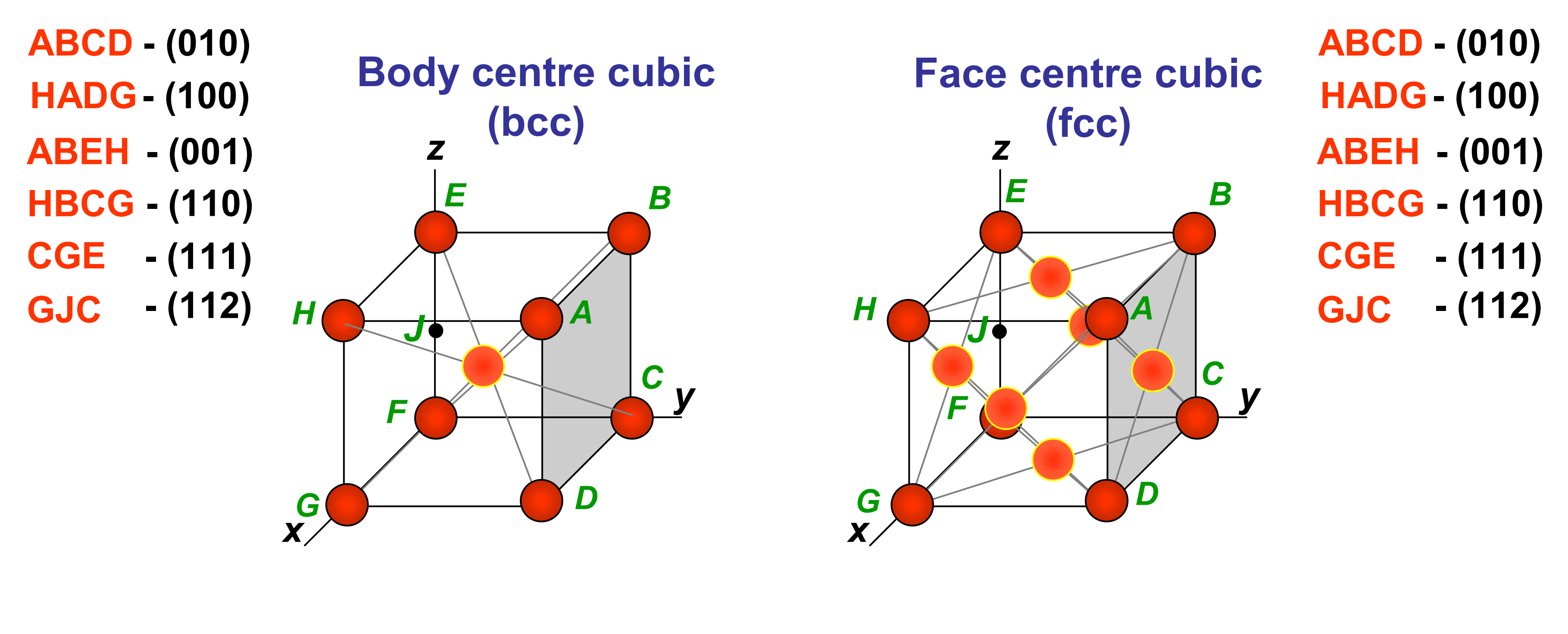
Simple cubic structure: NaCl

- A crystallographic plane is specified in terms of the ***length of its intercepts*** on the three axes.
- The reciprocal of these intercepts are used to identify the ***Miller indices*** **(hkl)** of the plane.
- ***Ex:*** plane ***ABCD*** is parallel to the ***x*** and ***z*** axes and intersect the ***y*** axis at one atomic distance a_o.→The indices of the plane are **1/∝ , 1/1, 1/∝ or (hkl) = (010).**
- There are ***six*** crystallographically equivalent planes on the ***cubic faces***;

(100)	(010)	(001)
$(\bar{1}00)$	$(0\bar{1}0)$	$(00\bar{1})$

→ $\{100\}$ **Family of planes**

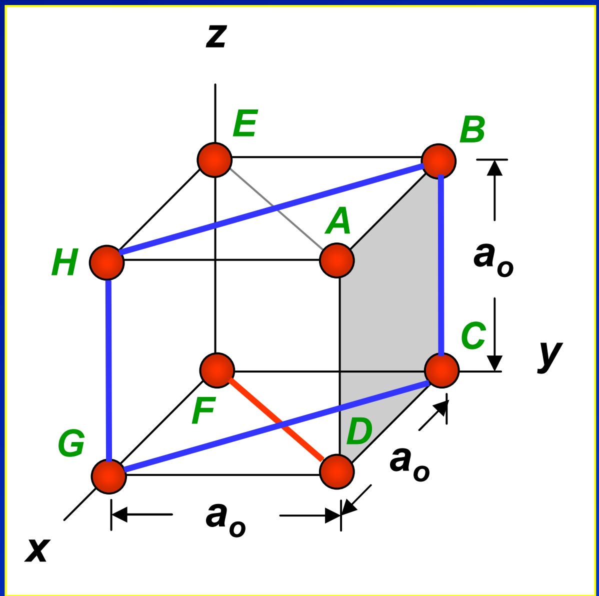
Crystallographic planes in cubic structures



Crystallographic planes in bcc and fcc structures.

Crystallographic directions

Crystallographic directions are indicated by *integers* in brankets: *[uvw].*



Simple cubic structure: NaCl

Ex: FD direction is obtained by moving out from the origin a positive distance a_o along the *x* and *y* axes.→ The direction indices are then **[110]** and the direction is always *perpendicular* to the plane having the same indices.

Ex: the **[110]** direction (*FD*) is perpendicular to **(110)** plane *BCGH*.

$$< 110 >$$
$$< uvw >$$

➡ **Family of directions**

Simple relationships between a direction and a plane

*For cubic system there are simple relationships between a **direction [uvw]** and a **plane (hkl).***

1) **[uvw]** is normal to **(hkl)** when $u = h; v = k, w = l$.
   [111] is normal to **(111).**
2) **[uvw]** is parallel to **(hkl),** when $hu + kv + lw = 0$.
3) Two planes $(h_1k_1l_1)$ and $(h_2k_2l_2)$ are normal if
   $h_1h_2 + k_1k_2 + l_1l_2 = 0$.
4) Two directions $u_1v_1w_2$ and $u_2v_2w_2$ are normal if
   $u_1u_2 + v_1v_2 + w_1w_2 = 0$.
5) Angles between planes $(h_1k_1l_1)$ and $(h_2k_2l_2)$ are given by

$$\cos\theta = \frac{h_1h_2 + k_1k_2 + l_1l_2}{(\sqrt{h_1^2 + k_1^2 + l_1^2})(\sqrt{h_2^2 + k_2^2 + l_2^2})}$$

Miller – Bravais system

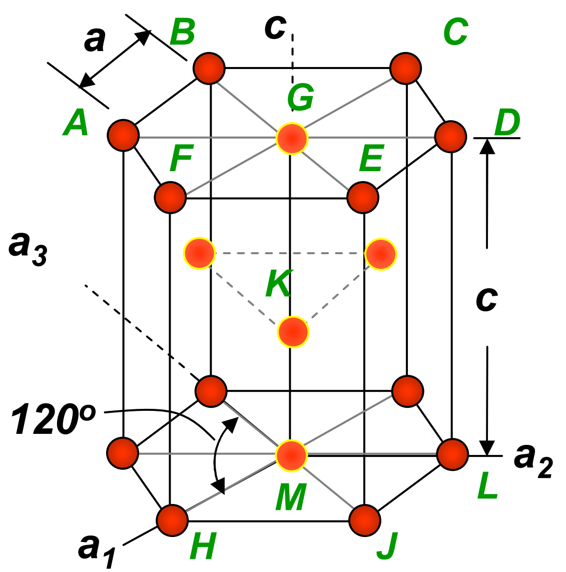
Miller Bravais indices *are used to specify planes and directions in the* ***hcp*** *structure, giving four indices* **(hkil).**

- ***These indices are based on four axes;***
  → three axes a_1, a_2, a_3 which are 120° apart in the basal plane.
  → the vertical c axis which is normal to the basal plane.

- The ***third index*** is related to the first two by the relation;

$$i = -(h + k)$$

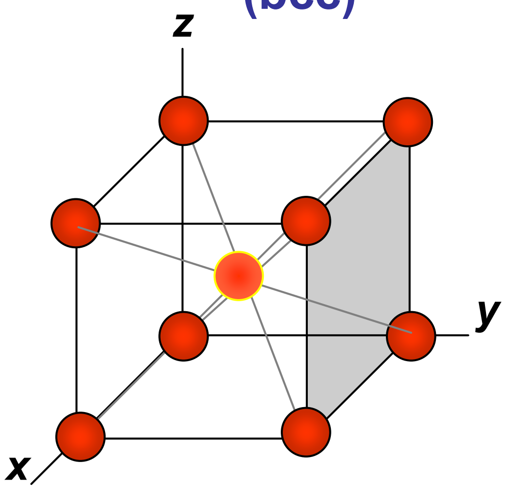
Ideal c/a is ~1.633



Basal plane ABCDEF	**- (0001)**
Prism plane FEJH	**- $(10\bar{1}0)$**
Pyramidal planes	
Type I, order 1 GHJ	**- $(10\bar{1}1)$**
Type I, order 2 KJH	**- $(10\bar{1}2)$**
Type II, order 1 GHL	**- $(11\bar{2}1)$**
Type II, order 2 KHL	**- $(11\bar{2}2)$**
Diagonal axis FCG	**- $(11\bar{2}0)$**

Number of atoms per unit cell

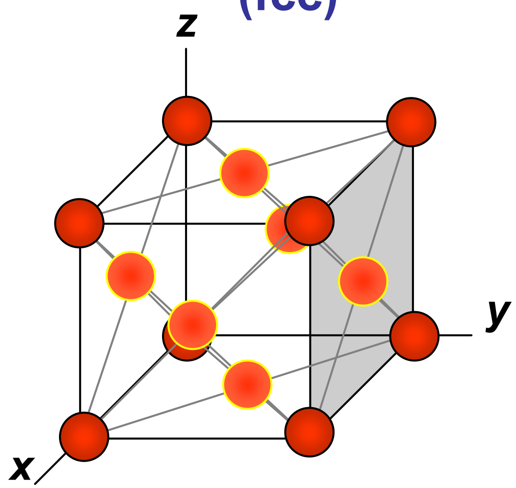
Body centre cubic (bcc)



Corners = 1/8 x 8 = 1 atom
Centre = 1 atom
Total atoms = 2 atoms

Ex: α - Fe, Ta, Cr, Mo, W

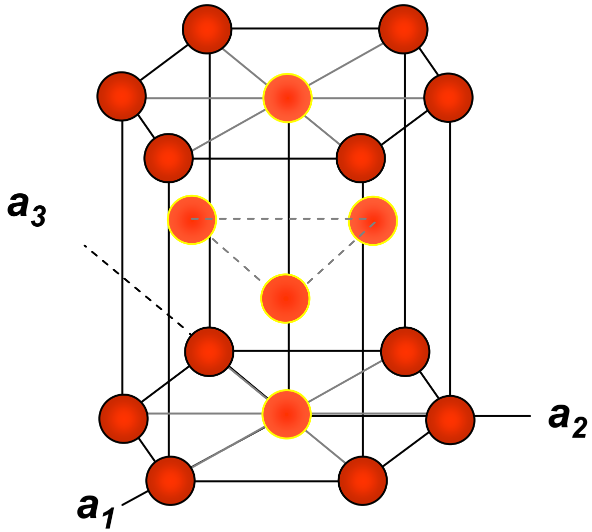
Face centre cubic (fcc)



Corners = 1/8 x 8 = 1 atom
Faces = 1/2 x 6 = 3 atoms
Total atoms = 4 atoms

Ex: Al, Cu, Pb, Au, Ni

Hexagonal close packed (hcp)



Corners = 1/6 x 6 = 1 atom
Centre = 1 atom
Total atoms = 2 atoms

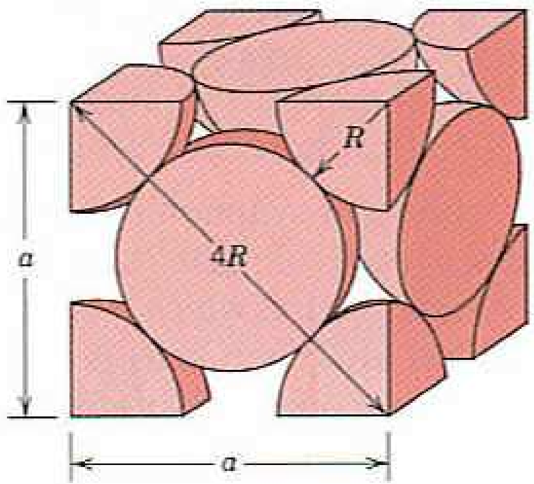
Ex: α - Ti , Zn, Mg, Cd

Atomic packing factor

Atomic packing factor (***APF***) is defined as the fraction of solid sphere volume in a unit cell.

$$APF = \frac{total\ \ sphere\ \ volume}{total\ \ unit\ \ cell\ \ volume} = \frac{V_s}{V_c}$$

FCC structure



volume = a^3

face diagonal length = 4R.

$$a^2 + a^2 = (4R)^2$$

$$a = 2\sqrt{2}R$$

FCC unit cell volume V_c ;

$$V_c = a^3 = (2\sqrt{2}R)^3 = 16\sqrt{2}R^3$$

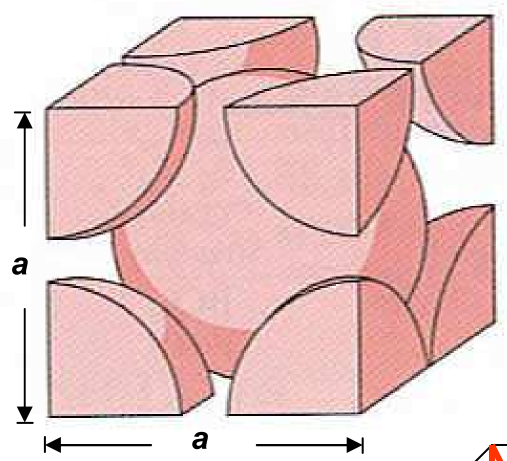
Total ***FCC*** sphere volume ;

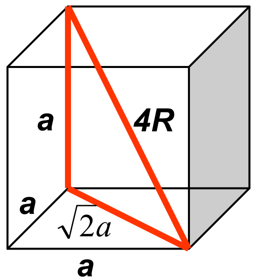
$$V_s = (4)\frac{4}{3}\pi R^3 = \frac{16}{3}\pi^3$$

$$APF = \frac{V_s}{V_c} = \frac{\left(\frac{16}{3}\right)\pi R^3}{16\sqrt{2}R^3} = 0.74$$

Atomic packing factor

BCC structure





volume = a^3

Diagonal length = 4R.

$$a^2 + 2a^2 = (4R)^2$$

$$a = 4R/\sqrt{3}$$

FCC unit cell volume $\boldsymbol{V_c}$;

$$V_c = a^3 = (4R/\sqrt{3})^3$$

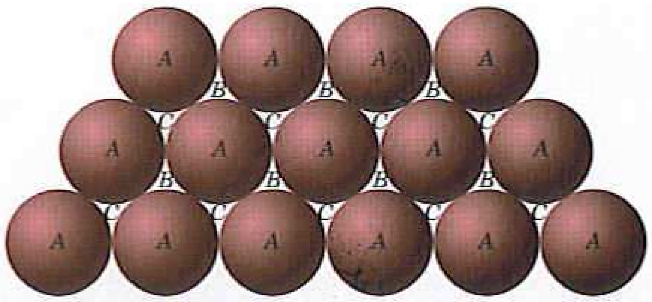
Total ***FCC*** sphere volume ;

$$V_s = (2)\frac{4}{3}\pi R^3 = \frac{8}{3}\pi R^3$$

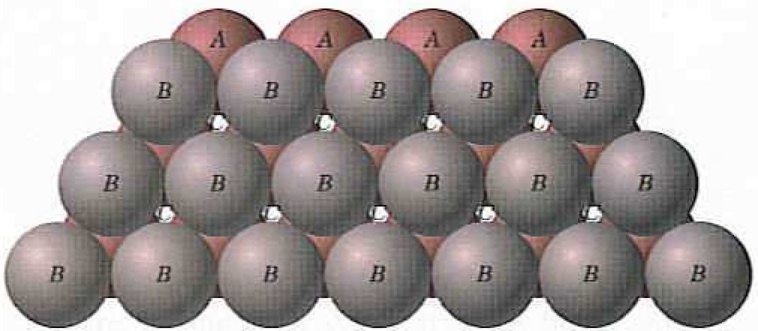
$$APF = \frac{V_s}{V_c} = \frac{\left(\frac{8}{3}\right)\pi R^3}{(4R/\sqrt{3})^3} = 0.68$$

Close packed structures

The ***fcc*** and the ***hcp*** structures are both ***close-packed structures*** ***APF*** = 0.74, whereas a ***bcc*** structure has ***APF*** = 0.68 and a ***simple cubic*** unit has ***APF*** = 0.52.

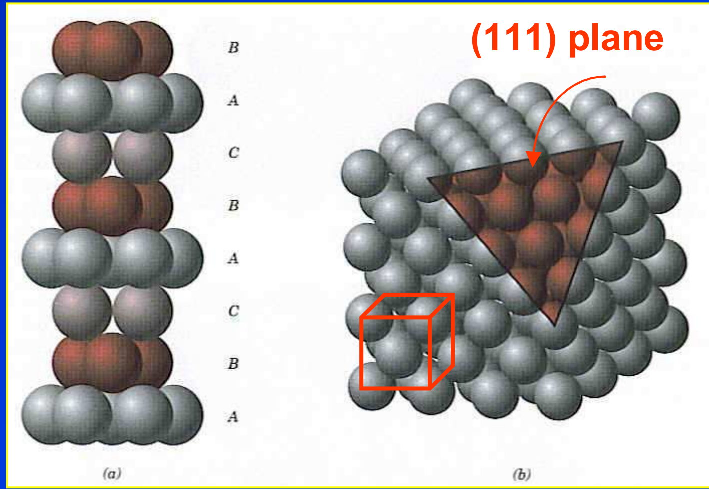


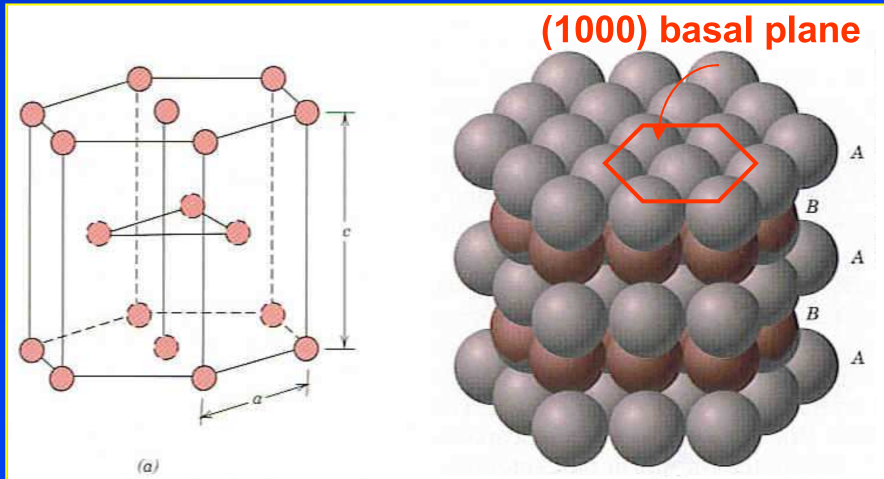
(a) Close-packed plane stacking ABC



(b) Close-packed plane stacking AB

Close-packed plane stacking sequence of FCC structure. ABCABC → {111} plane.

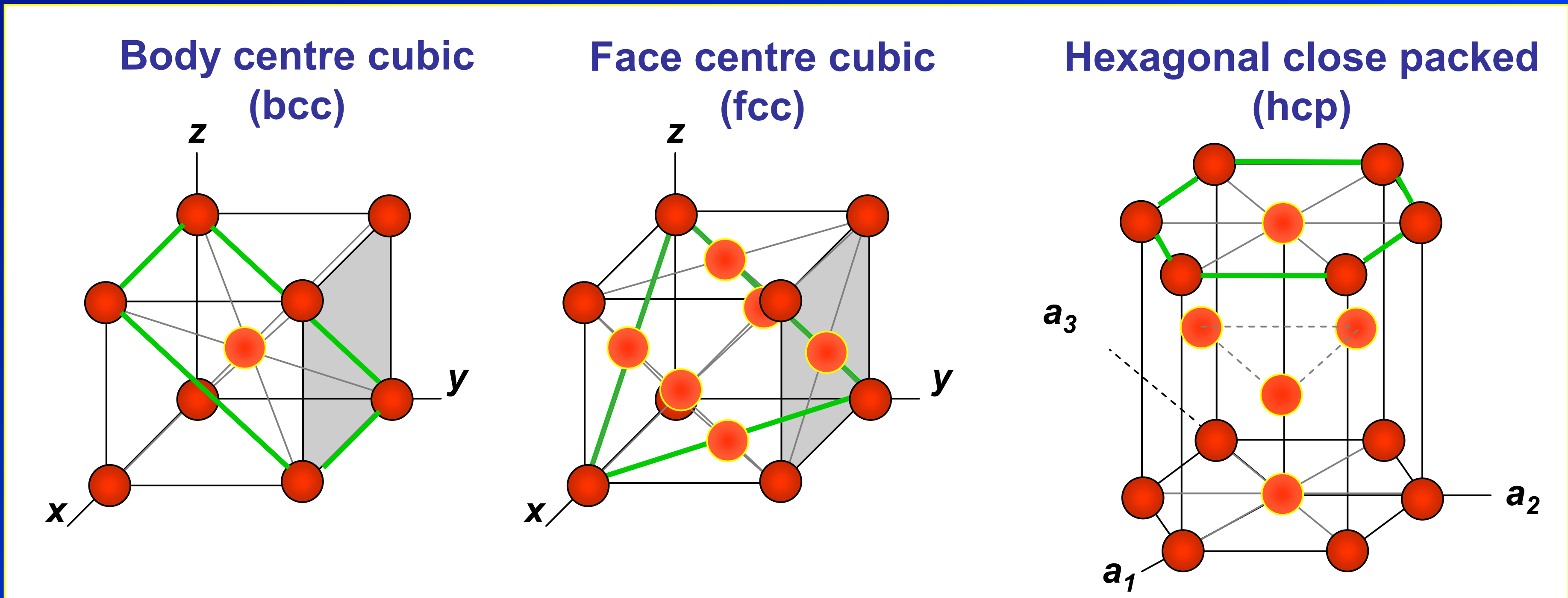




Close-packed plane stacking sequence of HCP structure. ABAB → (1000) basal plane.

Slip plane (low-index plane)

- Plastic deformation is generally confined to the ***low-index planes***, which has ***higher density of atom per unit area***.
- The planes of greatest atomic density also are the most widely spaced planes for the crystal structure.



Lattice defects

***Real crystal** is not perfect and has some defects.*
*In real materials → **structural sensitive**.*

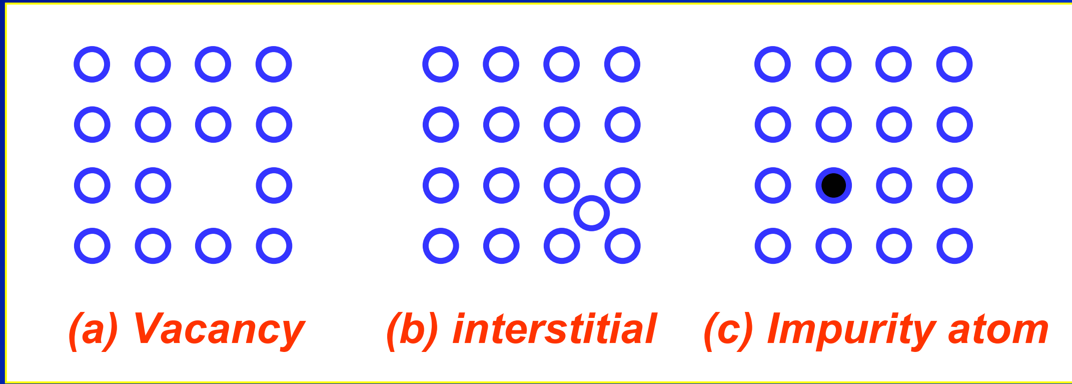
- All the ***mechanical properties*** are structural sensitive properties, i.e., yield stress, fracture strength, ductility etc.

- ***Defect or imperfection*** is used to describe any deviation from an orderly array of lattice points, which can be divided into;

1) Point defects

2) Line defects – dislocations

Point defects



a) Vacancy : an atom is ***missing*** from a normal lattice position. ***Due to*** :thermal excitation, extensive plastic deformation, high-energy particle bombardment.

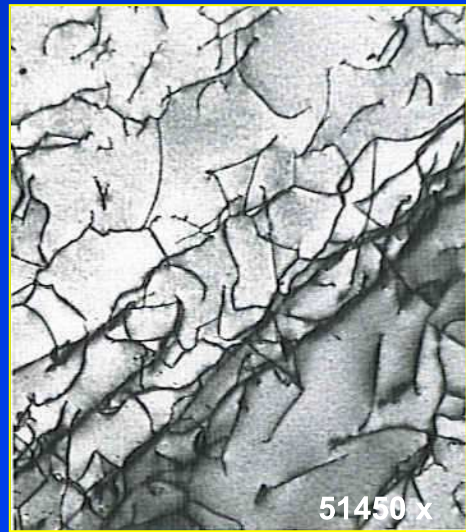
b) Interstitial atom : an atom that is ***trapped*** inside the crystal at a point intermediate between normal lattice positions. ***Due to*** radiation damage.

c) Impurity atom : Impurity atom which is present in the lattice, resulting in local disturbance of the lattice.

Line defects - dislocations

***Dislocation** is a linear or one-dimensional defect around with some of the atoms are misaligned.*

- ***Dislocations*** are responsible for the ***slip phenomenon***, by which most metals deform plastically.
- ***Dislocations*** are also intimately connected with nearly all other mechanical properties such as strain hardening, yield point, creep, fatigue and brittle fracture.



TEM of a Ti alloy (dark lines are dislocations)

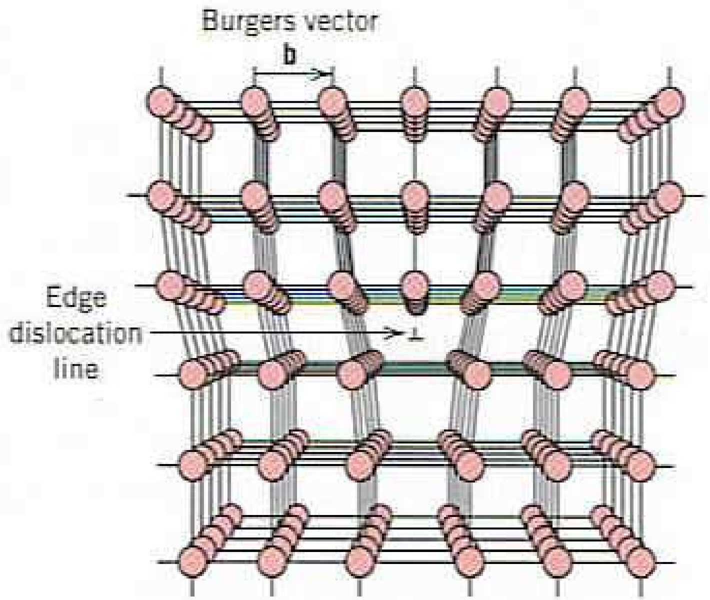
There are two basic types of dislocations;

1) ***Edge dislocation***
2) ***Screw dislocation***

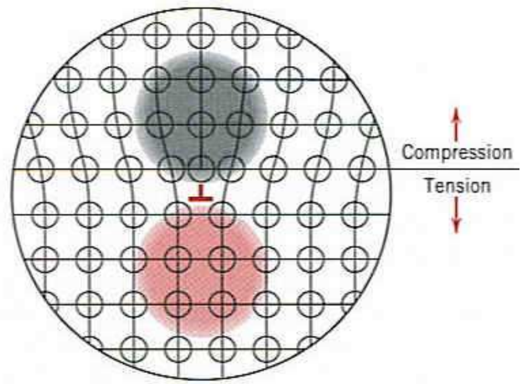
Edge dislocation

Edge dislocation *is a linear defect that centres around the line that is defined along the end of the extra portion of a plane of atoms (half plane),*

- Atoms above dislocation line are squeezed together (compressive), while those below are pulled apart (tensile), causing ***localised lattice distortion***.



Positive ⊥ **when the extra plane is *above* the slip plane.**
Negative ⊤ **when the extra plane is *below* the slip plane.**



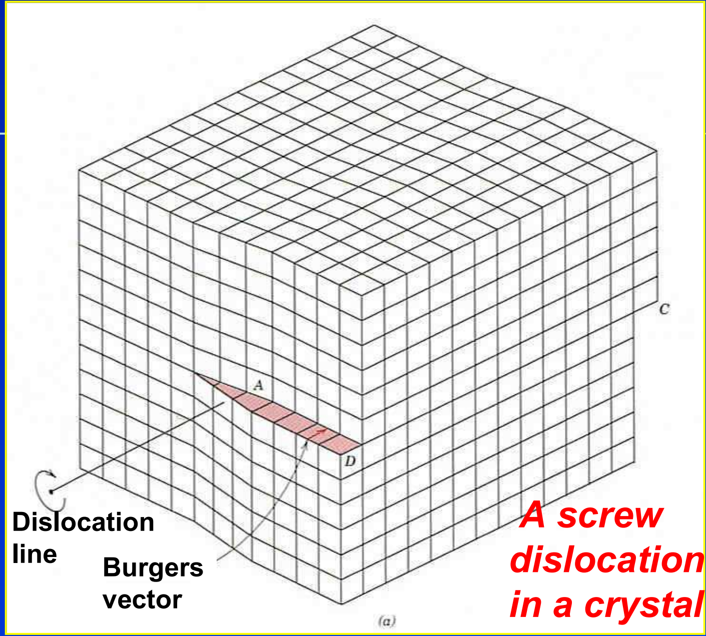
Atomic arrangement in a plane normal to an edge dislocation.

- The amount of displacement = the ***Burgers vector* b** of the dislocation, which is always perpendicular to the dislocation line.

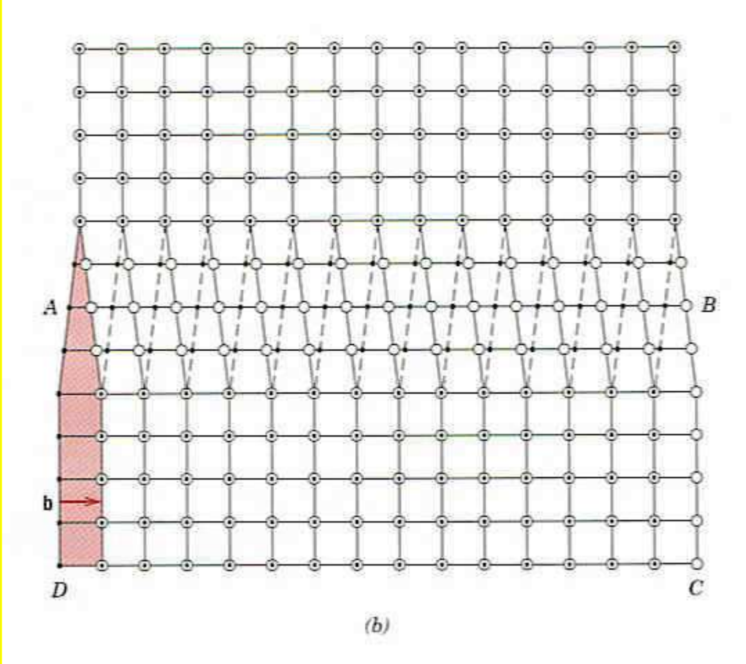
Screw dislocations

***Screw dislocation** may be thought of as being formed by applying a shear stress to produce a distortion.*

- The atomic distortion (a shift of one atomic distance to the right) is linear along the ***dislocation line***.
- The ***dislocation line*** is parallel to its ***Burgers vector*** **b** or slip vector.
- The symbol ↻ is sometimes used to designated the screw dislocation.
- Every time a circuit is made around the dislocation line, the end point is displaced one plane parallel to the slip plane in the lattice. → resulting in a ***spiral or staircase or screw***.

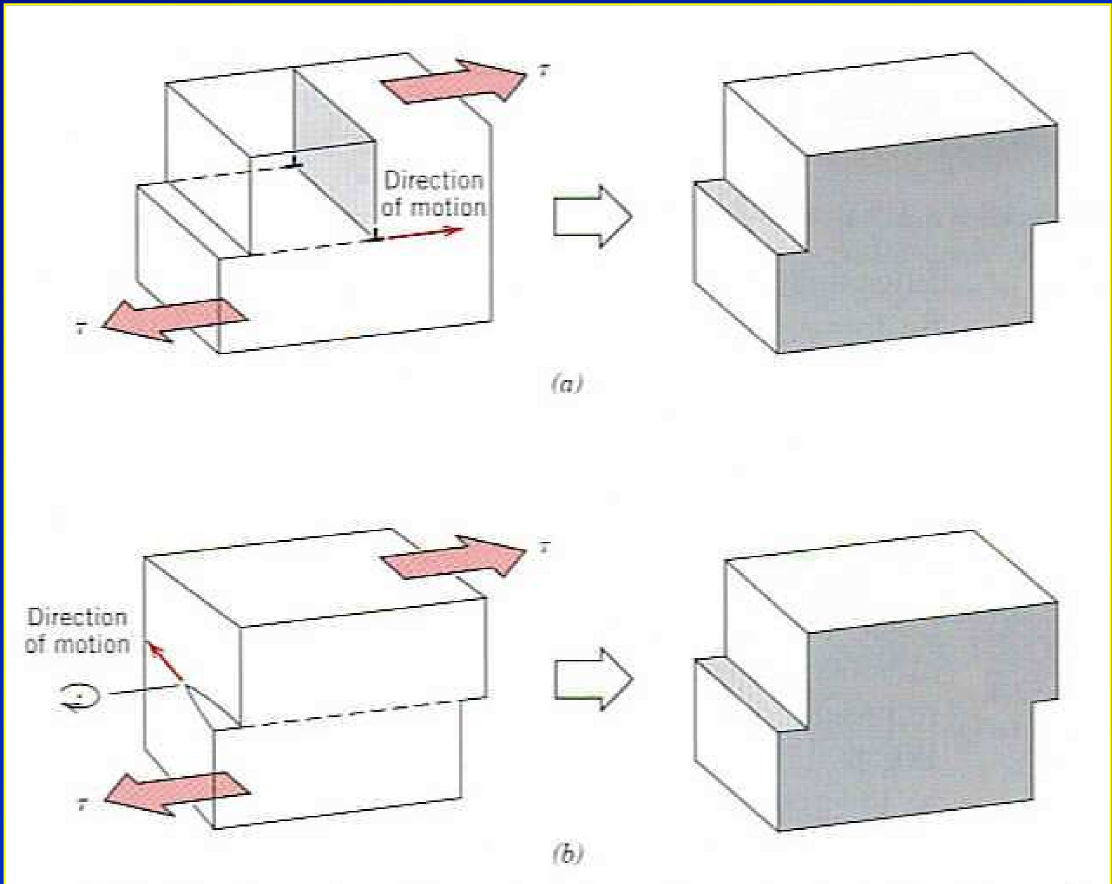


A screw dislocation in a crystal



Atomic arrangement around the screw dislocation (top view).

Movement of edge and screw dislocations



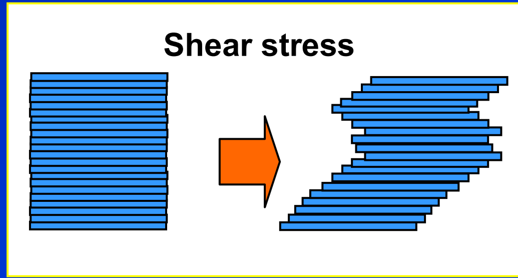
Formation of a step on the surface of a crystal by motion of

(a) An edge dislocation: the dislocation line moves in the direction of the applied shear stress τ.

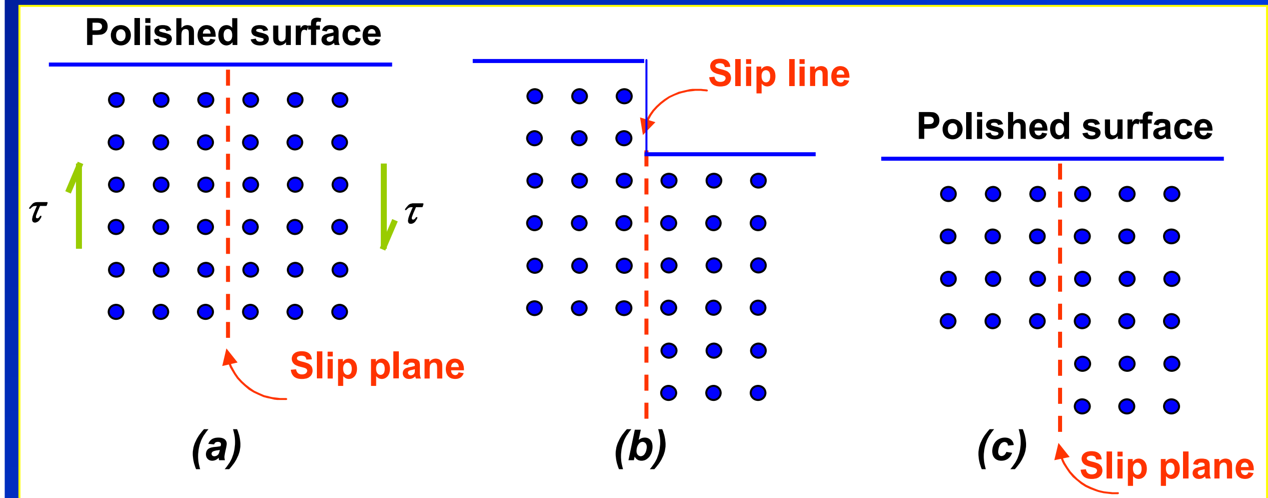
(b) A screw dislocation: the dislocation line motion is perpendicular to the stress direction.

Deformation by slip

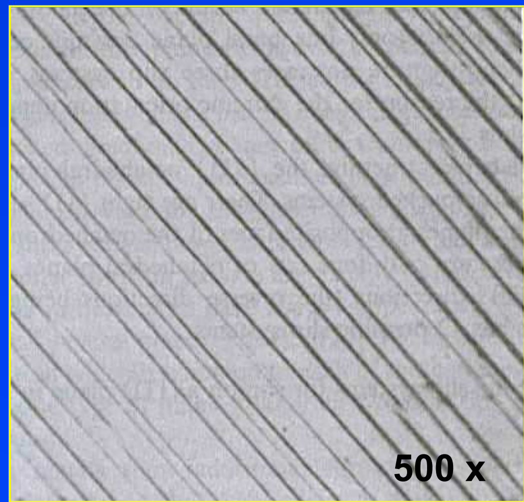
• Plastic deformation in metals is produced by movement of dislocations or slips, which can be considered analogous to the ***distortion produced in a deck of cards***.



• Slip occurs when the ***shear stress exceeds a critical value***. The atoms move an integral number of atomic distances along the slip plane, as shown in slip lines.



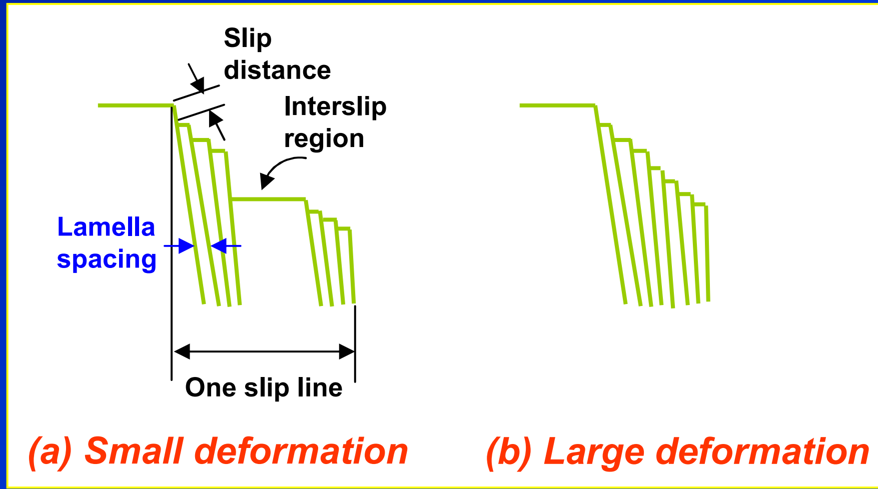
Classical ideal of slip



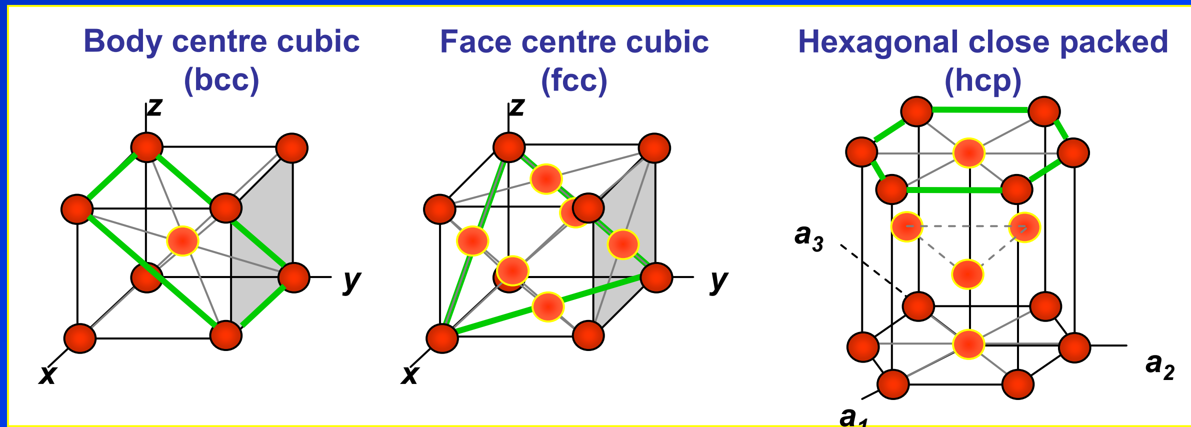
Straight slip lines in copper

Slip bands

- At high magnification, discrete slip lamellae can be found as shown.
- Slip occurs most readily on certain crystallographic plane or ***slip plane : the plane of greatest atomic density*** or low index plane and in the close packed direction.
- ***BCC*** structure: {110}, {112}, {123} planes and always in <111> direction.
- ***FCC*** structure: {111} plane and in <110> direction.
- ***HCP*** structure: (0001) basal plane and in <1120> direction.



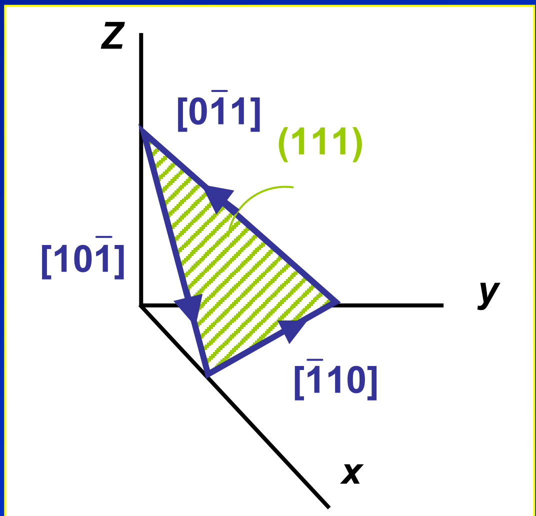
Schematic drawing of fine structure of a slip band



Slip planes in bcc, fcc and hcp structures

Example: *Determine the slip systems for slip on a (111) plane in a FCC crystal and sketch the result.*

Slip direction in ***fcc*** is <110> type direction. Slip directions are most easily established from a sketch of the (111) plane. To prove that these slip directions lie in the slip plane ***hu + kv + lw = 0***. → when **[uvw] // (hkl)**

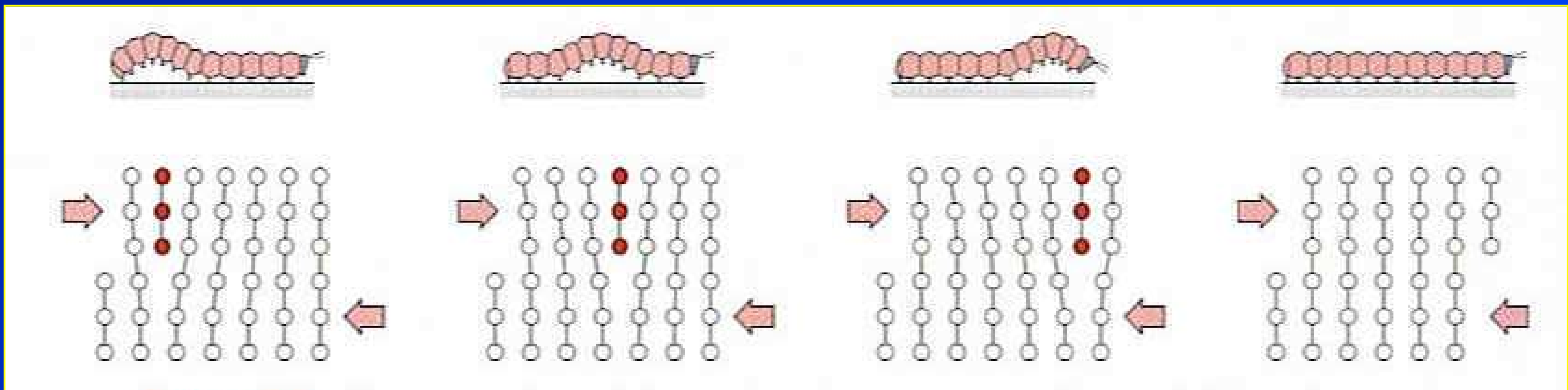


<110> → $[10\bar{1}]$, $[\bar{1}10]$, $[0\bar{1}1]$

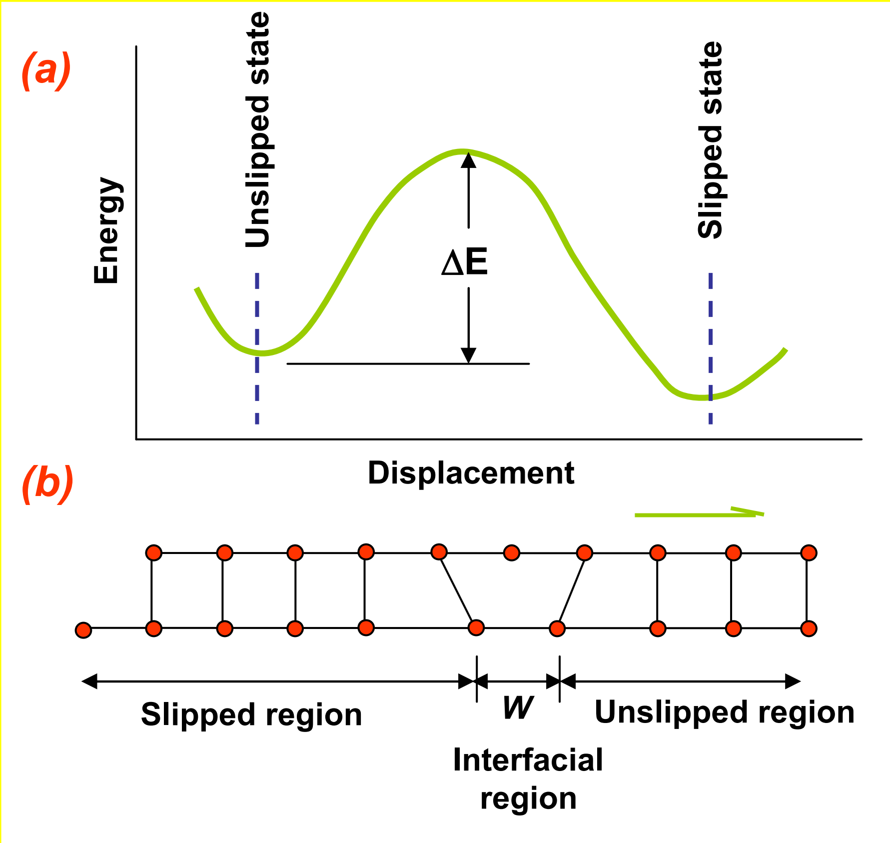
$$(1)(1)+(1)(0)+(1)(-1)=0$$
$$(1)(-1)+(1)(1)+(1)(0)=0$$
$$(1)(0)+(1)(-1)+(1)(1)=0$$

Slip by dislocation motion

- ***Slip*** is a plastic deformation process produced by ***dislocation motion***.
- ***Dislocation motion*** is analogous to the caterpillar movement model.
- The caterpillar forms a ***hump*** with its position and movement corresponding to those of ***extra-half plane*** in the dislocation model.



Energy change in slip



- *Cottrell* considers that plastic deformation is the transition from an unslipped to a slipped state by overcoming an energy barrier ΔE.
- The interfacial region is dislocation of the width ***w***.
- ***w*** ⇩ interfacial energy ⇩

  elastic energy ⇧

In ductile metals, the dislocation width is ~ 10 atomic spacing.

When the crystal is complex without highly close-packed planes and directions, dislocation tends to be ***immobile → brittleness.***

Critical resolved shear stress for slip

The ***extent of slip*** in a single crystal depending on:

1) The magnitude of the ***shear stress***
2) The geometry of the crystal structure
3) The number of active slip plane in the shear stress direction.

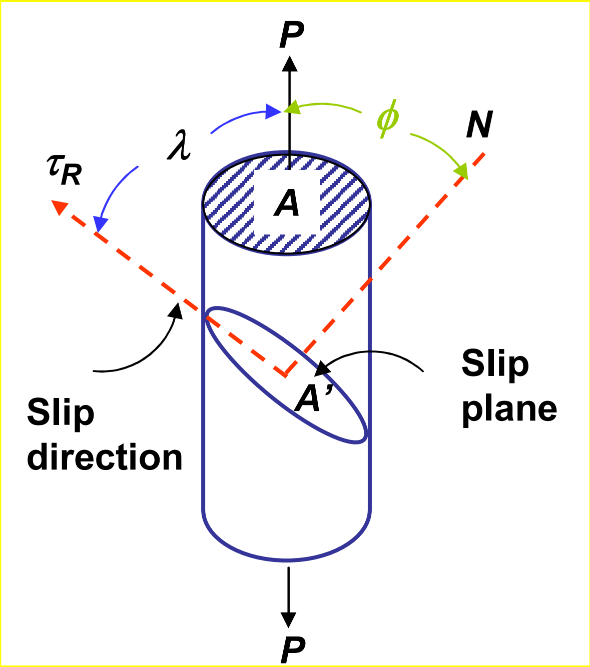
Slip occurs when the shearing stress on the slip plane in the slip direction reaches a critical resolved shear stress.

- ***Schmid*** calculated the ***critical resolved shear stress*** from a single crystal tested in tension.
- The area of the slip plane ***A' = A/cosϕ***.
- The force acting in the lip plane ***A'*** is ***Pcosλ***.
- The critical resolved shear stress is given by

…Eq. 1

$$\tau_R = \frac{P\cos\lambda}{A/\cos\phi} = \frac{P}{A}\cos\phi\cos\lambda$$

The maximum τ_R is when $\phi = \lambda = 45^o$.
Or $\tau_R = 0$ when $\phi = 0$ or $\lambda = 0$.



Example: Determine the tensile stress that is applied along the $[1\bar{1}0]$ axis of a silver crystal to cause slip on the $(1\bar{1}\bar{1})[0\bar{1}1]$ system. The critical resolved shear stress is 6 MPa.

The angle between the tensile axis $[1\bar{1}0]$ and normal to $(1\bar{1}\bar{1})$ is

$$\cos\phi = \frac{(1)(1)+(-1)(-1)+(0)(-1)}{\sqrt{(1)^2+(-1)^2+(0)^2}\sqrt{(1)^2+(-1)^2+(-1)^2}} = \frac{2}{\sqrt{2}\sqrt{3}} = \frac{2}{\sqrt{6}}$$

The angle between the tensile axis $[1\bar{1}0]$ and slip direction $[0\bar{1}1]$ is

$$\cos\lambda = \frac{(1)(0)+(-1)(-1)+(0)(-1)}{\sqrt{2}\sqrt{(0)^2+(-1)^2+(1)^2}} = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$$

From Eq.1

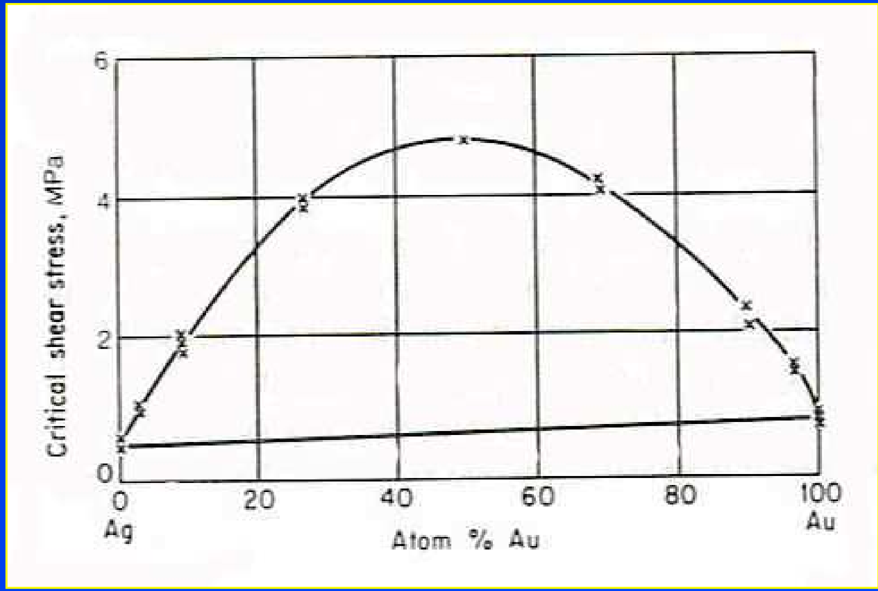
$$\sigma = \frac{P}{A} = \frac{\tau_R}{\cos\phi\cos\lambda} = \frac{6}{2\sqrt{6}\times\frac{1}{2}} = 14.7\ \ MPa$$

Critical resolved shear stress in real metals

Defects Vacancies Impurity atoms Alloying elements	↑	Critical resolved shear stress	↑

The ratio of the resolved shear stress to the axial stress is called the ***Schmid factor m***.

$$Schmid\ \ factor\ \ m = \cos\phi\cos\lambda$$

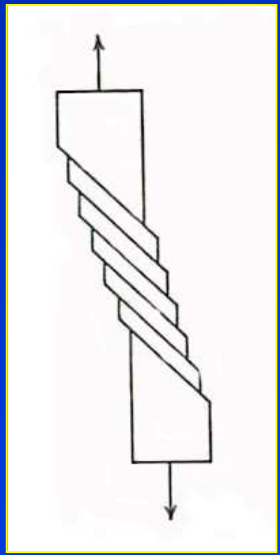


Variation of critical resolved shear stress with composition in Ag-Au alloy.

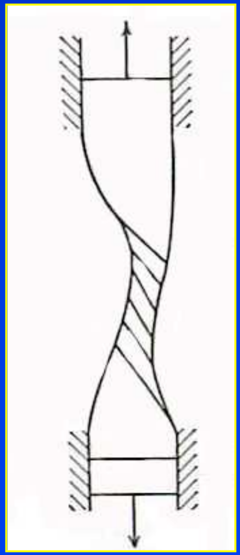
Deformation of single crystals

• When a single crystal is deformed freely by uniform glide on every slip plane along the gauge length ***without constraint***.

• In uniaxial tension, the grips provide constraint making the ***slip planes to rotate toward the tensile axis***.



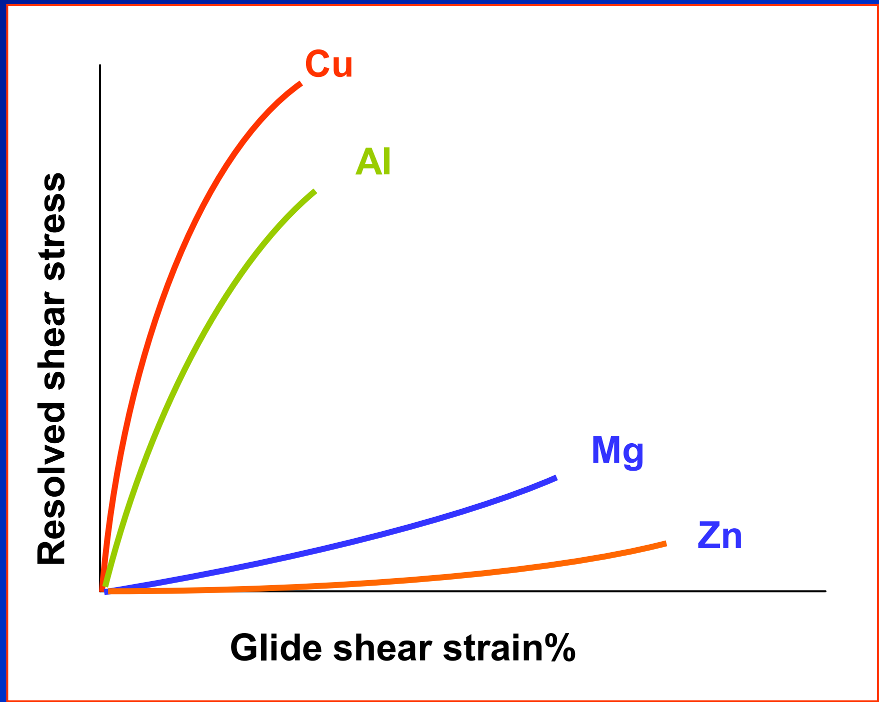
(a) Tensile deformation of single crystal without constraint.



(b) Rotation of slip planes due to constraint.

• *The increase in length of the specimen depends on the orientations of the active slip planes and the direction with the specimen axis.*

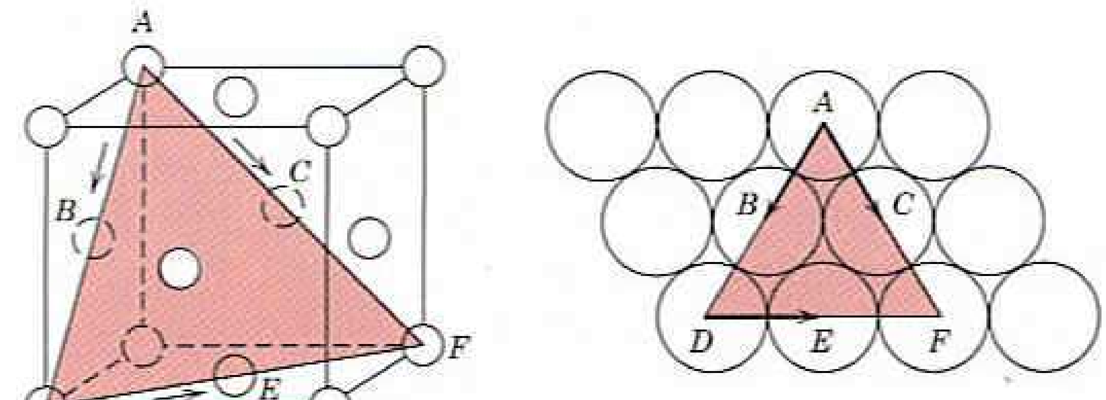
Single crystal stress-strain curves



Typical single-crystal stress-strain curves.

FCC metals exhibit greater strain hardening than ***HCP*** metals.

Deformation of FCC crystals



(a) A {111}<110> slip system in FCC unit cell.

(b) The (111) plane from (a) and three <110> slip directions.

Slip system in ***FCC*** metals is **{111}<110>.** → 4 sets of octahedral **{111}** planes and each of which has 3 **<110>** directions. → 12 potential slip systems.

Slip systems for FCC, BCC and HCP metals

Table 1 Slip Systems for Face-Centered Cubic, Body-Centered Cubic, and Hexagonal Close-Packed Metals

Metals	*Slip Plane*	*Slip Direction*	*Number of Slip Systems*
	Face-Centered Cubic		
Cu, Al, Ni, Ag, Au	$\{111\}$	$\langle 1\bar{1}0\rangle$	12
	Body-Centered Cubic		
α-Fe, W, Mo	$\{110\}$	$\langle \bar{1}11\rangle$	12
α-Fe, W	$\{211\}$	$\langle \bar{1}11\rangle$	12
α-Fe, K	$\{321\}$	$\langle \bar{1}11\rangle$	24
	Hexagonal Close-Packed		
Cd, Zn, Mg, Ti, Be	$\{0001\}$	$\langle 11\bar{2}0\rangle$	3
Ti, Mg, Zr	$\{10\bar{1}0\}$	$\langle 11\bar{2}0\rangle$	3
Ti, Mg	$\{10\bar{1}1\}$	$\langle 11\bar{2}0\rangle$	6

Metals with ***FCC*** and ***BCC*** crystal structures have a relatively large number of slip systems (at least 12). → Extensive plastic deformation → **Ductile**

Metals with ***HCP*** crystal structure have few active slip systems. → Less plastic deformation → **Brittle**

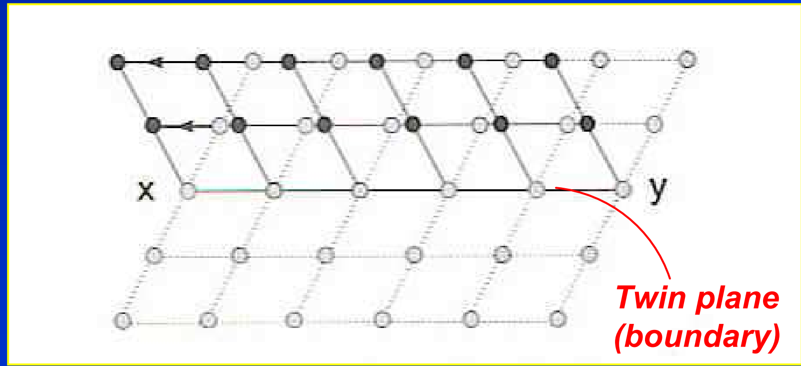
Deformation by twinning

Twinning occurs as atoms on one side of the boundary (plane) are located in ***mirror image positions*** of the atoms on the other side. The boundary is called ***twinning boundary***.

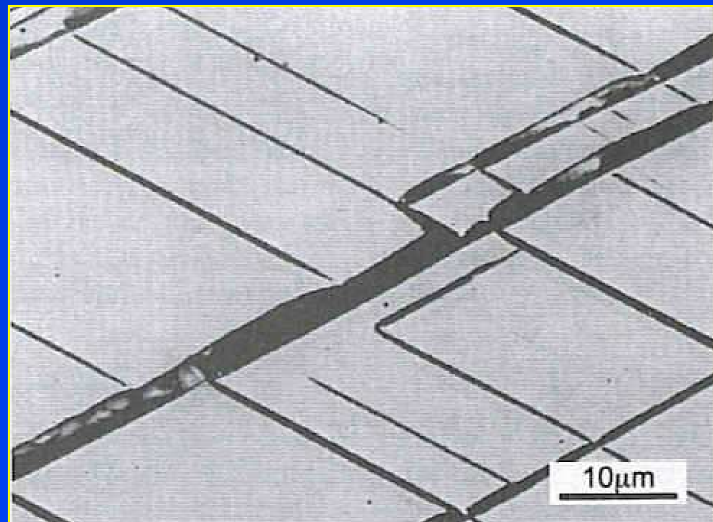
Twin results from atomic displacements produced from;

1) Applied mechanical shear force (***mechanical twin***) : in ***BCC, HCP***
2) During annealing heat treatment ***(annealing twin)*** : in ***FCC***.

<u>Note:</u> twinning normally occurs when slip systems are restricted or when the twinning stress > critical resolved shear stress.



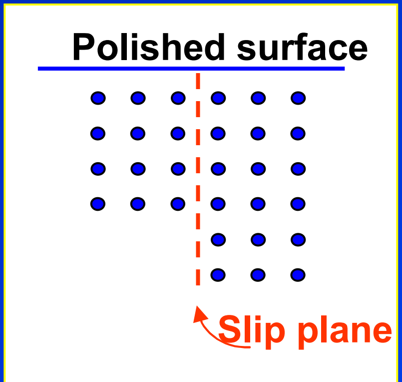
Schematic diagram of a twin plane and adjacent atom positions.

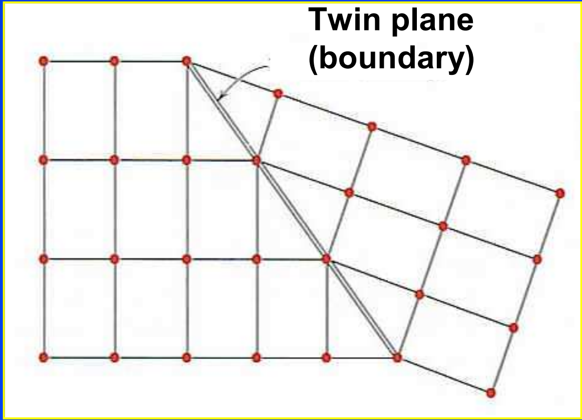


Deformation twins in 3.25% Si iron.

Comparisons of twinning and slip

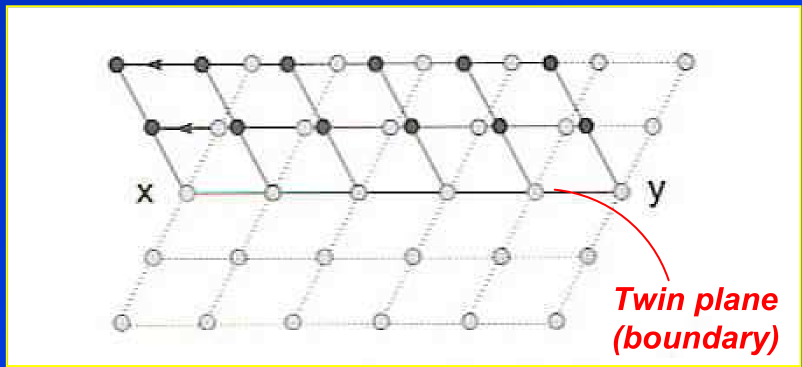
Slip	Twinning
• ***Similar*** orientations of the crystal above and below the slip plane.	• ***Different*** orientations of the crystal above and below the twinning plane.
• Slip normally occurs in discrete ***multiples of the atomic spacing***.	• Atom movements in twinning are much ***less than an atomic distance***.
• Slip occurs on relatively ***widely spaced plane***.	• Twinning occurs in a region of a crystal of ***every atomic plane*** involved in the deformation.





Other characteristics of twins

- Does not produce large amount of gross deformation due to ***small lattice strain***. → ***HCP*** metals therefore have low ductility.
- Does not largely contribute to plastic deformation but change the orientations which may place new (favourable) slip systems → ***additional slips can take place.***
- ***Twins*** do not extends beyond grain boundaries.
- The ***driving force*** for twinning is the ***applied shear stress***.



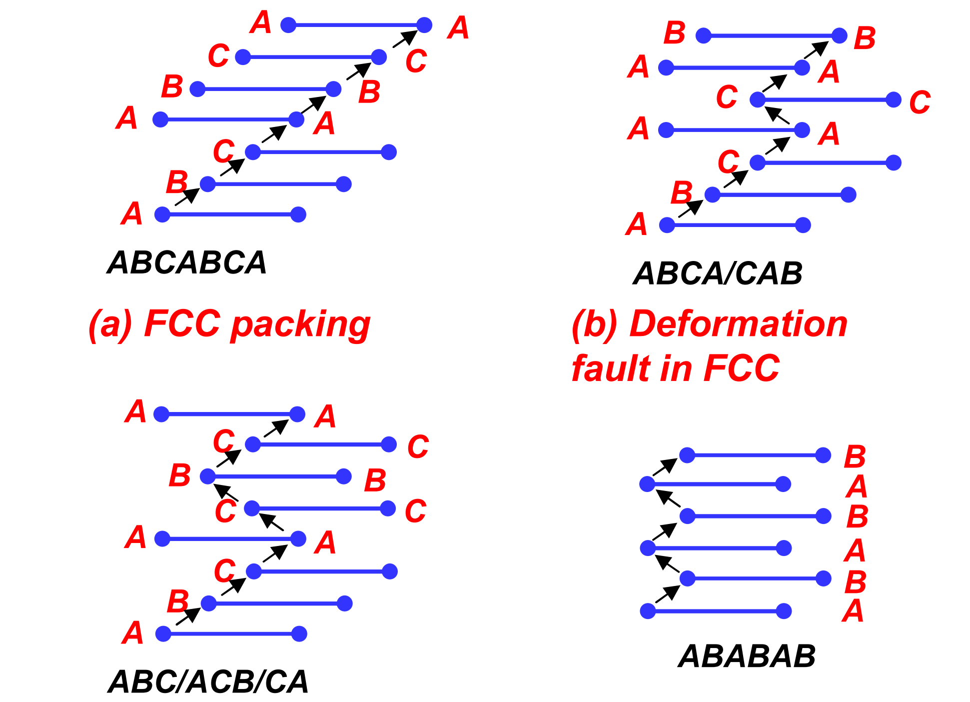
Schematic diagram of a twin plane and adjacent atom positions.

Stacking faults

Stacking faults can be found in metals when there is an interruption in the stacking sequence.

Examples:

- Stacking sequence in ***FCC*** is ABC ABC ABC … → ABC AC AB , *Fig (a) →(b).*
- Stacking sequence in ***HCP*** is AB AB AB … → AB BA AB , *Fig (d)*



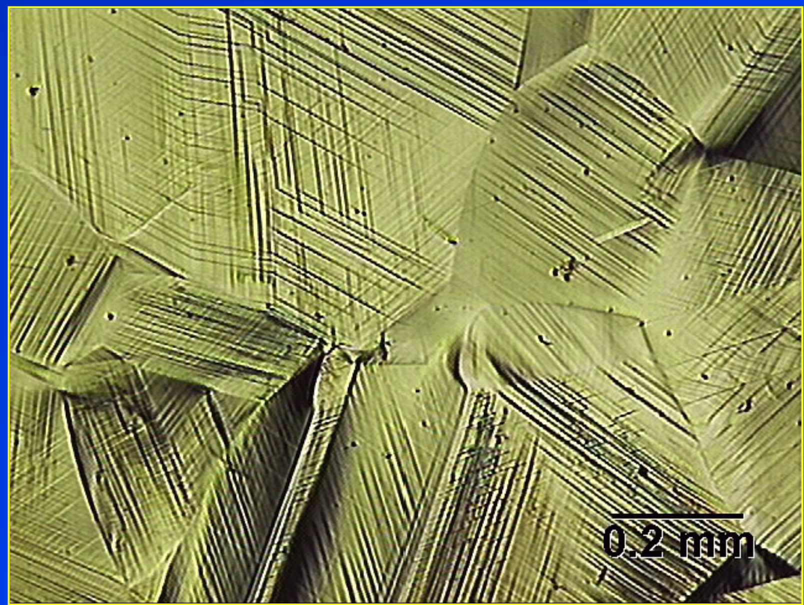
(a) FCC packing
(b) Deformation fault in FCC
(c) Twin fault in FCC
(d) HCP packing

Note: stacking faults influence plastic deformation.

Deformation bands

Deformation bands *consist of regions of different orientations and are formed when material is inhomogeneously deformed.*

- Polycrystalline specimens tend to form these deformation bands easier than a single crystal.
- Deformation bands are irregular in shape, poorly defined.
- Observed in ***FCC*** and ***BCC*** but not ***HCP***.



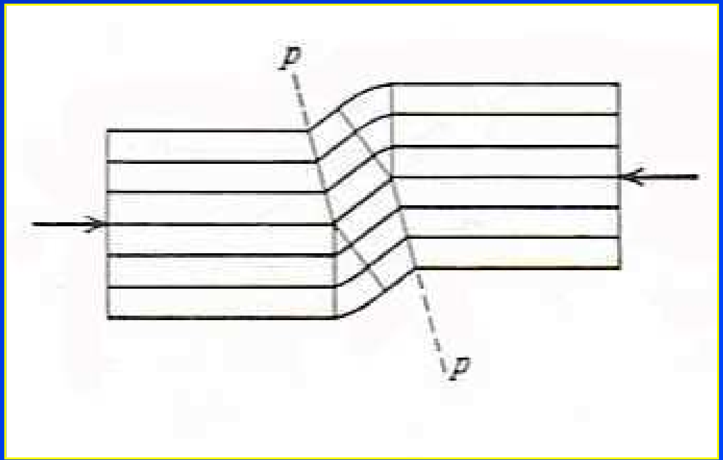
Deformation bands in specimen after tensile test

Kink bands

***Kinking or bulking** is observed when a **HCP** cadmium crystal is compressed with the basal plane nearly parallel to the crystal axis.*

- Horizontal lines represent ***basal planes*** and the planes designated ***p*** are the kink planes at which the orientation suddenly changes.

- The crystal is deformed by localised region and suddenly snapping into a tilted position with a sudden shorten of the crystal.



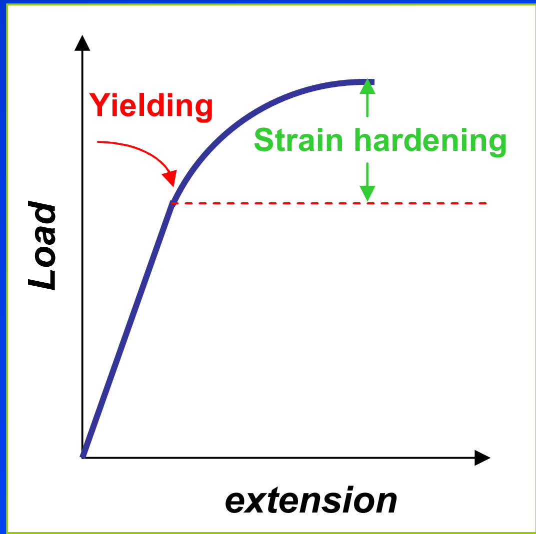
Kink band

Strain hardening of single crystals

Strain hardening *or work hardening is caused by* ***dislocations interacting*** *with each other and with barriers, which impede their motion through the crystal lattice.*

• ***Precipitate particles, foreign atoms*** serve as barriers which result in dislocation multiplication. → ***strain hardening***.

• ***Dislocation pile-ups at barriers*** produce a ***back stress*** which opposes the applied stress. → ***strain hardening***



• ***Dislocation density*** increases dramatically for example from 10^4 in annealed condition to 10^{10} in cold-worked condition.

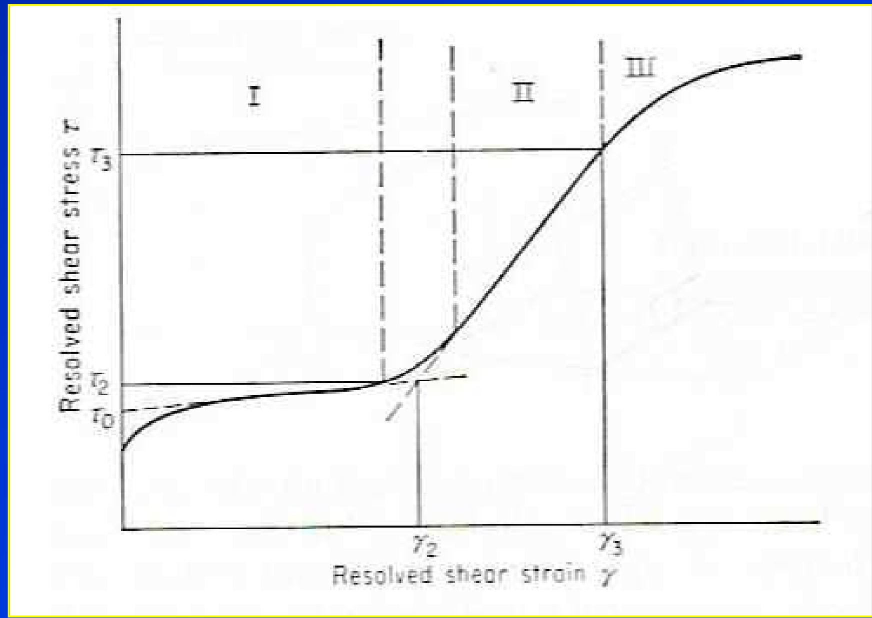
Flow curve for FCC single crystals

Stage I : *easy glide*

- Slips occur on only one slip system.
- Dislocation density is low.
- Crystal undergoes little strain hardening.
- Most dislocations escape from the crystal to the surface.

Stage II :

- Strain hardening occurs rapidly.
- Slips occur more than one set of planes. → much higher dislocation density.
- Dislocation tangles begin to develop.



Flow curve for FCC single crystal.

Stage I : *dynamic recovery*

- Decreasing rate of strain hardening.

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- W.D. Callister, ***Fundamental of materials science and engineering/ an interactive e. text***., 2001, John Willey & Sons, Inc., New York, ISBN 0-471-39551-x.
- Hull, D., Bacon, D.J., ***Introduction to dislocations***, 2001, Forth edition, Butterworth-Heinemann, ISBN 0-7506-4681-0.
- www.matsci.ucdavis.edu

Dislocation theory

Subjects of interest

- Introduction/Objectives
- Observation of dislocation
- Burgers vector and the dislocation loop
- Dislocation in the FCC, HCP and BCC lattice
- Stress fields and energies of dislocations
- Forces on dislocations and between dislocations

Dislocation theory

Subjects of interest (continued)

- *Dislocation climb*
- *Intersection of dislocations*
- *Jogs*
- *Dislocation sources*
- *Multiplication of dislocations*
- *Dislocation-point defect interactions*
- *Dislocation pile-ups*

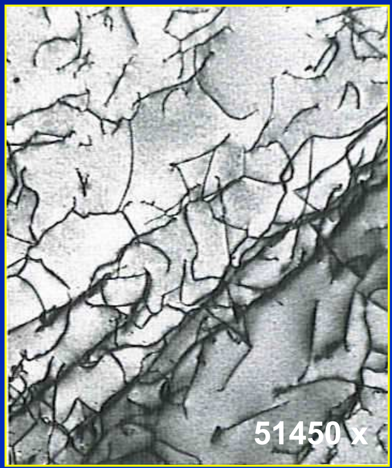
Objectives

• This chapter emphasises the understanding of the effects of dislocation behaviour on FCC, BCC and HCP crystal structures.

• This includes the interaction of dislocations such as climb, jogs, intersection and multiplication of dislocations and the roles of dislocations on plastic deformation of metals.

Introduction

Dislocations *introduce imperfection into the structure and therefore these could explain how real materials exhibit lower yield stress value than those observed in theory.*



Dislocations

Produce imperfection in crystal structures

- Lower the yield stress from theoretical values.
- Produce plastic deformation (strain hardening).
- Effects mechanical properties of materials.

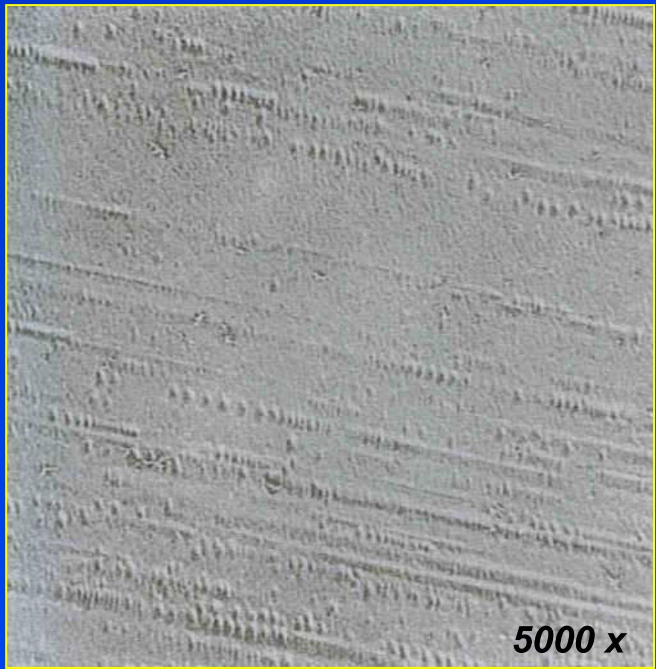
Observation of dislocations

A variety of techniques have been used to observe dislocations in the past 20 years to aid the better understanding of dislocation behaviour.

Chemical (etch–pit) technique

- Using ***etchant*** which forms a ***pit*** at the point where a dislocation intersect the surface.
- ***Preferential sites*** for chemical attack are due to ***strain field*** around dislocation sites (anodic).
- Can be used in bulk samples but limited in low dislocation density crystal (10^4 mm^{-2}).

Note: Pits are 500 A^o apart and with the dislocation density of 10^8 mm^{-2}.

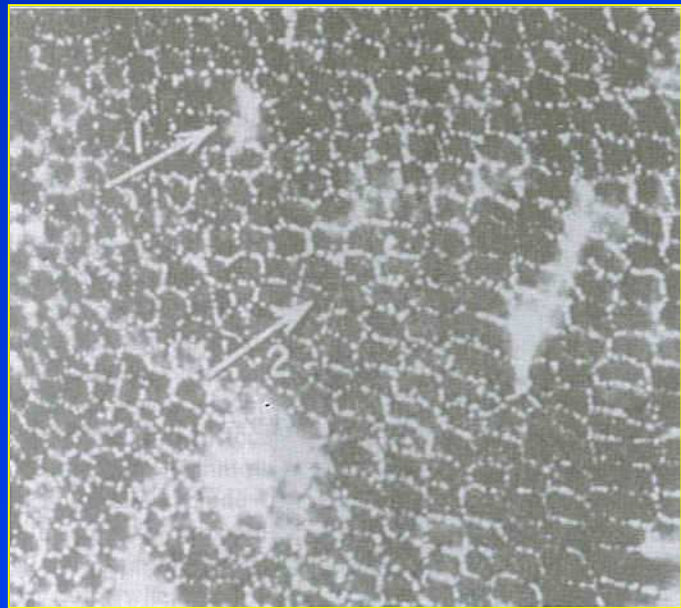


Etch pits on slip bands in alpha brass crystals

Decoration of dislocation technique

A small amount of impurity is added to form precipitates after suitable heat treatment to give internal structure of the dislocation lines.

- ***Hedges and Mitchell*** first used photolytic to decorate dislocation in ***AgBr.***
- Rarely used in metals but in ironic crystals such as ***AgCl, NaCl, KCl*** and ***CaF_2***.

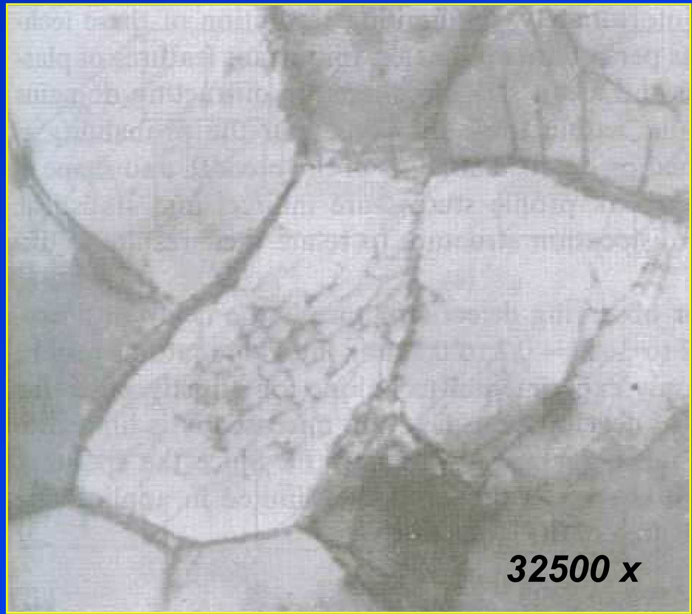


Hexagonal network of dislocations in NaCl detected by a decoration technique.

Transmission electron microscope (TEM)

***TEM** is the most powerful technique used to study dislocations.*

- A thin foil of 100 nm is prepared using electropolishing from a ~1 mm thick sheet.
- This thin foil is transparent to electrons in the electron microscope and this makes it possible to observed ***dislocation networks, stacking faults, dislocation pile-ups at grain boundaries***.
- By using the kinematic and dynamic theories of electron diffraction it is possible to determine the ***dislocation number, Burgers vectors and slip planes***.



Dislocation network in cold-worked aluminium.

Note: The sampling area is small therefore the properties observed cannot represent the whole materials.

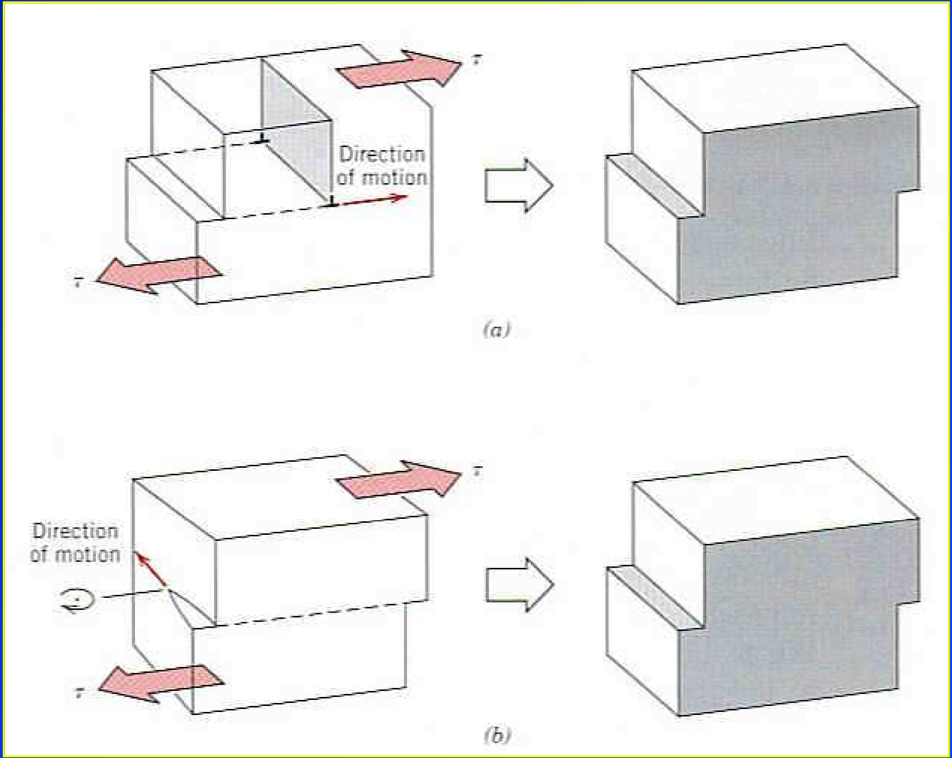
X-ray microscopy

- Using an X-ray technique to detect dislocation structure.
- The most common techniques are the ***Berg-Barret reflection method*** and the ***Lang topography method***.
- The resolution is limited to 10^3 dislocations/mm^2.

Burgers vector and the dislocation loop

Burgers vector *is the most characteristic feature of a dislocation, which defines the magnitude and the direction of slip.*

- Edge ***Burgers vector*** is ⊥ to the dislocation line.
- Screw ***Burgers vector*** is // to the dislocation line.
- Both shear stress and final deformation are identical for both situations.



Macroscopic deformation produced by glide of (a) edge dislocation and (b) screw dislocation.

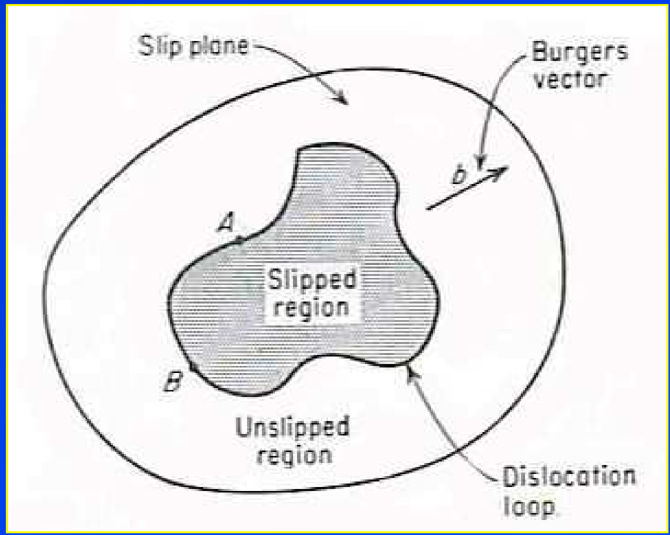
Note: Most dislocations found in crystalline materials are probably neither pure edge or pure screw but mixed.

Dislocation loops

*Dislocations in single crystals are straight lines. But in general, dislocations appear in **curves or loops**, which in three dimensions form and interlocking dislocation network.*

• Any small segments of the dislocation can be resolved into edge and screw components.

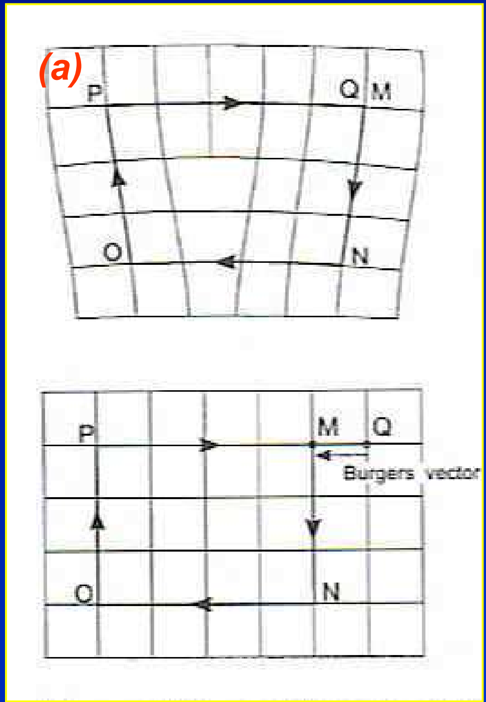
• ***Ex:*** pure screw at point ***A*** and pure edge at point ***B*** where along most of its length contains mixed edge and screw. But with the ***same Burgers vector***.



Dislocation loop lying in a slip plane.

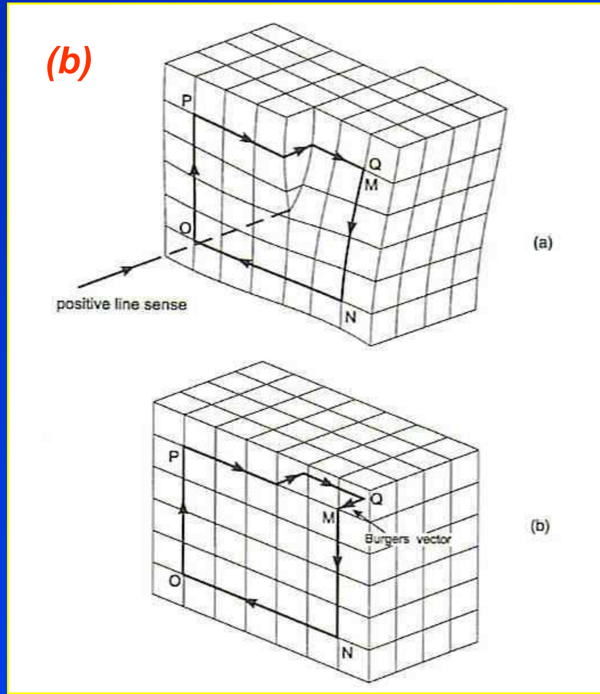
Burgers circuit

Burgers circuit is used to define the Burgers vector of dislocation.



Burgers circuits around edge dislocation

Burgers circuits around screw dislocation

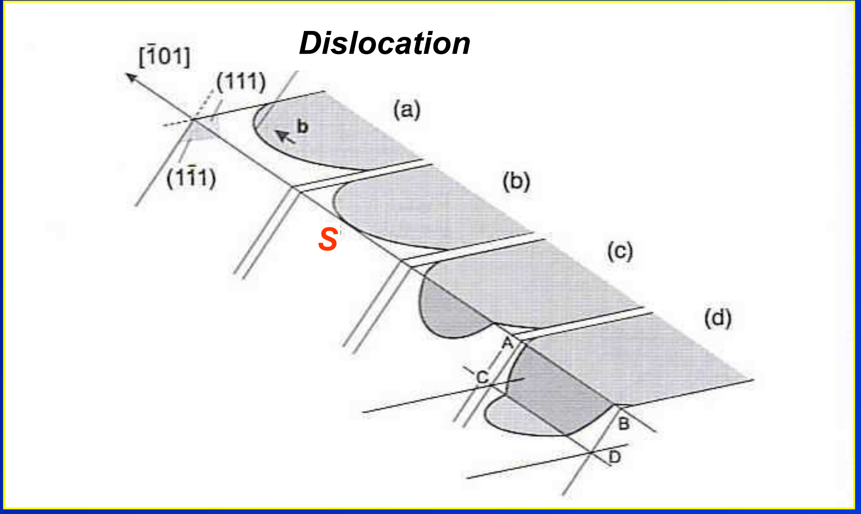


• If we trace a ***clockwise path*** from start to finish, the closure failure from finish to start is the ***Burgers vector* b** of the dislocation, see *fig (a).*

• ***A right-handed screw dislocation***, fig (b), is obtained when transversing the circuit around the dislocation line and we then have the helix one atomic plane into the crystal.

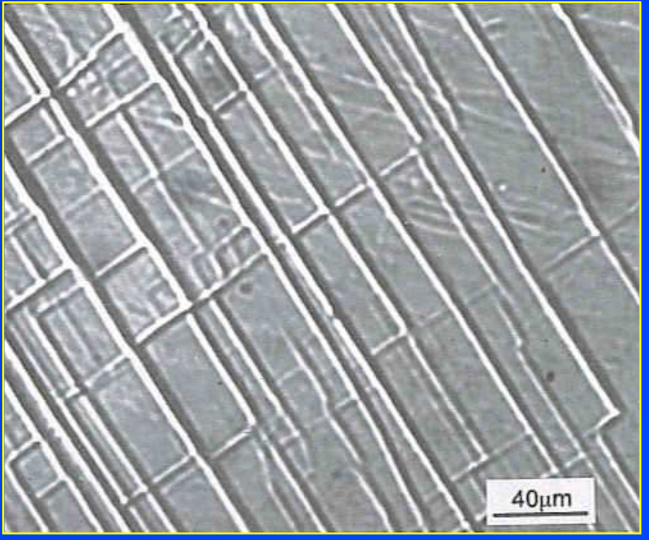
Cross slip

In **FCC** cubic metals, the ***screw dislocations*** move in **{111}** type planes, but can switch from one **{111}** type plane to another if it contains the direction of **b**. This process is called ***cross-slip***.



Cross slip in a face-centred cubic crystal.

- A screw dislocation at ***S*** is free to glide in either **(111)** or **(1$\bar{1}$1)** closed-packed planes.
- ***Double cross slip*** is shown in *(d).*



Cross slip on the polished surface of a single crystal of 3.25% Si iron.

Dislocation dissociation

Dislocation dissociation *occurs when the strength of dislocation is more than unity. The system becomes unstable → dislocation therefore dissociate into two dislocation.*

Note: *Dislocation of unit strength is a dislocation with a Burgers vector equal to one lattice spacing.*

The dissociation reaction $b_1 \rightarrow b_2 + b_3$ will occur when $b_1^2 > b_2^2 + b_3^2$.

- ***A dislocation of unit strength*** has a minimum energy when its ***Burgers vector*** is parallel to a direction of closest atomic packing.
- In close-packed lattices, dislocations with strength less than unity are possible. → therefore crystals always slip in the close-packed direction.

Dislocations in FCC lattice

• Slip occurs in the ***FCC*** lattice on the **{111}** plane in the **<110>** direction and with a Burgers vector **(a/2)[110].**

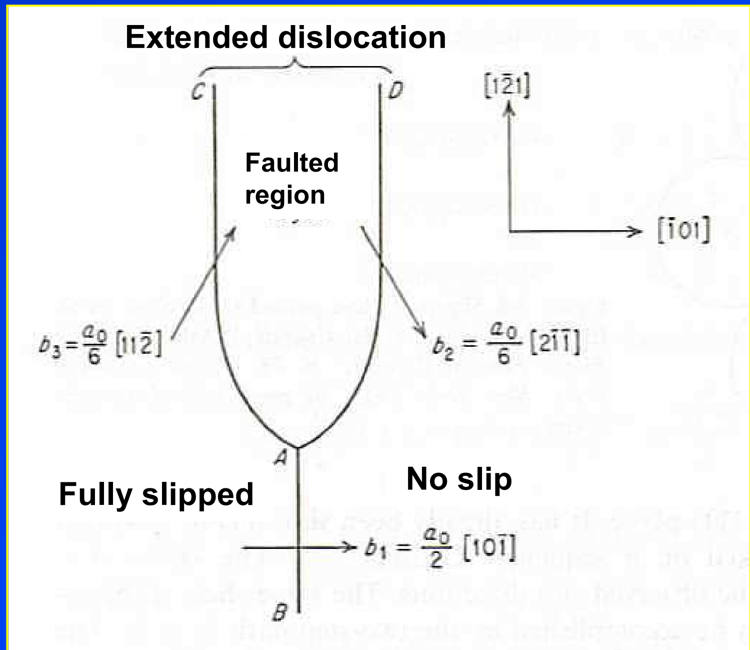
• The **{111}** planes are stacked on a close packed sequence ***ABCABC*** and vector **b = (a_o/2)[101]** defines one of the observed slip direction, which can favourably energetically ***decompose into two partial dislocations.***

$$b_1 \rightarrow b_2 + b_3$$

$$\frac{a_o}{2}[10\bar{1}] \rightarrow \frac{a_o}{6}[2\bar{1}\bar{1}] + \frac{a_o}{6}[11\bar{2}]$$

Shockley partials

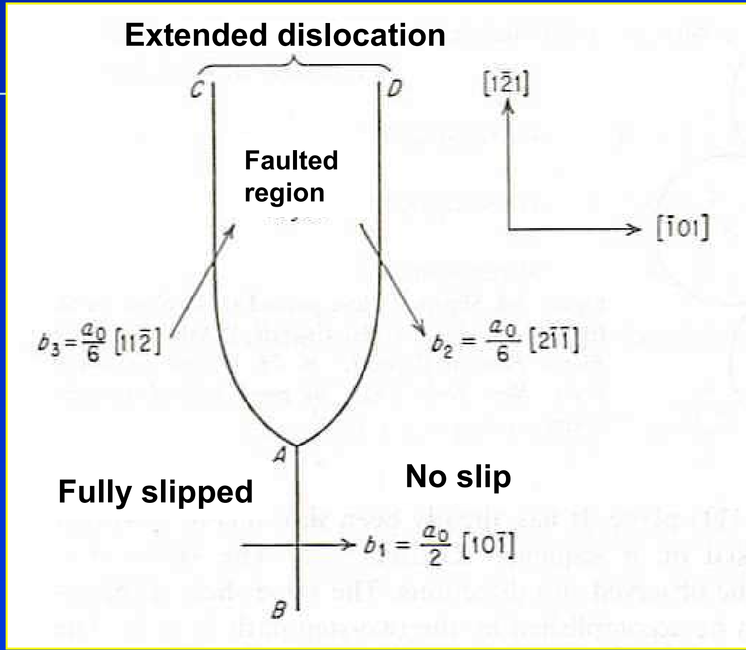
This ***Shockley partials*** creates a stacking fault ***ABCAC/ABC***.

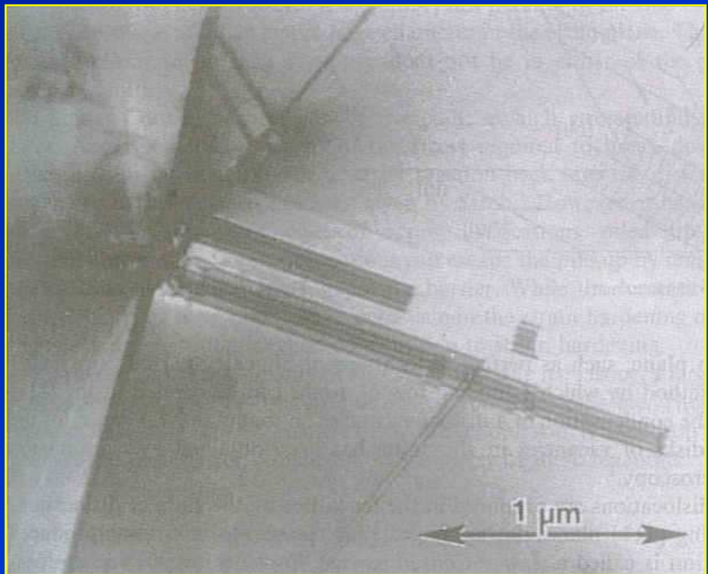


Dissociation of a dislocation to two partial dislocations.

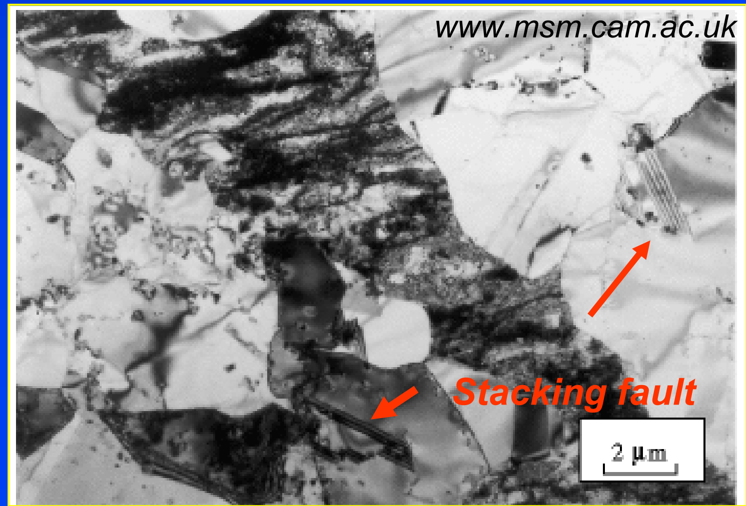
Dissociation of a dislocation into two partial dislocations

- The combination of the two partials ***AC*** and ***AD*** is known as an ***extended dislocation***.
- The region between them is a ***stacking fault*** which has undergone slip.
- The equilibrium of these partial dislocations depends on the stacking fault energy.



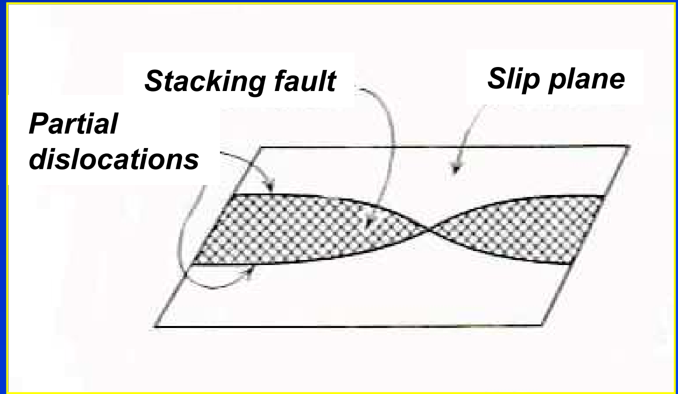


Group of stacking fault in 302 stainless steel stopped at boundary



Stacking faults

The wider region between partial dislocation, the lower stacking fault energy



Model of a stacking fault.

• ***Characteristics of metals with low SPF;***

1) Easy to strain harden
2) Easy for twin annealing to occur
3) Temperature dependent flow stress

• **Aluminium** – high stacking fault energy → more likely to cross slip.

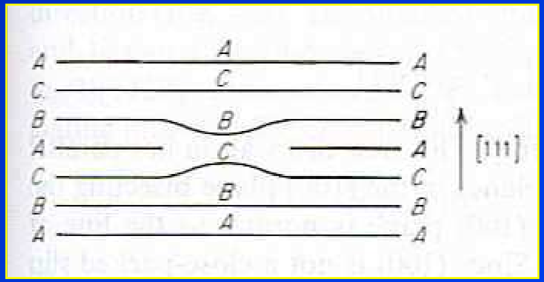
• **Copper** – lower stacking fault energy → cross slip is not prevalent.

Typical values of stacking fault energy

Metal	Stacking fault energy (mJ m^{-2})
Brass	<10
303 stainless steel	8
304 stainless steel	20
310 stainless steel	45
Silver	~25
Gold	~50
Copper	~80
Nickel	~150
Aluminium	~200

Frank partial dislocations

Frank partial dislocations are another type of partial dislocation in ***FCC*** lattice, which provide ***obstacles*** to the movement of other dislocations.



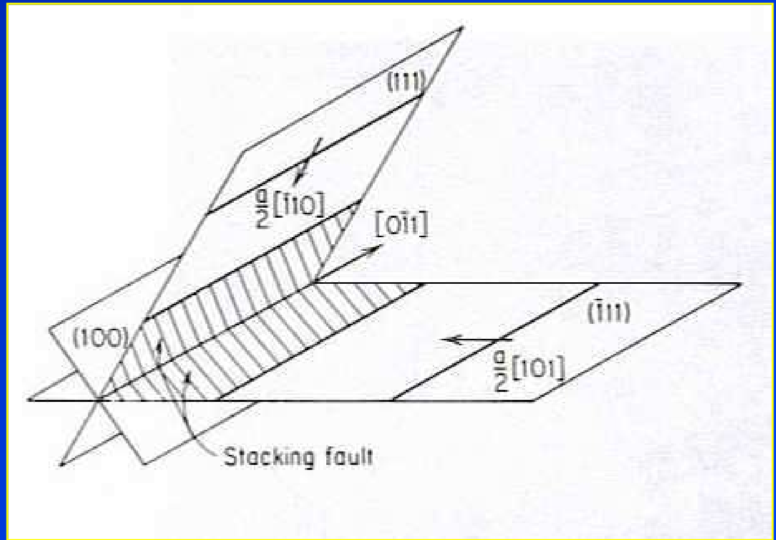
Frank partial dislocation or sessile dislocation.

- A set of **(111)** plane (viewed from the edge) has a missing middle ***A*** plane with a **Burgers vector (a_o/3) [111]** perpendicular to the central stacking fault.
- Unlike perfect dislocation, ***Frank partial dislocation*** cannot move by glide ***(sessile dislocation)*** but by diffusion of atom.

Lomer-Cortrell barrier

Intersection of **{111}** plane during duplex slip by glide of dislocations is called ***Lomer-Cortrell barrier***.

Ex: consider two perfect dislocations lying in different **{111}** planes and both parallel to the line of intersection of the **{111}** plane.



Lomer-Cortrell barrier

$$\frac{a_o}{2}[101] + \frac{a_o}{2}[\bar{1}10] \rightarrow \frac{a_o}{2}[011]$$

The new dislocation obtained has reduced energy.

Dislocations in HCP lattice

• Slip occurs in the ***HCP*** lattice on the basal **(0001)** plane in the **<11$\bar{2}$0>** direction.

• The basal **(0001)** plane the close packed of a sequence ***ABABAB*** and a Burgers vector **b = (a_o/3)[11$\bar{2}$0].**

• Dislocations in the basal plane can reduce their energy by dissociating into Shockley partials according to the reaction.

$$\frac{a_o}{3}[11\bar{2}0] \rightarrow \frac{a_o}{3}[10\bar{1}0] + \frac{a_o}{3}[01\bar{1}0]$$

The ***stacking fault*** produced by this reaction lies in the basal plane, and the extended dislocation which forms it is confined to glide in this plane.

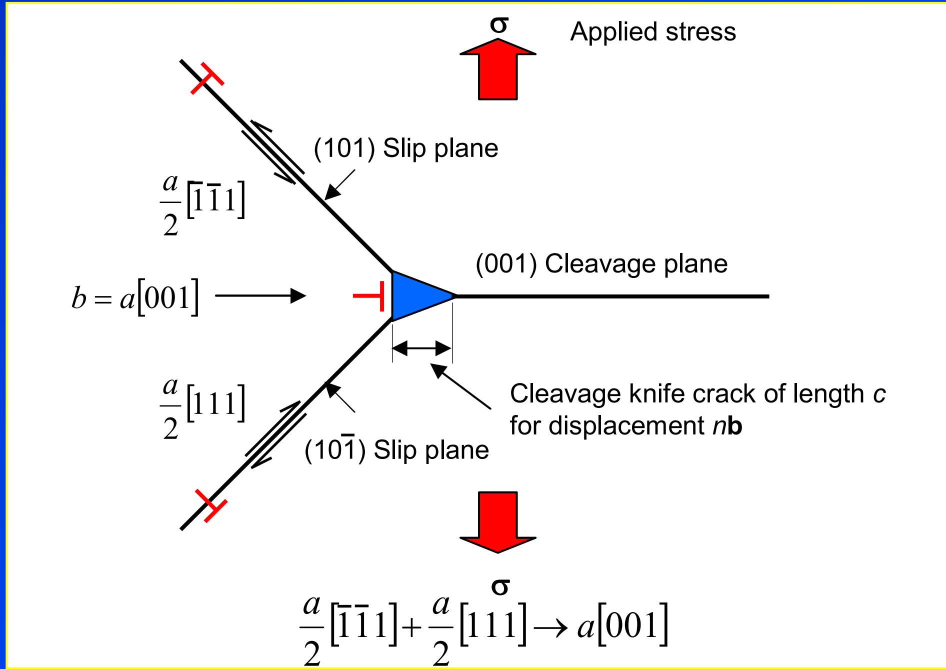
Dislocations in BCC cubic lattice

• Slip occurs in the ***BCC*** lattice on **{110}, {112}, {123}** planes in the **<111>** direction and a ***Burgers vector*** **b = $(a_o/2)[111]$.**

Cottrell has suggested a dislocation reaction which appears to cause ***immobile dislocations***. (a_o/2**[001]** in iron) → leading to a crack nucleus formation mechanism for brittle fracture.

$$\frac{a_o}{2}[\bar{1}\bar{1}1] + \frac{a_o}{2}[111] \rightarrow a_o[001]$$

the dislocation is ***immobile*** since the **(001)** is not a close-packed slip plane, the **(001)** plane is therefore the ***cleavage plane*** when brittle fracture occurs.

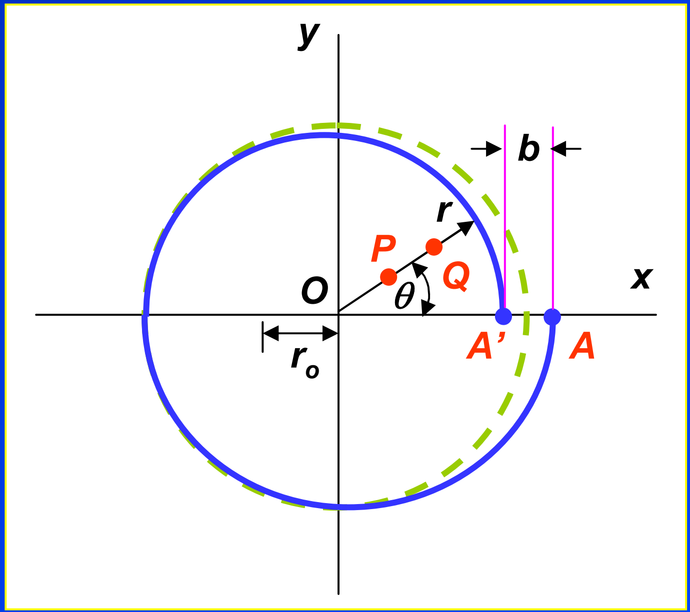


Slip on intersecting (110) plane.

Stress fields of dislocations

*A **dislocation** is surrounded by an **elastic stress field** that produces forces on other dislocations and results in interaction between dislocations and solute atoms.*

- The cross section of an elastic cylindrical piece (***dashed line***) has been distorted after an edge dislocation running through point **O** parallel to the **z** axis (***blue line***).
- The strain is zero in the **z** axis and therefore can be treated in plane strain **(x-y).**
- The ***stresses*** vary inversely with distance from the dislocation line and become infinite at ***r = 0***.



Deformation of a circle containing an edge dislocation.

$$\sigma_r = \sigma_\theta = \frac{-\tau_o b \sin\theta}{r} \quad \text{...Eq. 1}$$

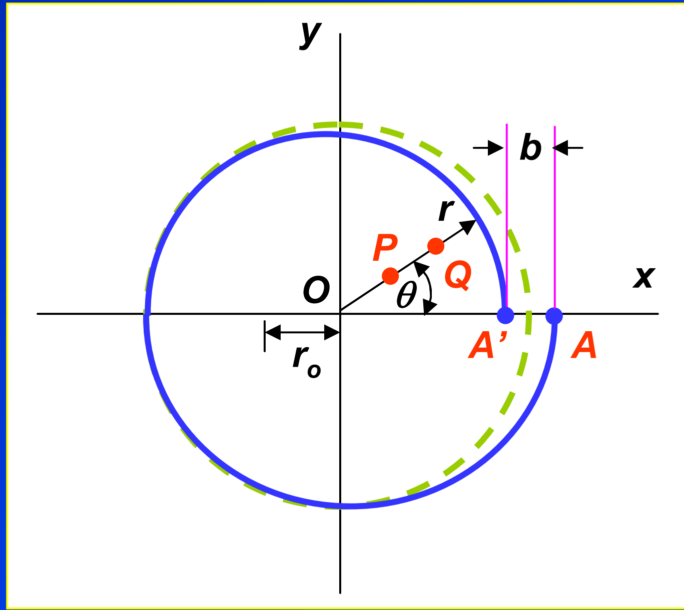
- The shear stress τ_{xy} is a maximum in the slip plane, when $y = 0$.

$$\tau_{xy} = \tau_o \frac{bx(x^2 - y^2)}{(x^2 + y^2)^2} \quad \text{...Eq. 2}$$

Strain energies of dislocations

The ***strain energy*** involved in the formation of an ***edge dislocation*** can be estimated from the work involved in displacement the cut ***OA*** a distance **b** along the slip plane.

$$U = \frac{Gb^2}{4\pi(1-\nu)} \ln \frac{r_1}{r_o} \quad \text{...Eq. 3}$$



Deformation of a circle containing an edge dislocation.

The ***strain energy*** of a ***screw dislocation*** is given by

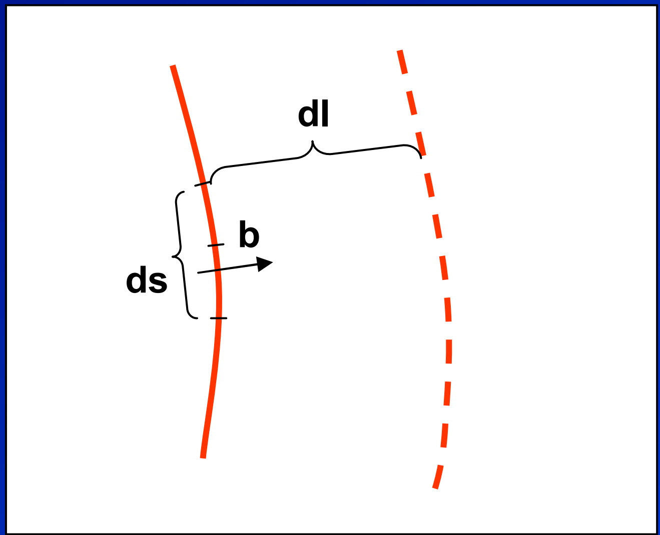
$$U = \frac{Gb^2}{4\pi} \ln \frac{r_1}{r_o} \quad \text{...Eq. 4}$$

Note: the total strain energy is the sum of elastic strain energy and the core energy of dislocation.

The dislocation energy per unit length simplifies to

$$U = \frac{Gb^2}{2} \quad \text{...Eq. 5}$$

Forces on dislocation



Force acting on a dislocation line.

• A dislocation line moving in the direction of its ***Burgers vector*** under the influence of a uniform ***shear stress*** τ.

• The force per unit length of dislocation ***F*** ;

$$F = \frac{dW}{dlds} = \tau b \qquad \text{...Eq. 6}$$

• This force is normal to the dislocation line at every point along its length and is directed toward the unslipped part of the glide plane.

• The Burgers vector is constant along the curved dislocation line.

Forces between dislocations

• Dislocations of ***opposite sign*** on the same slip plane will ***attract*** each other, run together, and annihilate each other.
• Dislocations of ***alike sign*** on the same slip plane will ***repel*** each other

The ***radial force F_r*** between two parallel ***screw dislocations***

$$F_r = \tau_{\theta z} b = \frac{Gb^2}{2\pi r}$$

…Eq. 7

Parallel screw (same sign) → +
Aniparallel screw (opposite sign) → -

The ***radial and tangential forces*** between two parallel ***edge dislocations***

$$F_r = \frac{Gb^2}{2\pi(1-\nu)}\frac{1}{r} \quad , F_\theta = \frac{Gb^2}{2\pi(1-\nu)}\frac{\sin 2\theta}{r}$$

…Eq. 8

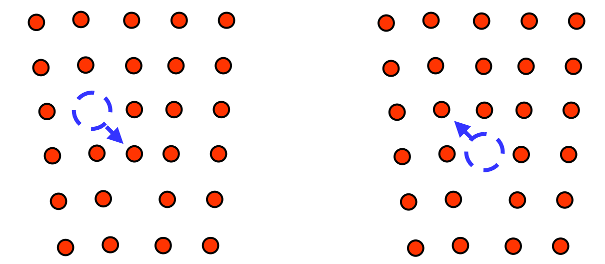
Dislocation climb

***Dislocation climb** is a non conservative movement of dislocation where and edge dislocation can move out of the slip plane onto a parallel directly above or below the slip plane.*

- ***Climb*** is ***diffusion-controlled*** (thermal activated) and occurs more readily at elevated temperature. → ***important mechanism in creep.***

- ***Positive direction of climb*** ⊥ is when the edge dislocation moves upwards. Removing extra atom (or adding vacancy around ⊥). ***Compressive force produces + climb.***

- ***Negative direction of climb*** ⊤ is when the edge dislocation moves downwards. Atom is added to the extra plane. ***Tensile forces to produce – climb.***



(a) Diffusion of vacancy to edge dislocation.

(b) Dislocation climbs up one lattice spacing.

*<u>Note:</u> Glide or slip of a dislocation is the direction parallel to its direction whereas climb of dislocation is in the **vertical** direction.*

Intersection of dislocations

*The **intersection of two dislocations** produces a sharp break (a few atom spacing in length) in dislocation line.*

*This break can be of **two types**;*

- ***Jog*** is a sharp break in the dislocation moving it out of the slip plane.
- ***Kink*** is a sharp break in the dislocation line which remains in the slip plane.

Note: Dislocation intersection mechanisms play an important role in the strain hardening process.

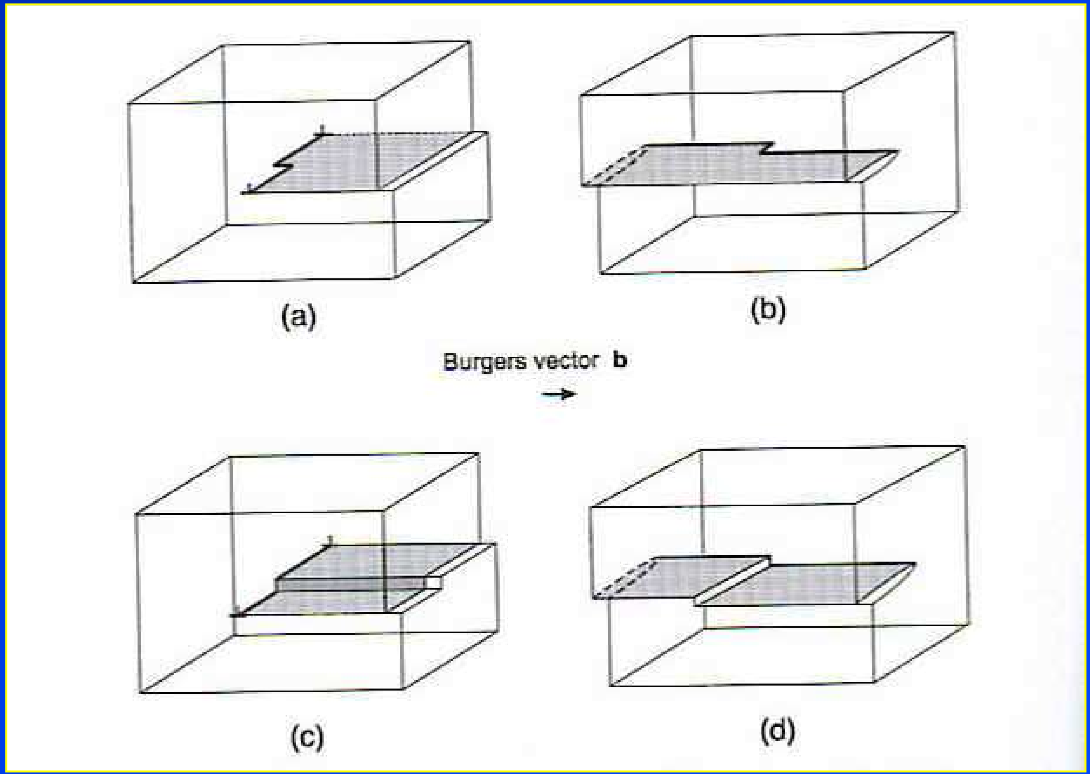
Jogs and Kinks

Jogs are steps on the dislocation which move it from one atomic slip plane to another.

Kinks are steps which displace it on the same slip plane.

(a), (b) Kinks in edge and screw dislocations

(c), (d) Jogs in edge and screw dislocations.

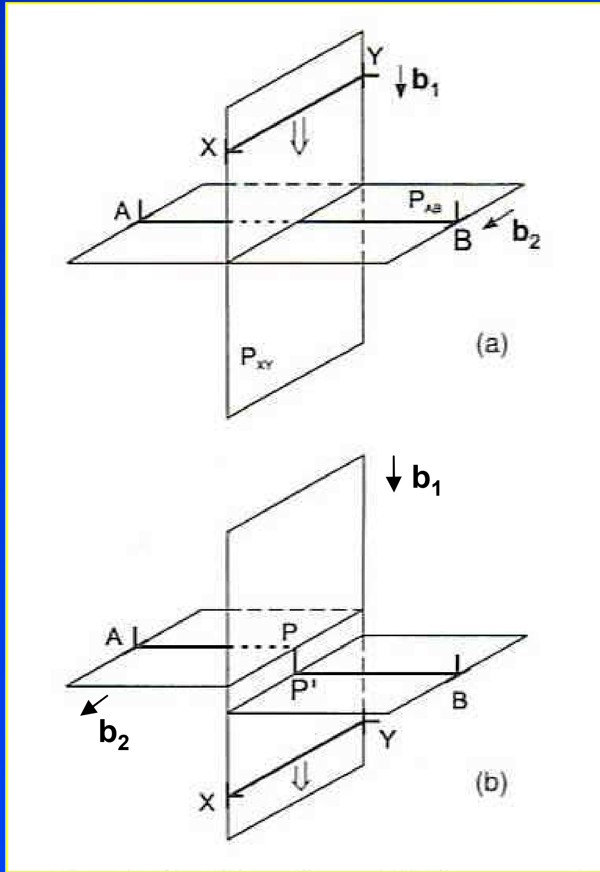


Intersection of two dislocations

1) Intersection of two dislocations with Burgers vectors at right angle to each other.

- An edge dislocation ***XY*** with ***Burgers vector*** **b_1** is moving on plane ***P_{xy}*** and cuts through dislocation ***AB*** with ***Burgers vector*** **b_2**.
- The intersection causes ***jog PP'*** in dislocation ***AB*** parallel to **b_1** and has ***Burgers vector*** **b_2**. and with the length of the ***jog*** = **b_1**.
- It can readily glide with the rest of dislocation.

<u>Note:</u> **b_1** is normal to ***AB*** and jogs ***AB***, while **b_2** is parallel to ***XY*** and no jog is formed.

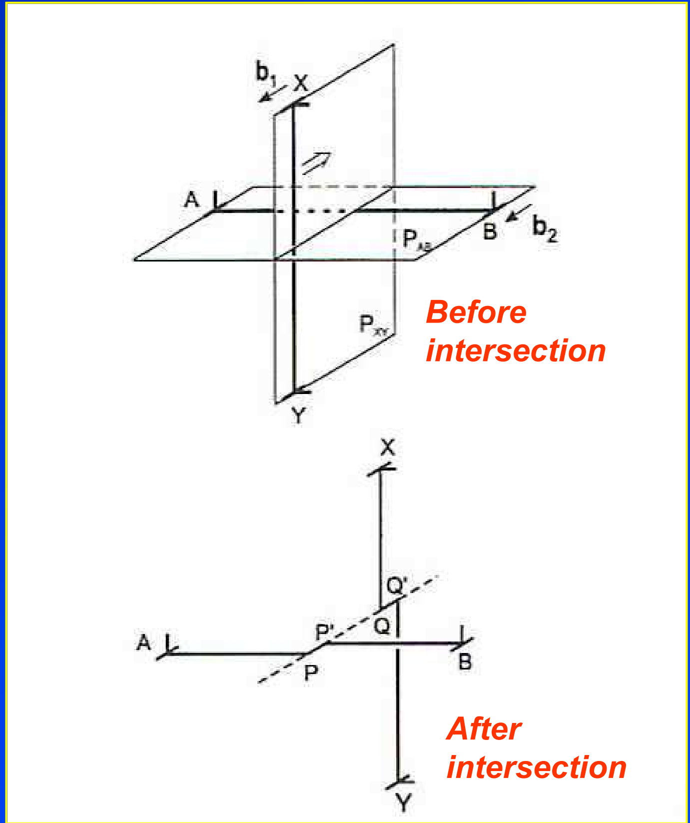


Intersection of two edge dislocations

Intersection of two dislocations

2) Intersection of two dislocations with Burgers vectors parallel to each other

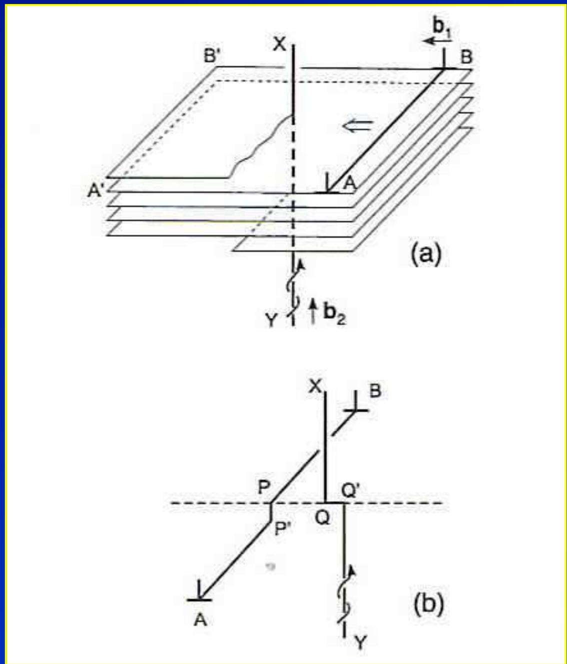
- Both dislocations are jogged.
- The length of *jog PP'* is b_1 and the length of *jog QQ'* is b_2.
- The *jogs* both have a screw orientation and lie in the original slip plane. This is called *Kink*. → not stable.



Intersection of edge dislocations with parallel Burgers vectors.

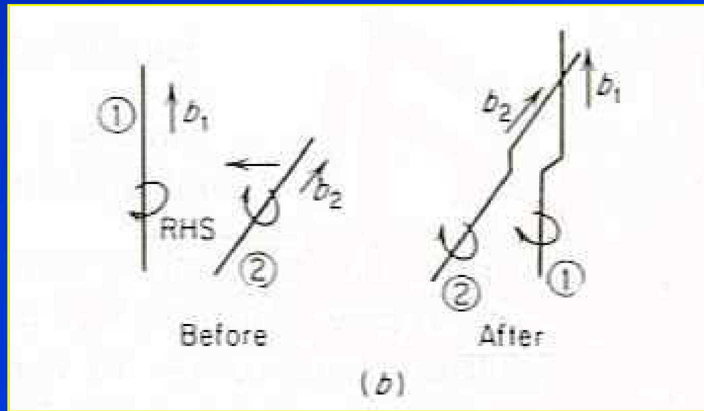
Intersection of two dislocations

3) Intersection of edge and screw dislocations.



Intersection produces a ***jog*** with an edge orientation on the edge dislocation and a ***kink*** with an edge orientation on the screw dislocation.

4) Intersection of two screw dislocations.



The intersection produces ***jogs*** of edge orientation in both screw dislocations. → very important in ***plastic deformation.***

Note: at temperature where climb cannot occur the movement of screw dislocation is impeded by jogs.

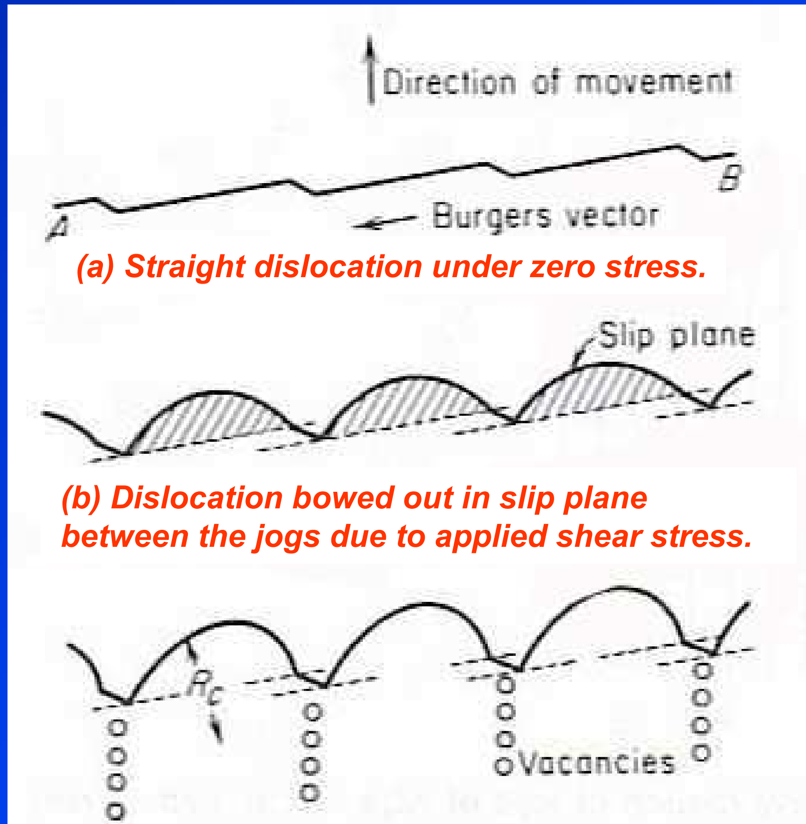
Jogs

• A stable jog ↑ length of the dislocation line → energy of the crystal ↑

(a) ***Many intersections*** occur when a screw dislocation encounter a forest of screw dislocations. → producing ***vacancy jogs*** and/or ***interstitial jogs.***

(b) Jogs act as ***pinning points*** and cause dislocations to ***bow out*** with the radius ***R*** when the ***shear stress*** τ is applied.

(c) At some critical radius R_c the τ required to further decrease R > the stress needed for non-conservative climb. Then the dislocation will move forward leaving a ***trail of vacancies (interstitials)*** behind each jog.



(a) Straight dislocation under zero stress.

(b) Dislocation bowed out in slip plane between the jogs due to applied shear stress.

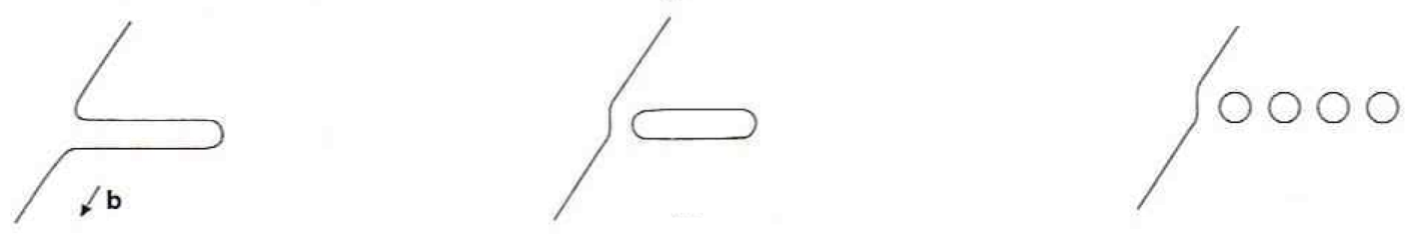
(c) Movement of dislocation leaving trails of vacancies behind the jogs.

Movement of jogged screw dislocation

Superjogs

Superjog *is a jog that has more than one atomic slip plane spacing high.*

As the stress increases, the dislocation ***bows out between the superjogs***, generating ***dislocation dipoles*** and later break into ***isolated loops***.



(a) Dislocation dipole. *(b) Elongated loop and jogged dislocation.* *(c) Row of small loops.*

Formation of dislocation loops from a dislocation dipole

Dislocation Sources

- All metals initially contain an appreciable number of dislocations produced from the ***growth of the crystal from the melt or vapour phase***.
- ***Gradient of temperature*** and ***composition*** may affect dislocation arrangement.
- ***Irregular grain boundaries*** are believed to be responsible for emitting dislocations.
- Dislocation can be formed by ***aggregation and collapse of vacancies*** to form disk or prismatic loop.
- Heterogeneous nucleation of dislocations is possible from high local stresses at ***second-phase particles*** or as a result of ***phase transformation***.

Multiplication of dislocations

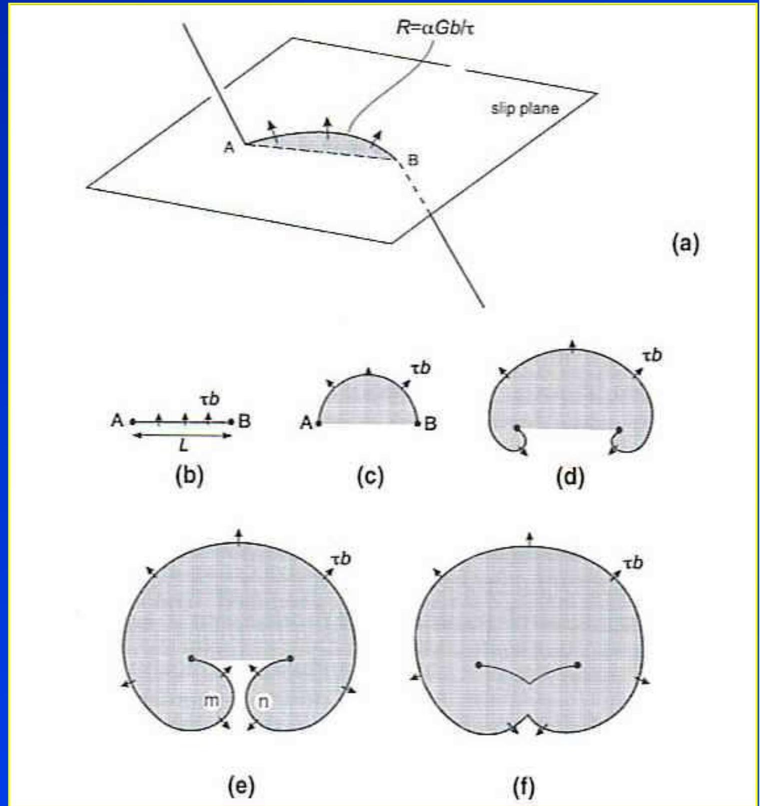
Frank & Read *proposed that dislocations could be generated from existing dislocations.*

- The ***dislocation line AB*** bulges out (***A*** and ***B*** are anchored by impurities) and produces ***slip*** as the shear stress τ is applied.
- The maximum τ for ***semicircle*** dislocation bulge, *fig (b)*

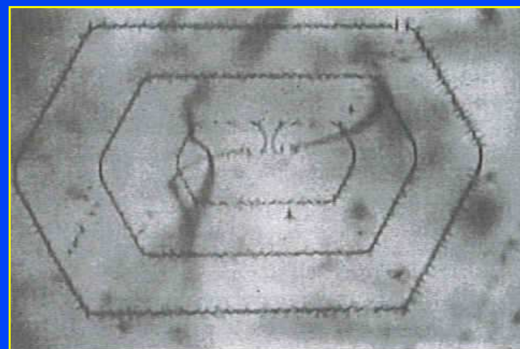
$$\tau \approx \frac{Gb}{2R} \approx \frac{Gb}{l}$$

- Beyond this point, the ***dislocation loop*** continues to expand till parts ***m*** and ***n*** meet and ***annihilate*** each other to form a ***large loop*** and a ***new dislocation***.

Note: Repeating of this process producing a dislocation loop, which produces slip of one Burgers vector along the slip plane.



The operation of Frank-Read source



Frank Read source in a silicon crystal

Dislocation-point defect interactions

Point defect and dislocation will interact elastically and exert forces on each other.

Negative interaction energy → attraction
Positive interaction energy → repulsion

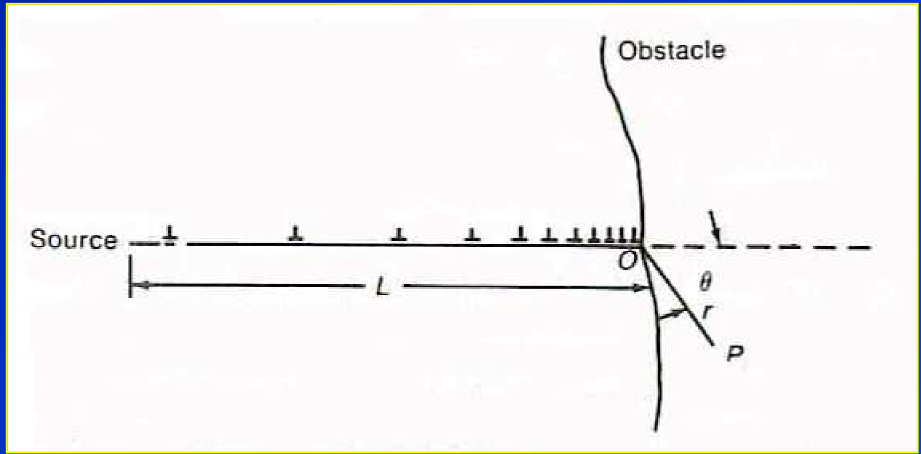
If the solute atom is	→	The atom will be
If the solute atom is larger than the solvent atom ***(ε > 1)***	→	***The atom will be repelled from the compressive side of a positive edge dislocation and will be attracted to the tension side.***
If the solute atom is smaller than the solvent atom ***(ε < 1)***	→	***The atom will be attracted to the compression side.***

- Vacancies will be attracted to regions of compression.
- Interstitials will be collected at regions of tension.

Dislocation pile-ups

Dislocations often pile up on slip planes at barriers i.e., grain boundaries or second phase particles.

High stress concentration on the leading dislocations in the pile-up.



Dislocation pile-ups at an obstacle.

If the pile-up stress > theoretical shear stress → yielding

A pile-up of ***n*** dislocations along a distance ***L*** can be considered as a giant dislocation with a ***Burgers vector*** ***n*****b**.

The breakdown of a barrier occur by

1) Slip on a new plane.
2) Climb of dislocation around the barrier.
3) Generation of high enough tensile stress to produce a crack.

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Strengthening mechanisms

Subjects of interest

- *Introduction/Objectives*
- *Grain boundary strengthening*
- *Yield-point phenomenon*
- *Strain ageing*
- *Solid-solution strengthening*
- *Strengthening from second phase particles*
- *Martensitic strengthening*
- *Strain hardening or cold working*
- *Bauschinger effect*
- *Preferred orientation (texture)*

Objectives

- Different types of strengthening mechanisms in metals which improve mechanical properties will be highlighted in this chapter.
- This also includes the nature of grain boundaries and their effects on the strengthening mechanisms, the influences of solute atoms, second phase particles, and fibre on the strengthening mechanisms.
- Other strengthening mechanisms such as strain hardening, martensitic hardening and cold working on the mechanical properties of the materials will also be discussed.

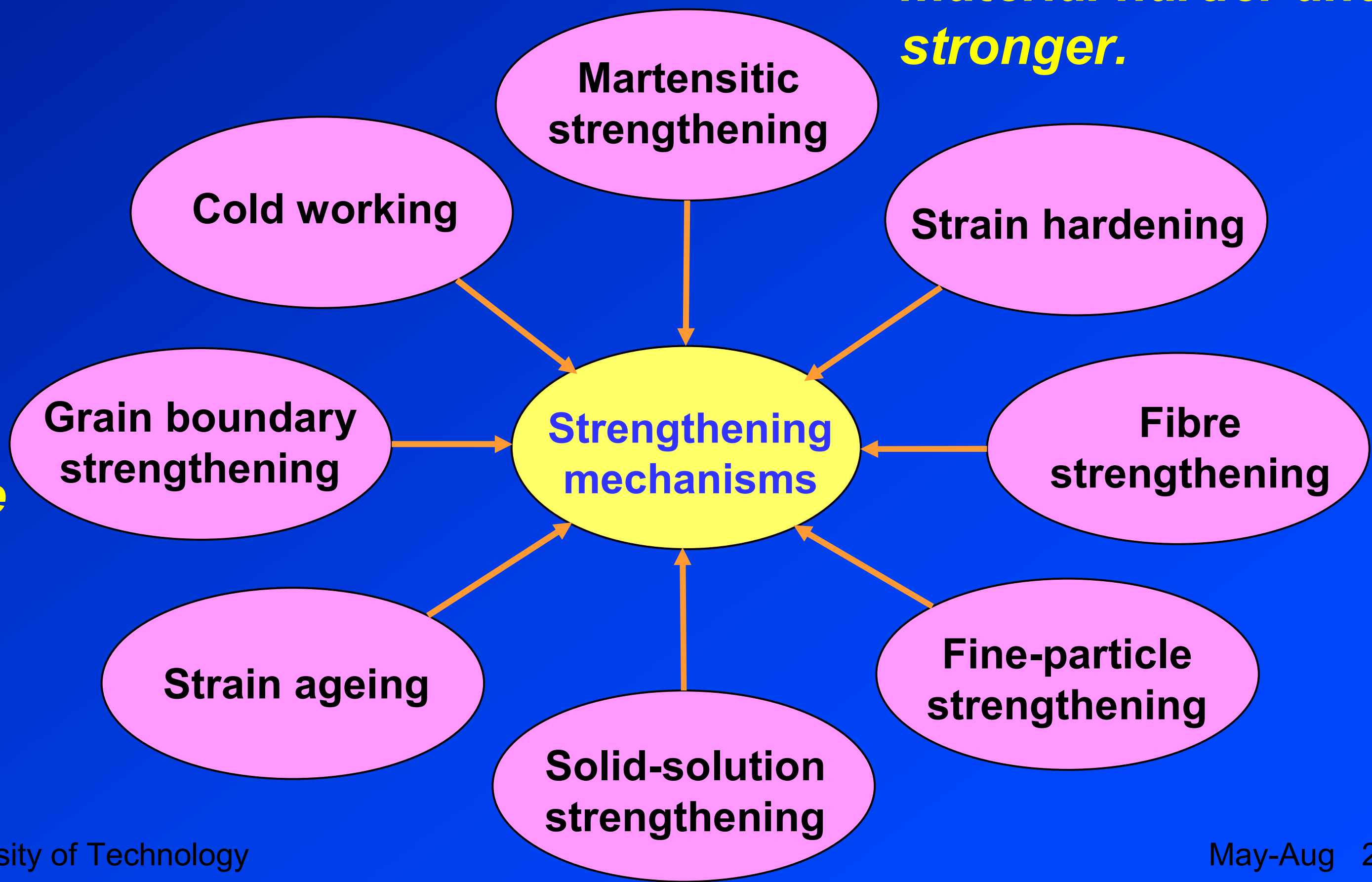
Introduction

The ability of a metal to plastically deform depends on the ability of dislocations to move. ⇨ *Strengthening techniques rely on restricting dislocation motion to render a material harder and stronger.*

To obtain material strength

⇩

Sometimes ductility or toughness are sacrificed.

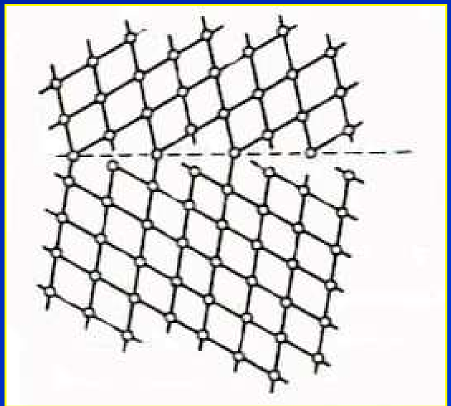


Grain boundary strengthening

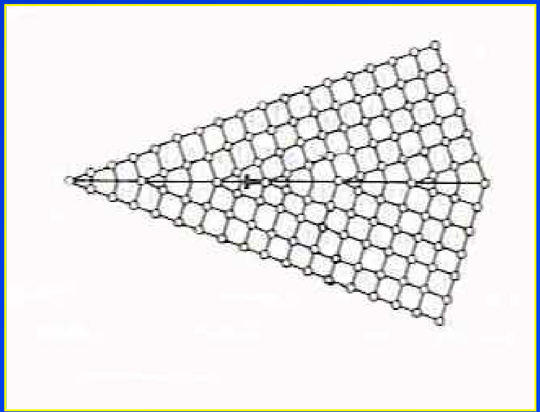
Grain boundaries

*Grain boundary separates two grains having **different crystallographic orientations**.*

*Grain boundary structure contains **grain boundary dislocations**, which are not mobile and produce extensive slip.*



Schematic atomic model of a grain boundary



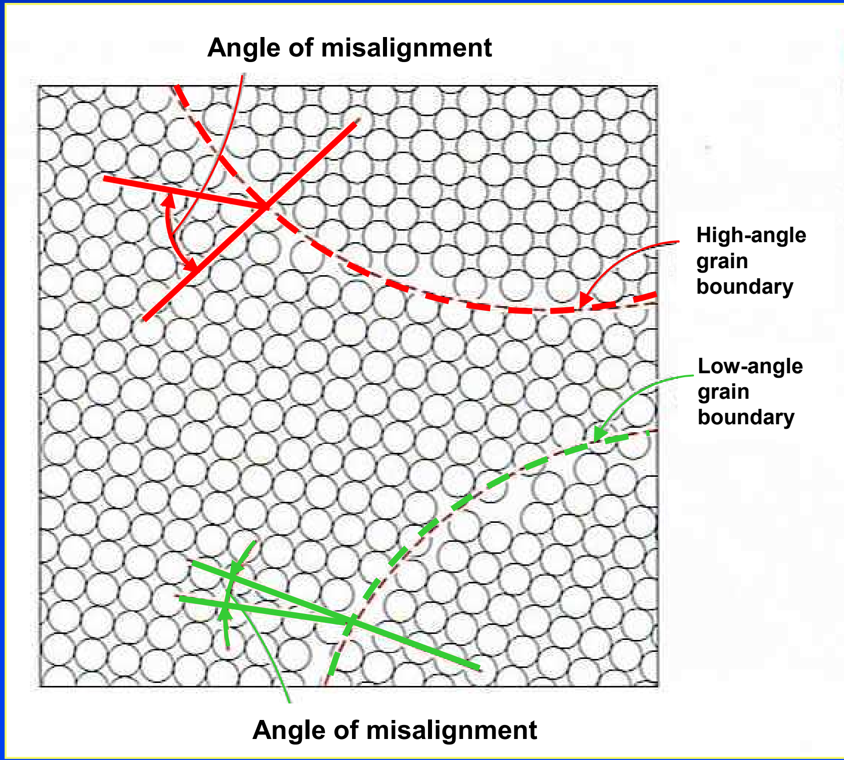
Dislocation model of grain boundary

High and low angle grain boundaries

High - angle grain boundary → high surface energy
Low - angle grain boundary → low surface energy

High energy grain boundary serves as ***preferential sites*** for solid state reactions;

1) Diffusion
2) Phase transformation
3) Precipitation



Schematic diagram showing low- and high-angle grain boundaries.

Low angle grain boundaries

- Along the boundary the ***atoms adjust their position*** by localised deformation to produce a smooth transition from one grain to the other.
- Where the atom planes end on the grain boundaries, it is therefore considered to have ***an array of dislocations***.
- The ***angular difference*** in orientation between the grain is θ.

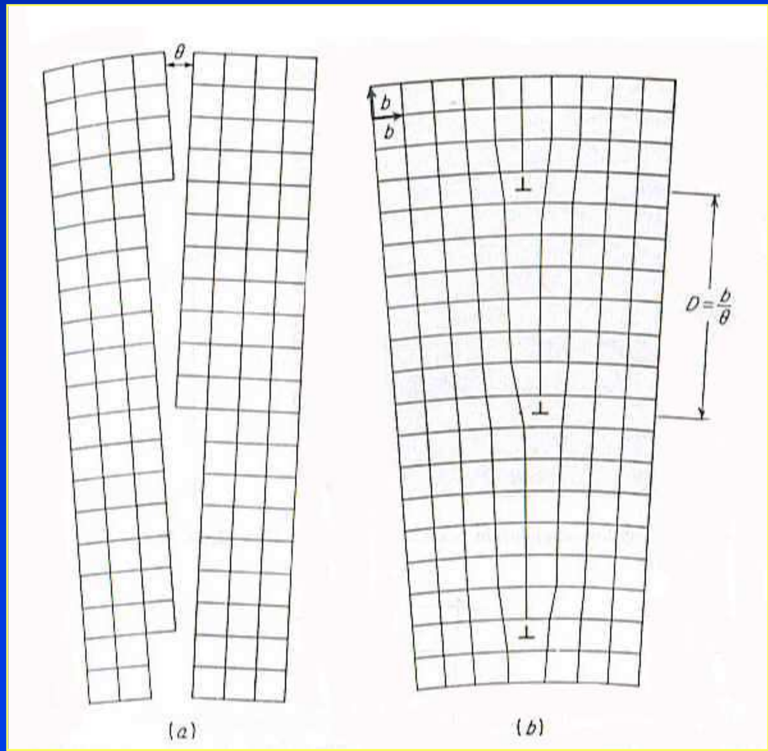
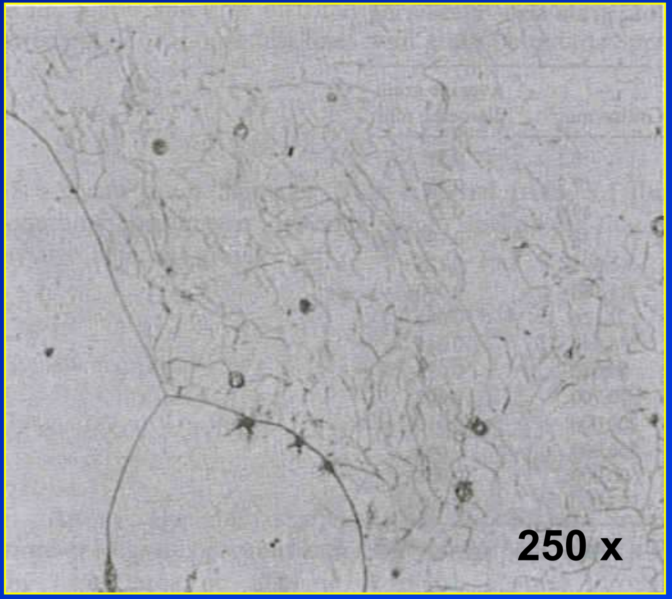


Diagram of low-angle grain boundary

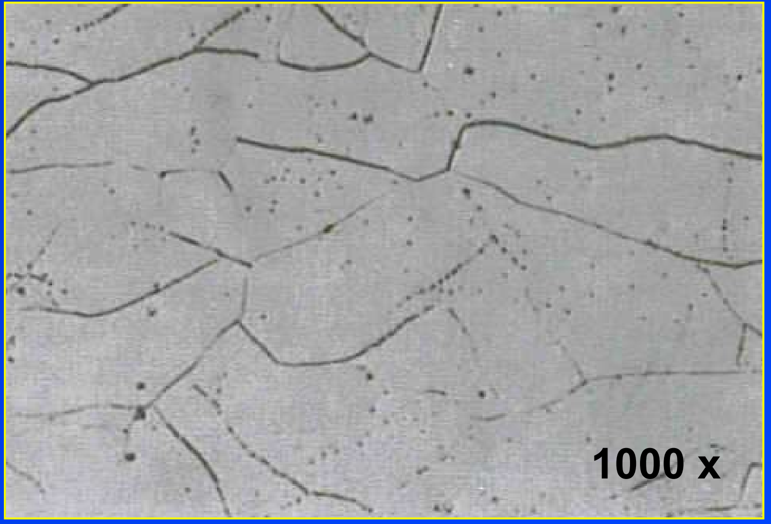
Subgrain boundaries

Subgrain boundaries are ***low-angle boundaries***, with ***lower-energy boundaries*** than the grain boundaries. → therefore etch less readily than grain boundaries.

If the angle θ is small the distance between dislocation is large. It is often possible to observe ***pits*** (corresponding to sites for ***edge dislocations***) along the boundaries, see *fig*.



Subgrain boundary network in Fe-3% alloy.



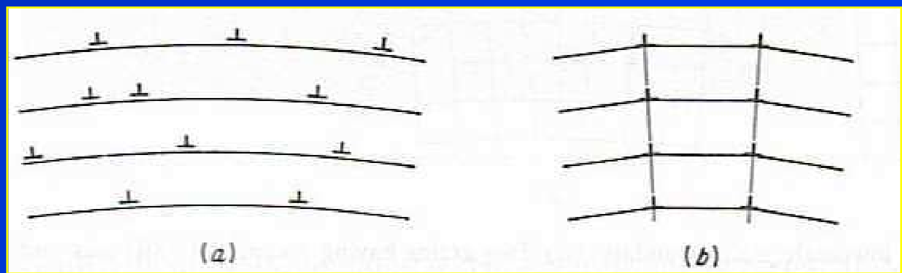
***Etch-pit structures along low-angle grain boundaries* in Fe-Si alloy.**

Polygonization

• ***Polygonization*** occurs when a single crystal is bent to a relatively small curvature and then annealed.

• Bending results in an excess number of dislocations of similar sign distributing along the bend-glide plane.

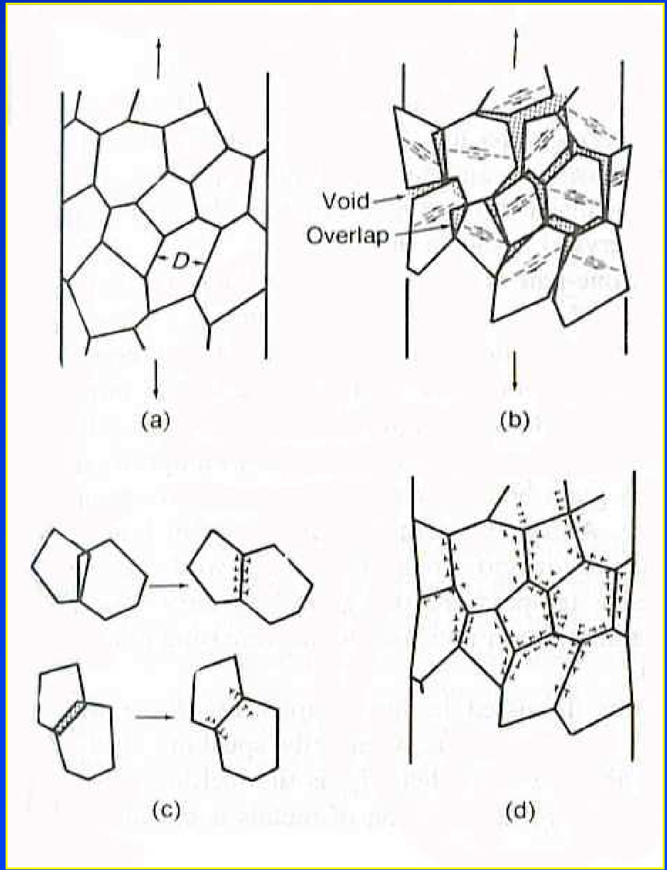
• After heating, dislocations group themselves into the lower-energy configuration of a low-angle boundary, forming a ***polygonlike network***.



Movement of dislocations to produce polygonization.

Deformation of grain boundaries.

- Discontinuity due to grain boundaries leads to ***more complex deformation mode*** in polycrystals than in single crystals.
- Individual grain is constrained since mechanical integrity and coherency are maintained along the grain boundaries, causing different deformation between neighbouring grains.
- A polycrystal macroscopically deforms as the stress is applied. Slips operate in each grains which produces ***overlaps*** and ***voids*** at boundaries, *fig (a),(b).*
- These overlaps and voids can be corrected by introducing ***geometrically necessary dislocations*** at (c),(d).



Ashby's model of deformation of a polycrystal.

Plastic deformation of polycrystalline metals

- Due to random crystallographic orientations of the numerous grains, the direction of ***slip varies from one gain to another***.
- *Fig.* shows two slip systems operate in each grain and variation in grain orientation is indicated by the ***different alignment of slip lines***.

Note: More slip systems are usually operate near the grain boundary, the material is usually harder near the boundary than the grain interior.

Slip lines on the surface of a deformed polycrystalline copper

Grain boundary sliding

At $T > 0.5T_m$, deformation can occur by sliding along the grain boundaries.

Temperature ↑

Strain rate ↓

→ ***Tendency for grain boundary sliding (as in creep).***

Equicohesive temperature

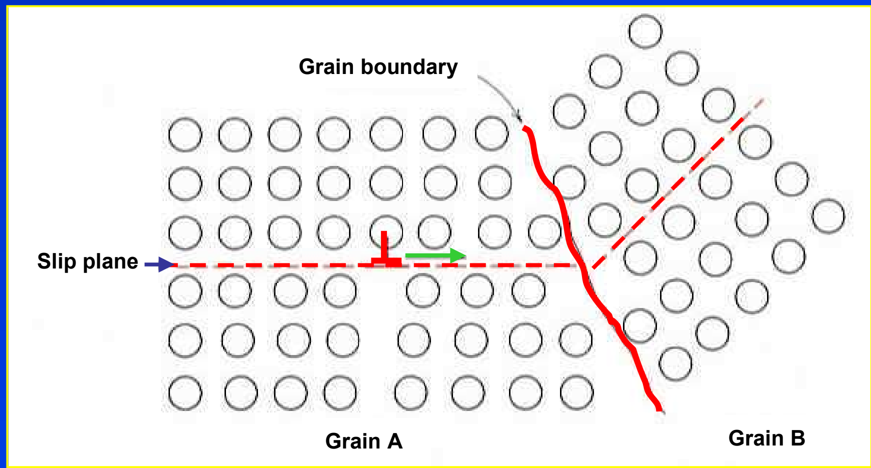
Above the ***equicohesive temperature***, the grain boundary region is weaker than the grain interior.

→ ***Strength increases with increasing grain size.***

Strengthening from grain boundaries

There are two important roles of the grain boundary which acts as a barrier to dislocation motion;

1) Difficulty for a dislocation to pass through ***two different grain orientations*** (need to change direction).
2) The atomic disorder within a grain boundary region contributes to a ***discontinuity of slip planes from one grain to another***.



The motion of a dislocation as it encounters a grain boundary.

Hall-Petch relation

A fine-grained material is harder and stronger than one that is coarse grained since greater amounts of grain boundaries in the fine-grained material impede dislocation motion.

The general relationship between the ***yield stress*** (tensile strength) and ***grain size*** was proposed by ***Hall*** and ***Petch***.

$$\sigma_o = \sigma_i + kD^{-1/2} \quad \text{...Eq. 1}$$

Where σ_o = the yield stress

σ_i = the 'friction stress' or resistance to dislocation movement

k = the 'locking parameter' or hardening contribution from grain boundary.

D = grain diameter

Hall - Petch relation and dislocation pile-up model

- The ***dislocation model*** for the ***Hall-Petch equation*** was originally based on the idea that grain boundaries act as ***barriers*** to dislocation motion.

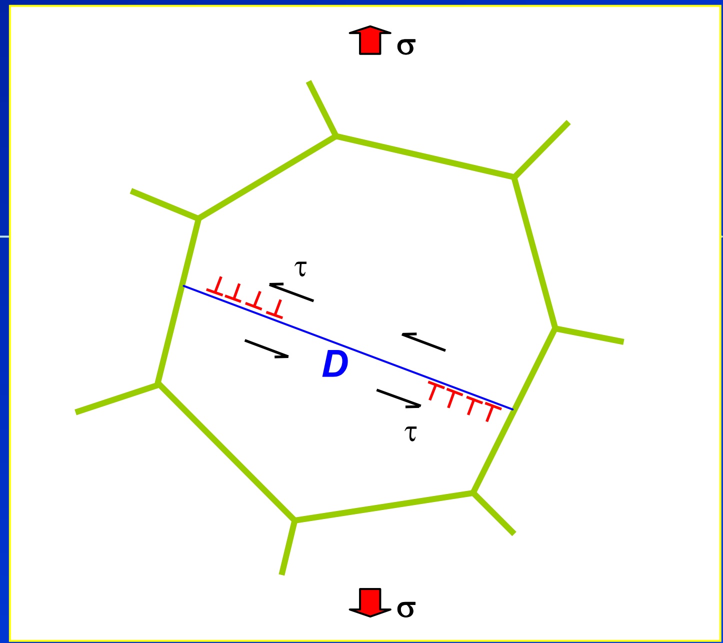
- Dislocations will be sent out from the source at the centre of a grain of ***D*** diameter to ***pile up*** at grain boundary.

- The number of dislocations at the pile-up is

$$n = \frac{k\pi\tau_s D}{4Gb} \qquad \text{...Eq. 2}$$

Where τ_s is the average resolved shear stress

k is a factor close to unity



The ***stress at the tip of the pile-up*** must exceed some critical shear stress τ_c to continue slip past the grain-boundary barrier

Then

$$\tau_c = n\tau_s = \frac{\pi\tau_s^2 D}{4Gb}, \quad \tau_s = \tau - \tau_i$$

$$\tau = \tau_i + \left(\frac{\tau_c 4Gb}{\pi D}\right)^{1/2}$$

$$\tau = \tau_i + kD^{-1/2} \qquad \text{...Eq. 3}$$

Note: for large pile-ups

Grain size determination

• Since the size of the grain is usually associated with mechanical properties of the materials, determination of the grain size is therefore of importance.

• ***There are a number of techniques utilised for grain size measurement;***

1) Intercept method
2) ASTM standard comparison charts (grain number)
3) Image analyser

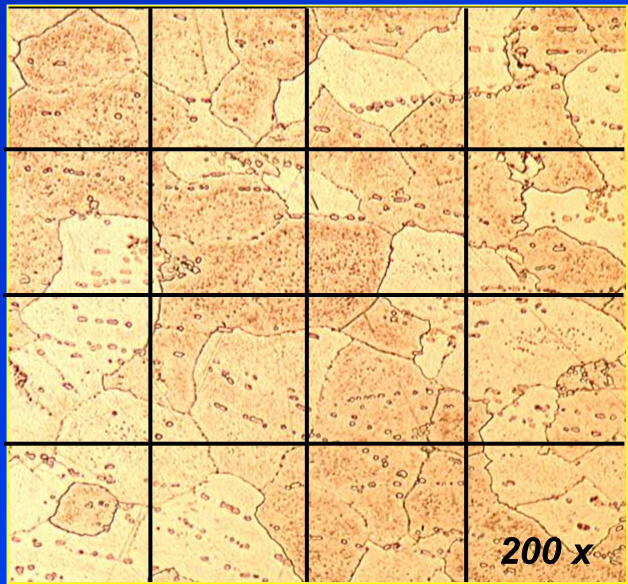
• ***The obtained parameters can be specified in terms of;***

1) Average grain volume
2) Average grain diameter
3) Average area
4) Maximum diameter
5) Minimum diameter
6) Aspect ratio

Intercept method

Intercept method *is carried out by measuring the mean number of intercepts of random test lines with grain boundaries per unit length of test line* N_L.

- Strength lines of the same length ***L*** are drawn though several photomicrograph with a known scale.
- The grain intersected N_L by each line segment are counted.
- The ***average grain diameter*** is obtained by



$$D = \frac{L}{N_L M}$$

…Eq. 4

Where ***M*** is a ***linear magnification*** of the photomicrograph.

Note: The grain size obtained by this method will be somewhat ***smaller*** *than the actual grain size. In some case, a factor 2/3 is used.*

ASTM standard comparison chart

Comparison of the grains at a fixed magnification with the ***American Society for Testing and Materials (ASTM)*** *grain size charts.*

The ***ASTM*** grain-size number ***G*** is related to n_a, the number of grains per mm^2 at a magnification of 1 x by the relationship

$$G = -2.9542 + 1.4427 \ln n_a$$

…Eq. 5

Table 6-1 Comparison of grain-size measuring systems†

ASTM no.	Grains/mm²	Grains/mm³	Average grain diameter, mm
−1	3.9	6.1	0.51
0	7.8	17.3	0.36
1	15.5	49.0	0.25
2	31.0	138	0.18
3	62.0	391	0.125
4	124	1,105	0.090
5	248	3,126	0.065
6	496	8,842	0.045
7	992	25,010	0.032
8	1,980	70,700	0.022
9	3,970	200,000	0.016
10	7,940	566,000	0.011
11	15,870	1,600,000	0.008
12	31,700	4,527,000	0.006

† "Determining the Standard Grain Size," ASTM Standard E112, 1985.

Example: *If a steel has a value of σ_i = 150 MPa and **k** = 0.70 MPa.m$^{1/2}$, what is the value of the yield stress if the grain size is ASTM no.6.*

From *Eq.6*

$$\ln n_a = (G + 2.9542)/1.4427$$

$$n_a = \exp\left(\frac{6 + 2.9524}{1.4427}\right) = 496\ \ mm^{-2}\ \ = 496 \times 10^6\ \ m^{-2}$$

Grain diameter

$$D \approx \sqrt{1/n_a}\ \ or\ \ D^2 \approx 1/n_a$$

$$D^2 \approx 20 \times 10^{-10}\ \ m^2,\ \ D \approx 44.7 \times 10^{-6}\, m$$

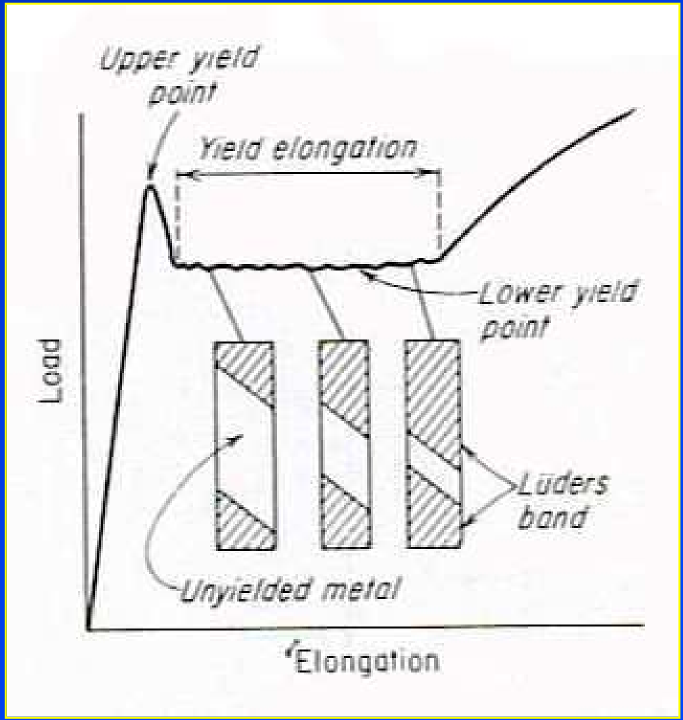
$$\frac{1}{\sqrt{D}} = 149\ \ m^{-1/2}$$

$$\sigma_o = \sigma_i + kD^{-1/2} = 150 + (0.70)(149) = 254.3\ \ MPa$$

Yield point phenomenon

*Metals, particularly **low-carbon steel**, show a localised heterogeneous transition from elastic to plastic deformation. → **Yield point elongation**.*

- The load after the ***upper yield point*** suddenly drop to approximately constant value (***lower yield point***) and then rises with further strain.
- The elongation which occurs at constant load is called the ***yield-point elongation***, which are heterogeneous deformation.
- ***Lüder bands*** or stretcher strains are formed at approximately 45^{o} to the tensile axis during yield point elongation and propagate over the specimen.



Yield point behaviour in BCC metals

Note: The yield point phenomenon has also been observed in other metals such as Fe, Ti, Mo, Cd, Zn, Al alloys.

The upper yield point

*The **upper yield point** is associated with small amounts of **interstitial** or **substitutional** impurities.*

- The solute atoms (***C*** or ***N***) in low carbon steel, ***lock*** the dislocations, → ***raise the initial yield stress***.

- The breakaway stress required to pull a dislocation line away from a line of solute atoms is

$$\sigma \approx \frac{A}{b^2 r_o^2} \quad \text{...Eq. 6}$$

Where ***A*** is ***4Gba³ε***, a is atomic radius

r_o is the distance from the dislocation core to the line of solute atoms ~ 0.2 nm.

- When the dislocation is ***pulled free*** from the solute atoms, slip can occur at lower stress. → the ***lower yield point***.

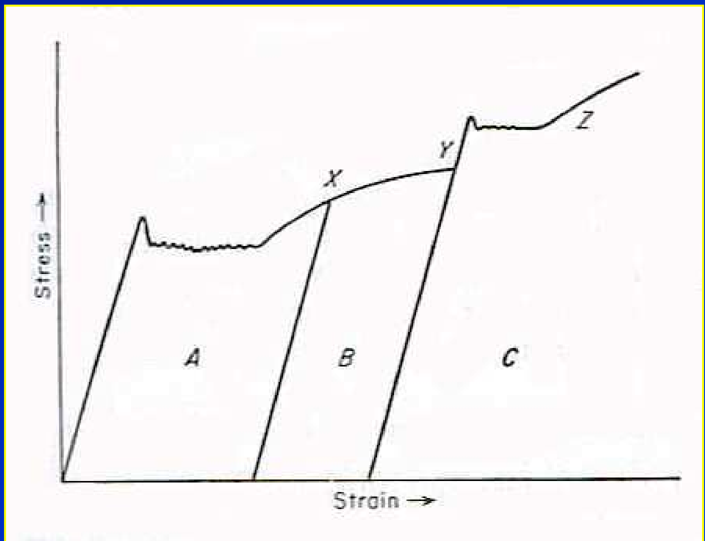
- The magnitude of the yield-point effect depends on ***interaction energy***, ***concentration of solute atoms***.

<u>Note:</u> Upper yield point is promoted by using elastically rigid machine, careful axial alignment of specimen (free from stress concentrations, high strain rate, low temperature.)

Strain ageing

Strain ageing *is a phenomenon in which the metal* ***increase in strength while losing ductility*** *after being heated at relatively low temperature or cold-working.*

The ***reappearing of the (higher) yield point*** after ageing is obtained, see *fig*.

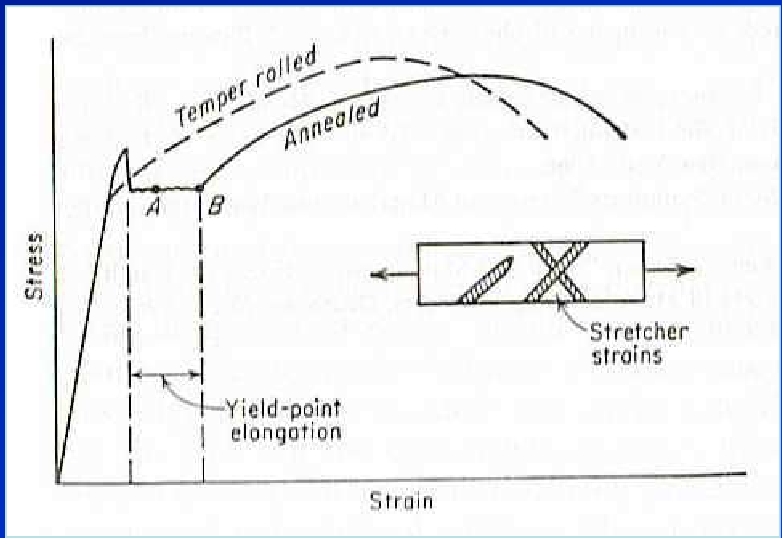


Strain ageing in low-carbon steel.

- Reloading at ***X*** and straining to ***Y*** does not produce yield point.
- After this point if the specimen is reloading after ageing (RT or ageing temp) the ***yield point will reappear*** at a higher value.
- This reappearance of the yield point is due to the ***diffusion of C and N atoms to anchor the dislocations***.
- ***N*** has more strain ageing effect in iron than ***C*** due to a higher ***solubility*** and ***diffusion coefficient***.

Stretcher strains

- **Strain ageing** should be eliminated in **deep drawing steel** since it leads to surface marking or **stretcher strains**.
- To solve the problem, the amount of ***C*** and ***N*** should be lowered by adding elements such as ***Al, V, Ti, B*** to form ***carbides*** or ***nitrides***.



Relation of stretcher strain in stress-strain curve

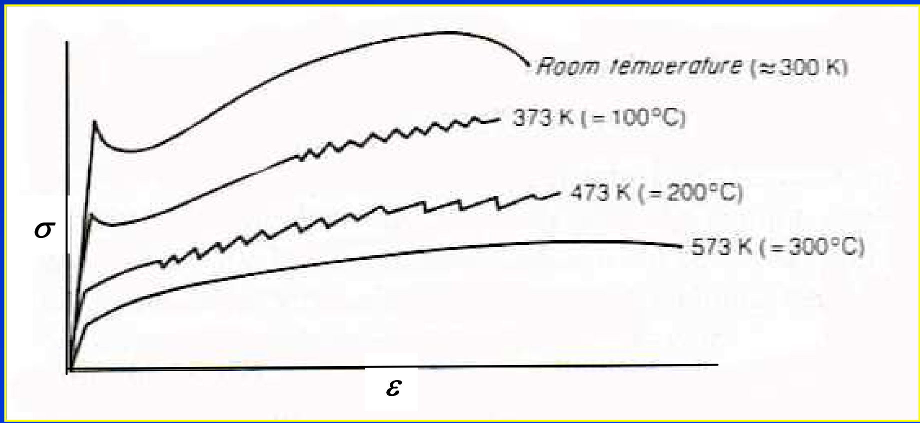
Stretcher strain in low-carbon steel

Serrated stress strain curves

• Strain ageing increases yield point but ***lower ductility***.

• Strain ageing is also associated with ***serrated stress-strain curves*** or ***repeated yielding***, due to high speed of diffusion of solute atoms to catch and lock dislocations.

• This dynamic strain ageing is also called ***Portevin-LeChatelier effect***.



Portevin-LeChaterier effect.

Blue brittleness

Blue brittleness *occurs in plain carbon steel in which* ***discontinuous yielding*** *appears in the temperature range 500 to 650 K.*

During this blue brittleness region, steels show

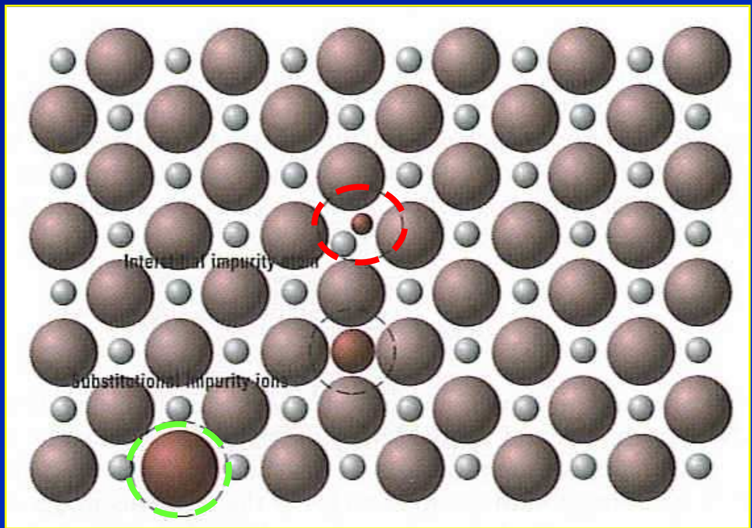
- Decreased tensile ductility.
- Decreased notched-impact resistance.
- Minimum strain rate sensitivity.

Note: This is just an accelerated strain aging by temperature.

Solid-solution strengthening

Solute atoms are introduced into the matrix (solvent atoms).

There are two types of solid solutions;



1) ***<u>Substitutional solid solution</u>***: the solute and solvent atoms are ***similar in size***, rendering the solute atoms to occupy lattice point of the solvent atoms.

2) ***<u>Interstitial solid solution:</u>*** The solute atoms are of ***smaller size*** than the solvent atom, rendering the solute atoms to occupy the interstitial sites in the solvent lattice.

<u>Note:</u> solid solution is compositionally homogeneous, the solute (impurity) atoms are randomly distributed throughout the matrix.

Factors affecting solubility of solute atoms

The ***solubility of the solute atoms*** in the host matrix (solvent) can be determined by several factors;

1) ***Atomic size factors :*** Solid solution is appreciable when the difference in atomic radii between the tow atoms is < ***~15%,*** otherwise → creating substantial lattice distortion.
2) ***Crystal structure :*** ***Similar crystal structure*** of metals of both atom types types are preferred.
3) ***Electronegativity :*** The more ***electropositive*** one element and the more ***electronegative*** the other, the more tendency to form an intermetallic compound than solid solution.
4) ***Valences :*** A metal will have more of a tendency to dissolve another metal of ***higher valency*** than one of a lower valency.

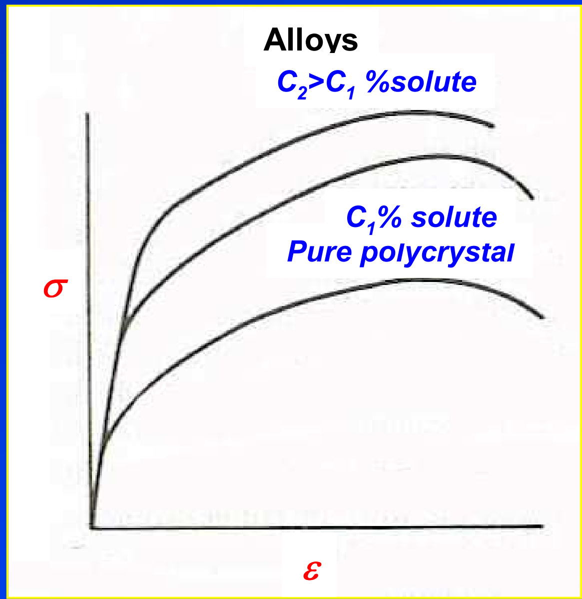
Effects of solute alloy additions on stress-stain curve

• The addition of ***solute atoms*** raises the ***yield stress*** and the ***stress-stain curve*** as a whole.

• Therefore from *Eq.1*

$$\sigma_o = \sigma_i + kD^{-1/2}$$

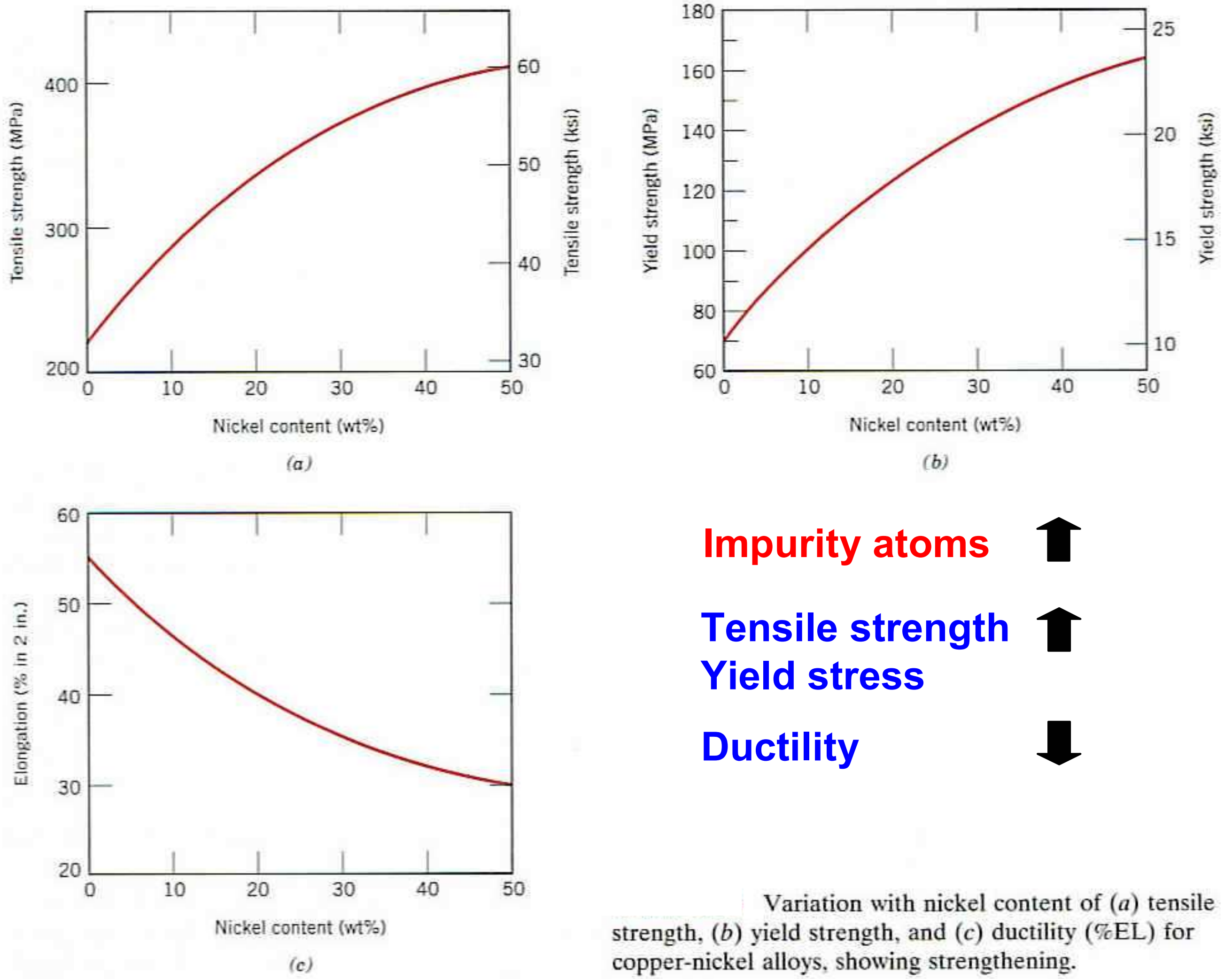
• The ***solute atoms*** should have more influence on the ***frictional resistance to dislocation motion*** σ_i than the locking of dislocation ***k***.



Effects of solute atoms on stress-strain curves.

Solute atoms ↑ ***Strengthening effect*** ↑

Effects of solute alloy additions on tensile properties



Impurity atoms ⬆

Tensile strength ⬆
Yield stress

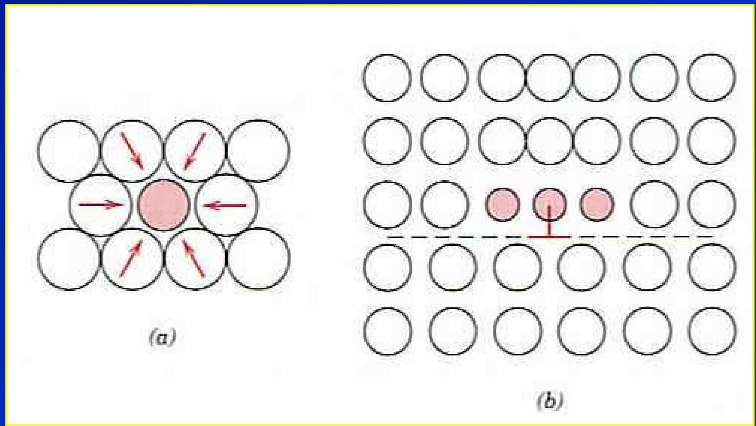
Ductility ⬇

Variation with nickel content of (*a*) tensile strength, (*b*) yield strength, and (*c*) ductility (%EL) for copper-nickel alloys, showing strengthening.

Lattice strain due to solute atoms

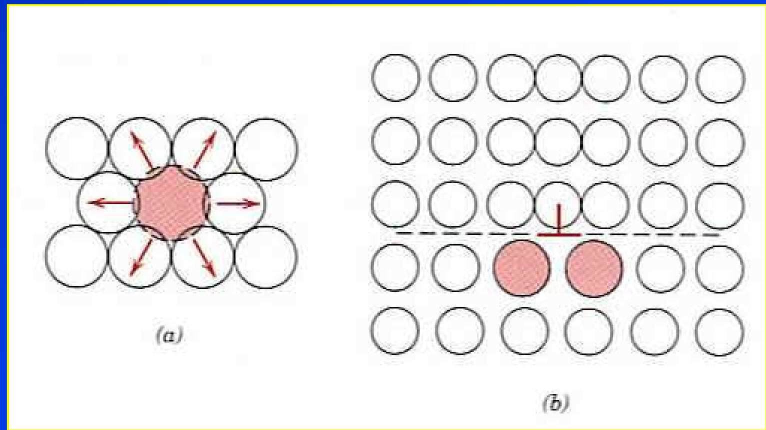
Lattice strains produced by the introduction of solute atoms can be divided into:

1) Tensile lattice strain



Smaller solute atoms are introduced, imposing tensile lattice strain to the host atoms.

2) Compressive lattice strain



Larger solute atoms are introduced, imposing compressive lattice strain to the host atoms.

Interactions between solute atoms and dislocations

Solute atoms can interact with dislocations by the following mechanisms:

1) ***Elastic interaction***
2) ***Modulus interaction***
3) ***Stacking-fault interaction***
4) ***Electrical interaction***
5) ***Short-range order interaction***
6) ***Long-range order interaction***

Note: 1, 2, 6 are insensitive to temperature and influence at about $0.6T_m$.

Elastic interaction

• Strengthening due to elastic interaction is proportional to the ***misfit between solute atoms and dislocations*** giving elastic field around surrounding them.

Modulus interaction

• The presence of the solute atom locally alter the modulus of the crystal. Solute atom with ***small shear modulus*** will reduce the energy of the strain field.

Stacking-fault interaction

Solute atoms within the stacking fault ↑ → Stacking fault energy ↓ → Separation between partial dislocations ↑

Electrical interaction

• The solute atoms having charge can interact with dislocation which have electrical dipoles. → ***weak***.

Short-range order interaction

• Strengthening by short-range order interaction is due to more work which has to put in when dislocations try to move pass through the short range ordered atoms.

Long-range order interaction

• Alloys having a long-range periodic arrangement of dissimilar atoms develop ***superlattice***. The stress required to move a dislocation through a long-range region is high and the rate of ***strain hardening*** is higher in the ordered condition than the disordered state.

Strengthening from second phase

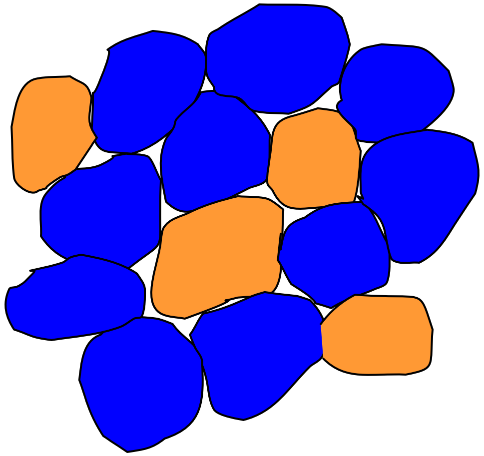
Many commercial alloys are composed of two or more metallurgical phases which provide strengthening effects:

- **Two phase aggregates**
- **Second phase/intermetallic particles**
- **Precipitation hardening**
- **Fibering structure**

Note: *1)These are heterogeneous on a microscopic scale or maybe homogeneous on a macroscopic scale.*

2) Strengthening from second phases is normally additive to the solid solution strengthening produced in the matrix.

Strengthening by two-phase aggregates



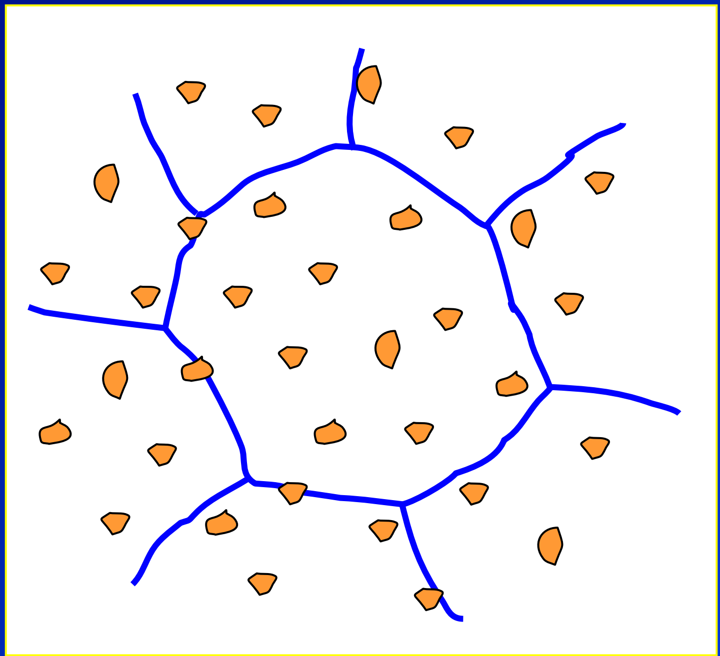
Two-phase aggregates

The size of the second phase particles are of similar size to that of the matrix.

Examples ;

- Beta brass particles in an alpha brass matrix
- Pearlite colonies in the ferrite matrix in annealed steels

Strengthening by second phase particles

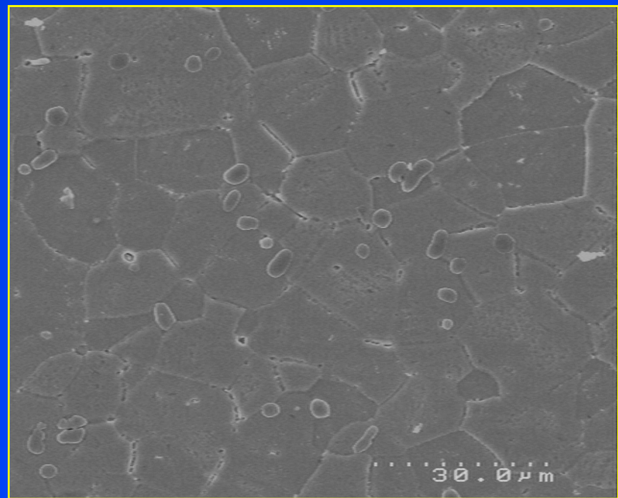


Dispersed second-phase particles in the matrix.

- The ***second phase or intermetallic particles*** are much finer (down to submicroscopic dimensions) than the grain size of the matrix.
- The second phase particles produce ***localised internal stresses*** which alter the ***plastic properties*** of the matrix.

Examples ;

- Second phase particles in matrix.



Factors influencing second-phase particle strengthening

Particle size

Particle shape

Number (V_f)

Distribution (interparticle spacing)

}

Strength

Ductility

Strain hardening

Note: Its almost impossible to vary these factors independently in experiments.

If the contributions of each phase are independent, the properties of the multiple phase alloy is the summation of a weighted average of individual phases.

For example;

Stress $$\sigma_{avg} = V_1\sigma_1 + V_2\sigma_2 + ...V_n\sigma_n$$ *…Eq. 7*

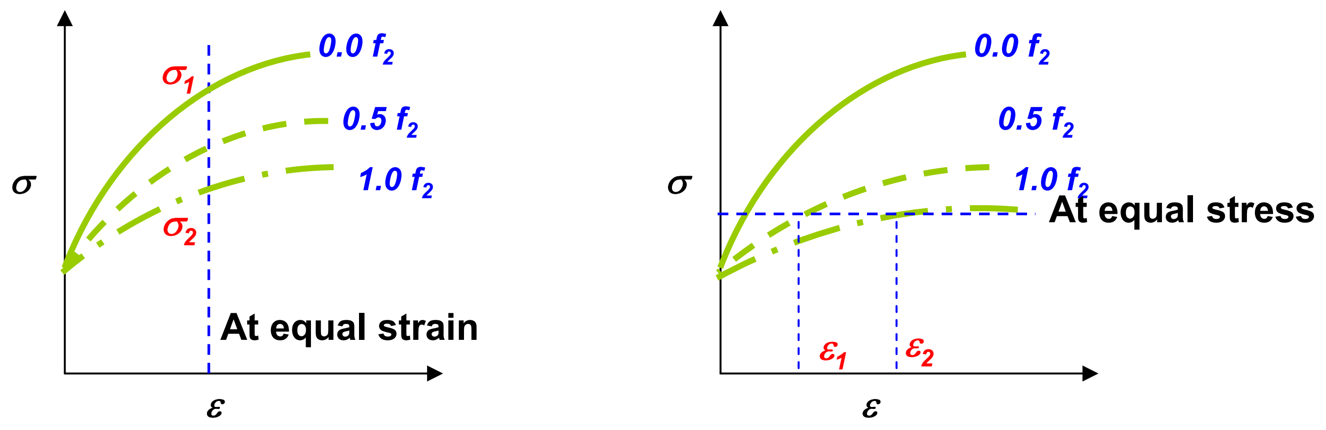
Strain $$\varepsilon_{avg} = V_1\varepsilon_1 + V_2\varepsilon_2 + ...V_n\varepsilon_n$$ *…Eq. 8*

Where the volume fraction ***V***

$V_1 + V_2 + ... + V_n = 1$

Estimate flow stress of two-phase alloy

- The average property in the ***two-phase alloy*** will increase with the volume fraction V_f of the strong phase.



(a) At equal strain *(b) At equal stress*

- It is more often that the ***second phase is stronger than the matrix*** but not all second-phase particles produce strengthening effects.

- The ***strong bonding between particles*** and matrix is required to be able to produce strengthening effects.

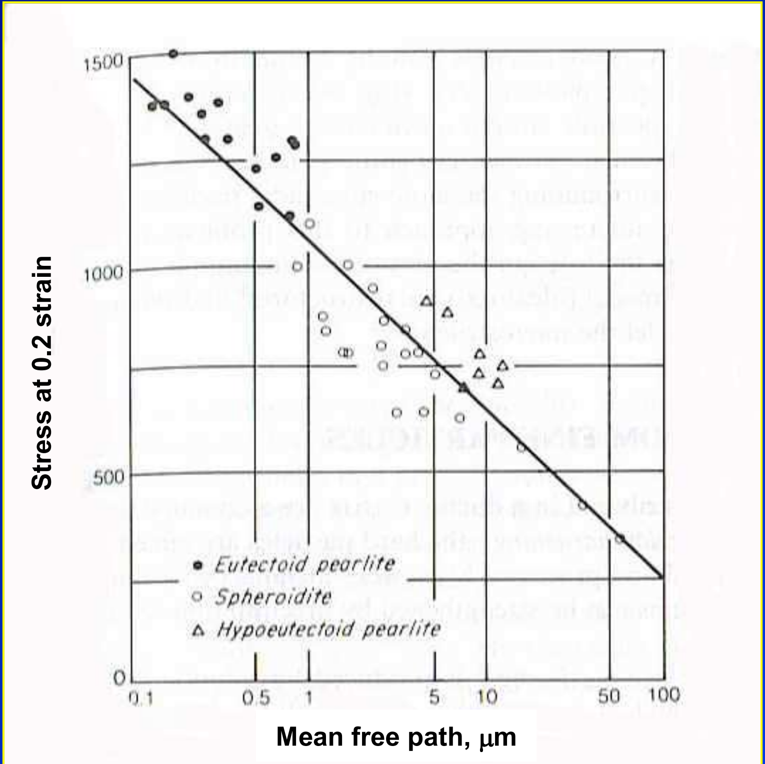
Deformation in two ductile phase alloys

- Depending on the V_f of the two phases and the total deformation.
- Slip will occur first in the weaker phase
- Not all second phase particles produce strengthening effects.

Deformation in alloy of a ductile and brittle phase

- Mechanical properties depends on how the hard brittle phase distribute throughout the softer matrix.
- Homogeneously distributed hard particles promote strength.
- Continuously distributed along the grain boundaries leads to brittle fracture.→ ***reduce strength***.

Microstructure dependence of yield stress in steels



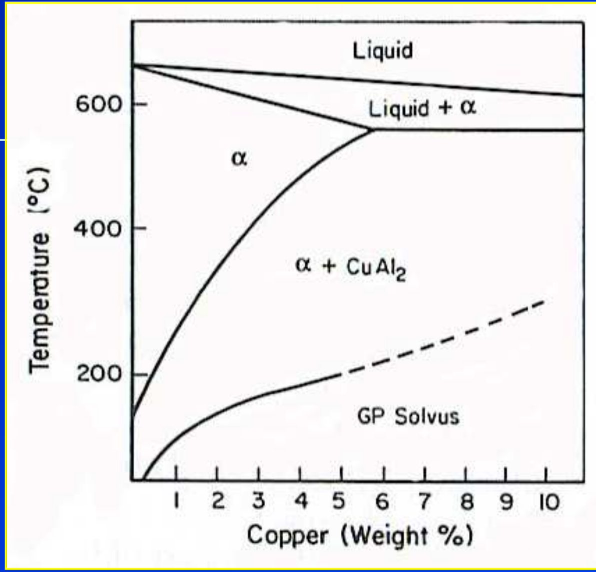
Flow stress vs. log of mean free ferrite path in steels.

- ***Gensamer et al*** studied the influence of different microstructures obtained from ***annealed, normalised and spheroidized steels*** (aggregates of ***cementites*** and ***ferrite***).
- The $\sigma_{0.2\%}$ was inversely proportional to the logarithm of ***mean free ferrite path*** (interparticle spacing).

Interparticle spacing ⇩ **Yield stress** ⇧

Precipitation hardening

Precipitation hardening or age hardening requires the second phase which is ***soluble at high temperature*** but has ***a limited solubility at lower temperatures***.



Al-Cu phase diagram

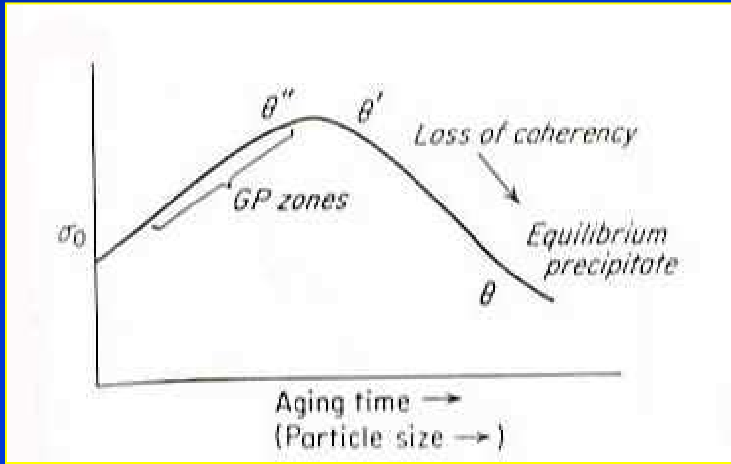
Solution treating at high temperature, then quenching	→	**Second phase is in solid solution.**
Ageing at low temperature	→	**Precipitation of the second phase, giving strengthening effect.**

Example: Age hardening aluminium alloys
Copper-beryllium alloys

Note: In precipitate-hardened system, there is coherency between the second-phase particle and the matrix.
But in dispersion-hardened system, there is no coherency.

The formation of coherency precipitate

A number of steps occurs during precipitation hardening.



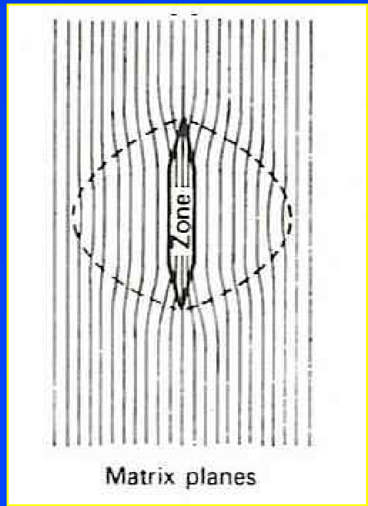
Variation of yield stress with ageing time.

- After quenching from solid solution the alloy contains ***areas of solute segregation or clustering.*** → ***GP zone.*** This clustering is GP[1] produces local strain giving higher hardness than the matrix.

- The hardness of the ***GP zone*** increases with ageing time, developing GP[2] or ***θ''.***

- Precipitate ***θ'*** is ***coherent*** with the matrix. → further ***increase in hardness.***

- Further ageing produces *θ*, (***not coherent*** with the matrix). → ***lowering the hardness***.



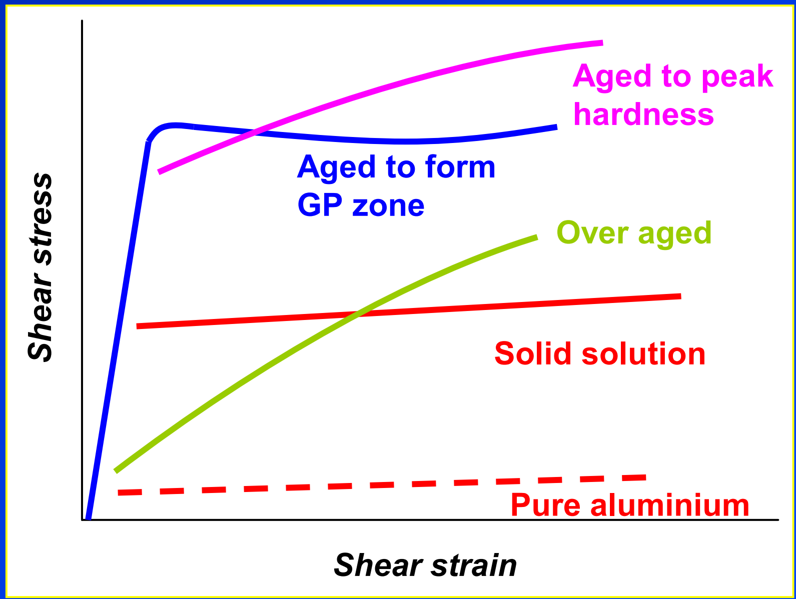
Deformation of alloys with fine particle strengthening

Case study in deformation of Al-4.5%Cu single crystal

• After solution treated and quenched, ***copper is in supersaturated solid solution***, giving ***higher yield stress*** than pure aluminium.

• The yield stress increases when the crystal is aged to form coherent ***GP zone***. Yield drop and low strain hardening suggest that dislocations cut through the zone once the stress reaches a high enough value.

• ***Strain hardening*** significantly increase when the crystal is aged to ***peak hardness***. Dislocations are short and move around particles.



• ***Over-aged condition*** produces coarse noncoherent particles, giving low yield stress, high strain hardening.

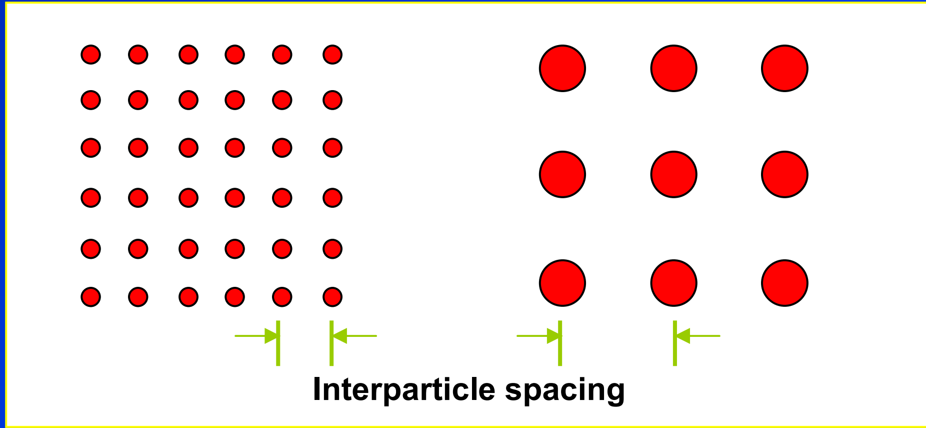
Factors affecting precipitation hardening

Particle size, shape, volume fraction and distribution are ***key factors*** in improving precipitation hardening (cannot vary independently).

- High strength alloys seem to consist of ***fine strong particles well distributed in deformed matrix.***
- Fine hard particles increase strength by cutting dislocations → dislocation tangles → increasing strain hardening.
- Deformed matrix bares the load which makes fracture more difficult.

Example: For a given V_f

Particle size ↑ ***Interparticle spacing*** ↑



Interparticle spacing λ

$$\lambda = \frac{4(1-V_f)r}{3V_f} \qquad \ldots Eq.\ 9$$

Where V_f is the volume fraction of spherical particles of radius r.

Properties affecting strengthening mechanisms by particles

Coherency strain

• Misfit between particles and matrix produces strain field → improving strength.

Modulus effect

• Modulus difference between the matrix and the particles produces strength but it is not the case in most alloys.

Stacking-fault energy

• Yield stress increases with the difference in stacking fault energy between the particle and the matrix.

Interfacial energy and morphology

• High particle-matrix surface energy leads to higher strength. (rely on surface-to-volume ratio or morphology)

Ordered structure

• introduce anti-phase boundaries.
• good high temperature strength.

Lattice friction stress

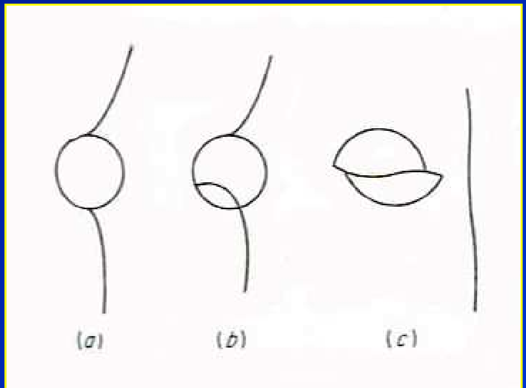
• Peierls stress in particle and matrix produce strengthening effect.

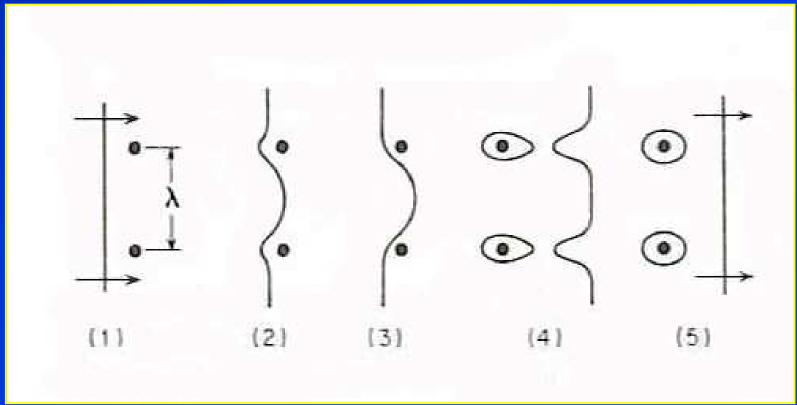
Interaction between fine particles and dislocations

***Second phase particles** act in two distinct ways to retard the motion of dislocations.*

1) Particles maybe cut by dislocation

2) Particles allow dislocation to bypass/bow around them.





Orawan's mechanism of dispersion hardening.

- When the particles are small / soft.

- In over aged noncoherent precipitates. Bowing of dislocations around particles leaving dislocation loops behind.

- *Stress required to force dislocation between particles;*

$$\tau_o = \frac{Gb}{\lambda} \quad \text{...Eq. 10}$$

Role of slip character

The slip character can be characterised in to;

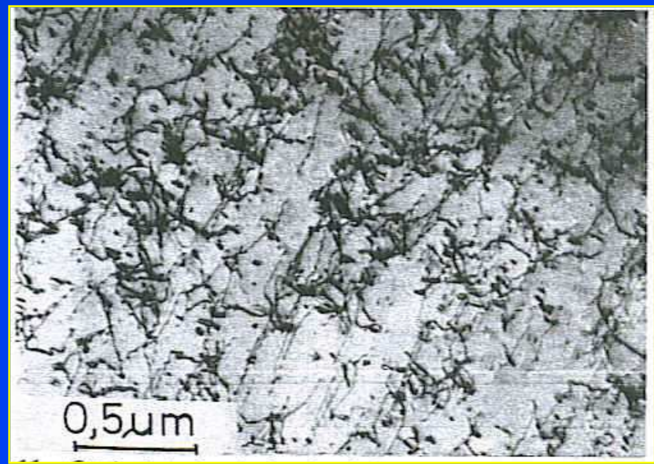
- ***Planar or wavy***
- ***Coarse or fine***

- ***Coarse planar slip*** → promotes brittle failure. Particles which is easily sheared by dislocations tend to produce coarse planar slips.

- ***Fine wavy slip*** → homogeneous deformation, giving best ductility at a given strength level. Particles which are by passed by dislocations lead to fine wavy slip.

Planar slips in aged hardenable Al alloy.

Coplanar bands in warm rolled nitrogen-alloyed austenistic stainless steel

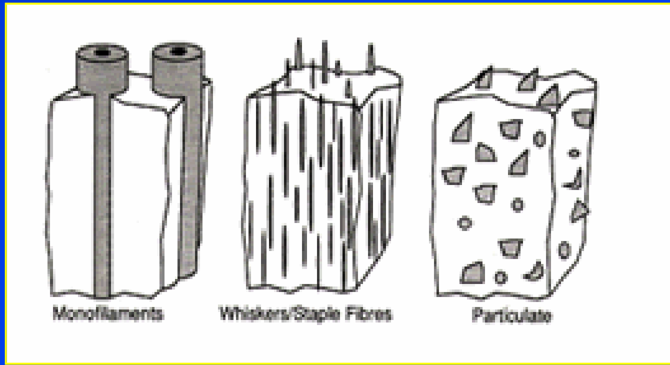


Wavy slips

Fibre strengthening

- ***Ductile metals*** can be reinforced using relatively ***stronger fibres***.
- Very high strength whiskers of Al_2O_3, or ***SiC*** fibres have been used for this purpose.
- ***Fibre-reinforced materials*** (metal or polymer as matrix) are also known as composite materials.

- The matrix transmits the load to the fibres.
- protect fibres from surface damage.
- separate individual fibres and blunt crack from fibre breakage.



- High modulus fibres in Fibre-reinforced metals carry more load than dispersion-reinforced metals.
- Fibre-reinforced materials are highly anisotropic.

Note: Variation of stress between fibres and matrix is complex.

Strength and moduli of composites

The ***rule of mixtures*** is used to approximate the modulus and strength of a fibre-reinforced composite.

If a tensile force ***P*** is applied in the direction of the fibre, and assuming that the strain of fibre and matrix are similar, $e_f = e_m = e_c$.

$$P = \sigma_f A_f + \sigma_m A_m$$ ...*Eq. 11*

Where A_f and A_m are the cross-sectional areas of fibre and matrix.

The average composite strength σ_c is

$$\sigma_c = \frac{P}{A_c} = \frac{\sigma_f A_f}{A_c} + \frac{\sigma_m A_m}{A_c}$$

$$\sigma_c = \sigma_f V_f + \sigma_m V_m$$ ...*Eq. 12*

where

$$A_c = A_f + A_m$$

$$V_f + V_m = 1$$

Likewise

$$E_c = E_f V_f + E_m V_m$$ ...*Eq. 13*

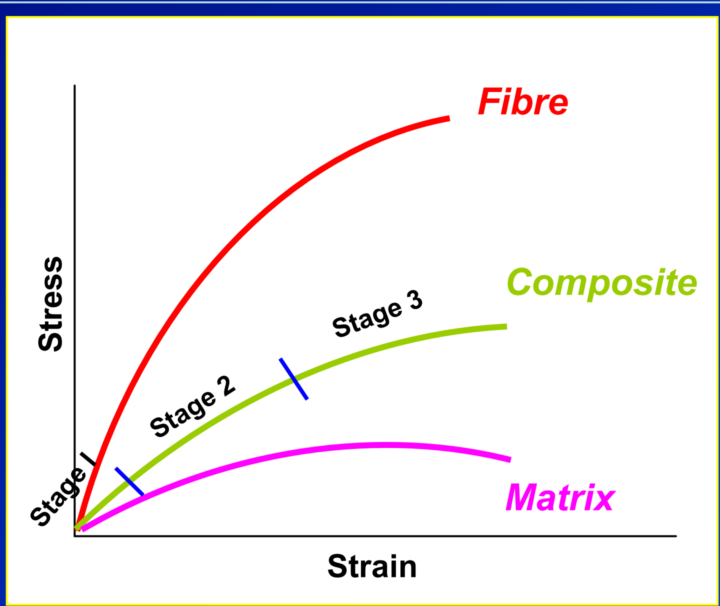
Example: *Boron fibre, E_f = 380 GPa, are made into a unidirectional composite with an aluminium matrix, E_m = 60 GPa. What is the modulus parallel to the fibres for 10 and 60 volume%.*

$$E_c = E_f V_f - (1 - V_f) E_m$$

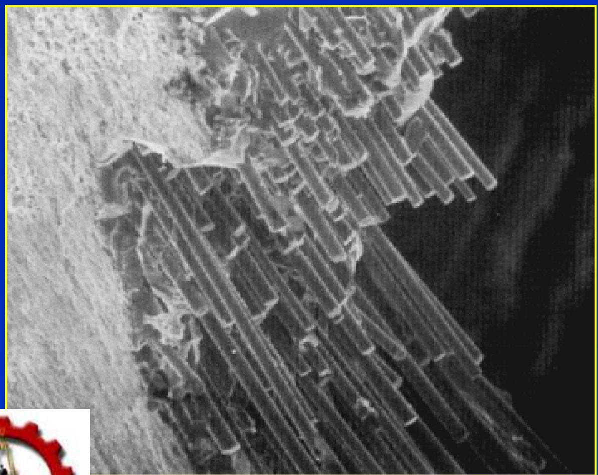
$$V_f = 0.10 \quad , E_c = 380(0.10) + 0.9(60) = 92 \;\; GPa$$

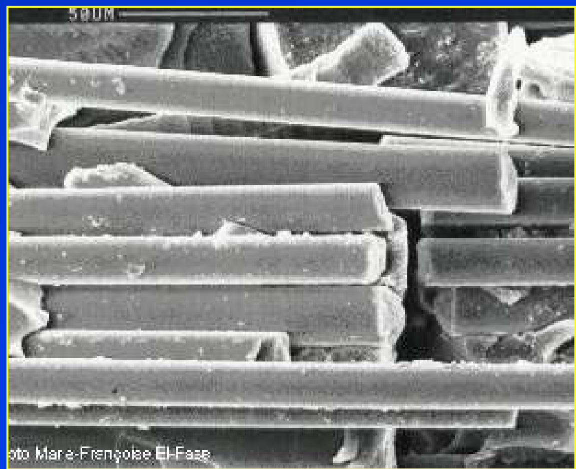
$$V_f = 0.60 \quad , E_c = 380(0.60) + 0.4(60) = 252 \;\; GPa$$

Stress-strain curves of the fibre, matrix and fibre-reinforced composite



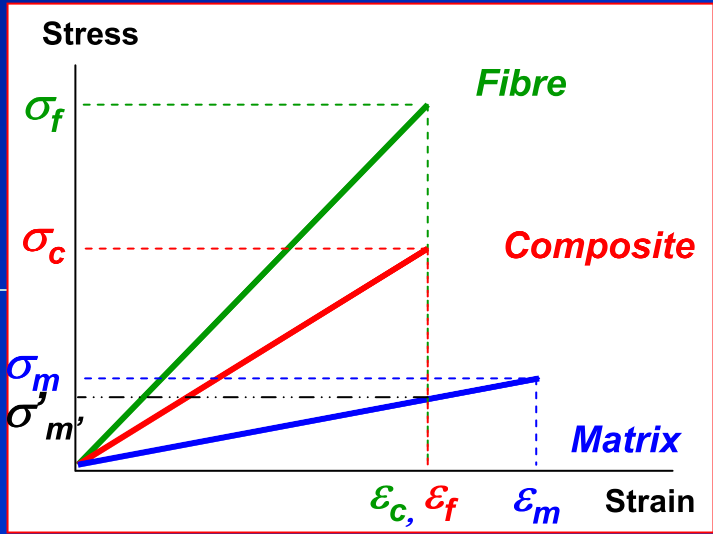
- ***Stage 1 :*** Both fibres and matrix undergo elastic deformation.
- ***Stage 2 :*** Matrix deforms plastically but fibres deform elastically.
- ***Stage 3 :*** Both matrix and fibres undergo plastic deformation.





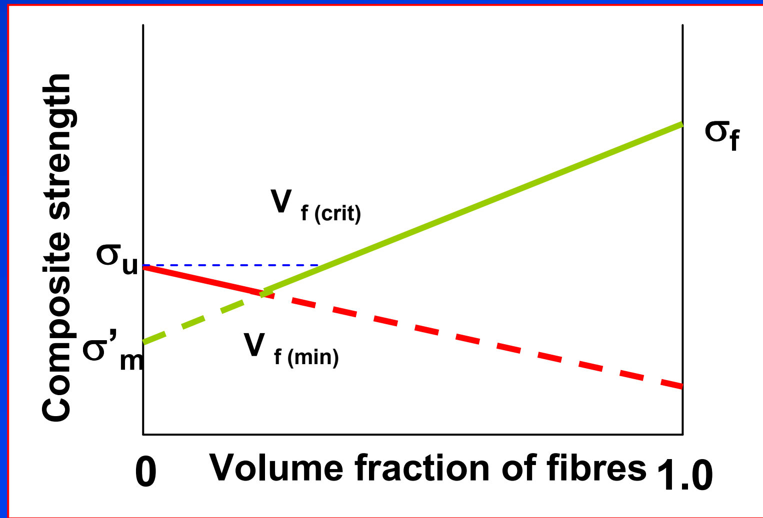
- The load is transferred from ductile matrix to strong fibres.
- Breakage or pull-out of fibres increase the strength.

Theoretical variation of composite strength with volume fraction of fibres



- The ***critical fibre volume*** which must be exceeded for fibre strengthening to occur.

$$V_{f\,(crit)} = \frac{\sigma_{mu} - \sigma'_m}{\sigma_{fu} - \sigma'_m}$$ *…Eq. 14*



- The ***minimum volume fraction*** of fibre which must be exceeded to have real reinforcement.

$$V_{f\,(\min)} = \frac{\sigma_{mu} - \sigma'_m}{\sigma_{fu} + \sigma_{mu} - \sigma'_m}$$ *…Eq. 15*

σ_u is the ultimate tensile strength of the composite

σ_f is the strength of the fibre

σ'_m is the flow stress in the matrix

Martensite strengthening

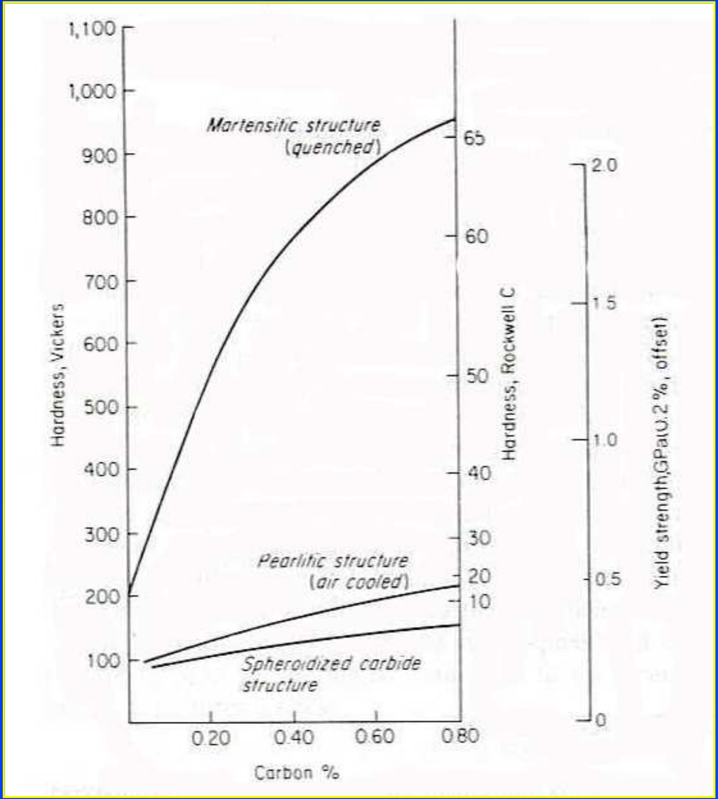
- ***Martensitic strengthening*** is obtained when austenite is transformed into martensite by a ***diffusionless shear-type process*** in quenching.

- ***Martensitic transformation*** occurs in many alloy systems but ***steels*** has shown the most pronounced effect.

High strength of martensite is due to two main contributions;

- ***Slip barriers*** *from (1) conventional* ***plate martensite structure*** *with a unique habit plane and an internal parallel twins of each 0.1 mm thick within the plate and (2)* ***Block martensite structure*** *containing a high dislocation density of* 10^{9} *to* 10^{10} mm^{-2}.

- ***Carbon contents*** *(<0.4%) lead to carbon atom clustering and dislocation interaction → increased strength and hardness, see fig.*



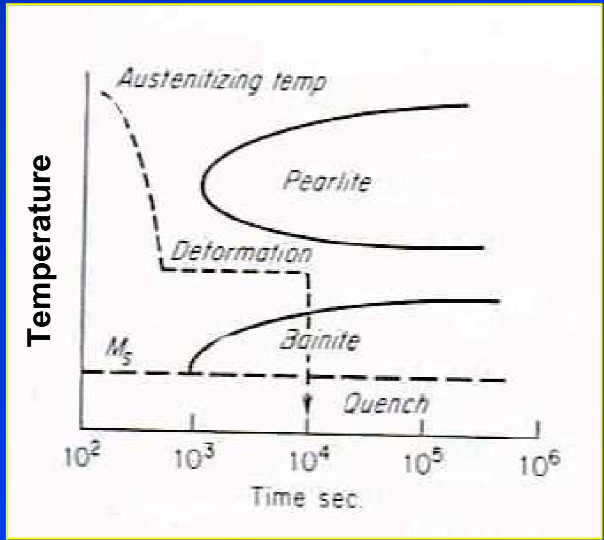
Hardness in various products in steel.

Ausforming process

***Ausforming** is a thermo-mechanical process where steel is plastically deformed (>50%) usually rolling and then quenched to below the M_s to form **martensite**.*

- Plastic deformation of austenite should be done ***without transformation to pearlite or bainite***.

- ***Highest strengths*** are achieved by the greatest possible deformation at the lowest temperature at transformation does not occur.



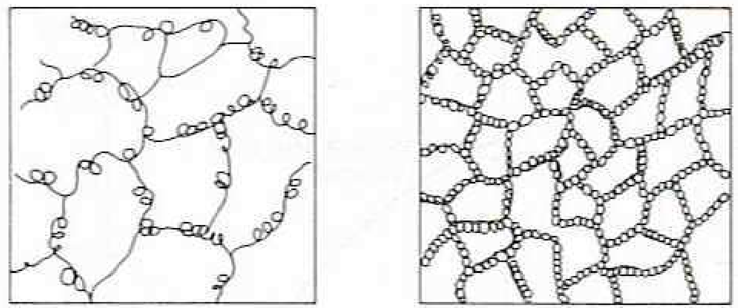
TTT diagram showing steps in ausforming process.

- Uniformly distributed dislocations of high density (10^{11} mm^{-2}) and precipitation provides sites for ***dislocation multiplication*** and ***pinning***, contribute to very high strength (2-3 GPa) with 40-20% RA.

Strain hardening or cold working

Cold-work structure occurs when plastic deformation carried out at in a temperature region and over a time interval such that the strain hardening is not relieved.

- ***Cold worked structure*** contains dislocation ~ 10^{11} mm^{-2}, while annealed structure possesses ~10^4 to 10^6 mm^{-2}.
- As the deformation proceeds, the high density dislocations tangles form the ***cell walls***.
- About 10% of energy input in cold work process is stored in the lattice.



(a) Deformed to 10% *(b) Deformed to 50%*

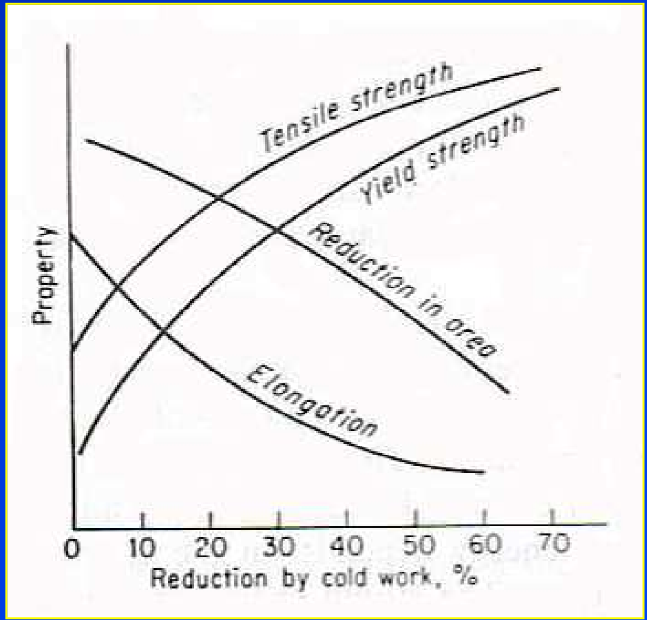
Dislocations in cell walls.

Temp ↓ **Strain rate** ↑ **Stored energy** ↑

Strain hardening

• Strain hardening or cold working is used to harden alloys that do not respond to heat treatment.

Cold work ↑	**Strength** ↑
	Ductility ↓



Variation of tensile properties with amount of cold-work.

• The rate of strain hardening is lower in ***HCP*** than in ***cubic metals***.

• The final strength of ***cold-worked solid solution alloy*** is almost always greater than that of the ***pure metal cold-worked to the same extent***.

Annealing of cold-worked metal

- ***Annealing*** of the cold worked structure at high temperature ***softens the metal*** and reverts to a strain-free condition.
- Annealing restores the ductility to a metal that has been severely strain hardened.

- Annealing can be divided into ***three distinct processes;***

1) Recovery

2) Recrystallisation

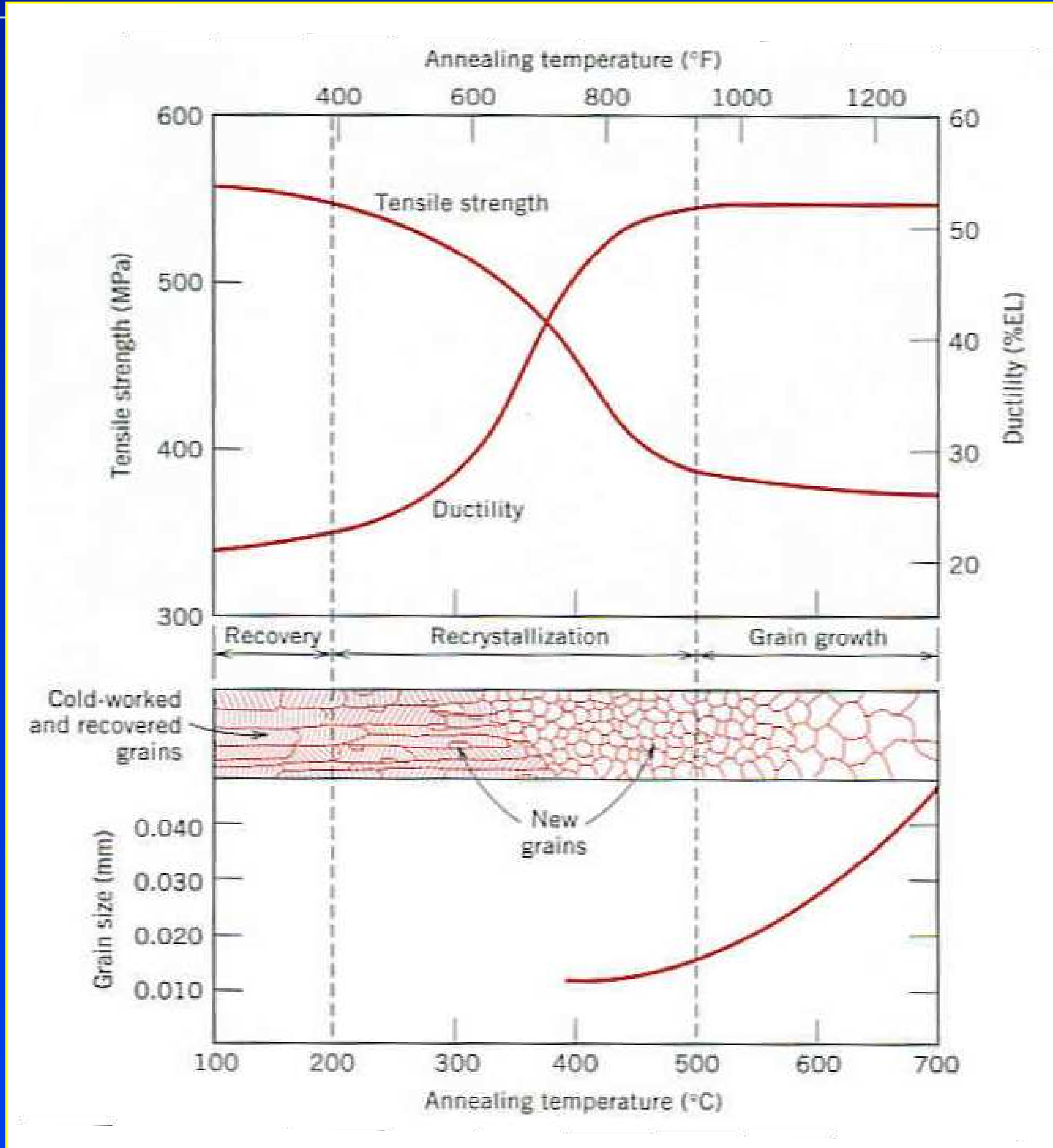
3) Grain growth

Recovery, recrystallisation, grain growth

- ***Recovery*** : the restoration of the physical properties of the cold-worked metal without any observable change in microstructure. *Strength is not affected.*

- ***Recrystallisation*** : the cold-worked structure is replaced by a new set of strain-free grains. *Hardness and strength decrease but ductility increases.*

- ***Grain growth*** : occurs at higher temperature where some of the recrystallised fine grains start to grow rapidly. *Grain growth is inhibited by second phase particles to pin the grain boundaries*.



Properties change during recovery, recrystallisation and grain growth

Variables affecting recrystallisation behaviour

There are six variables affecting recrystallisation behaviour.

1) The amount of prior deformation
2) Temperature
3) Time
4) Initial grain size
5) Composition
6) Amount of recovery prior to start the recrystallisation.

Degree of deformation ⇩ T_{recrys} ⇧

Degree of deformation ⇧

T_{anneal} ⇩ , GS_{recrys} ⇩

$GS_{original}$ ⇧ ***Cold work*** ⇧

- ***Impurity*** *decrease recrystallisation temperature.*
- ***Solid solution alloying*** *additions raise the recrystallisation temperature.*

Preferred orientation (texture)

• *Severe deformation produces a **reorientation of the grains** into a **preferred orientation**. Certain crystallographic planes tend to orient themselves in a preferred manner with respect to the maximum strain direction.*

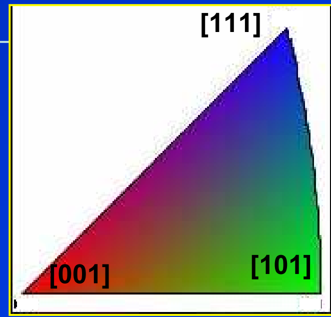
• The ***preferred orientation*** resulting from plastic deformation is ***strongly dependent on the available slip and twining systems***, but not affected by processing variable such as die angle, roll diameter, roll speed, etc.

Table 7.3 Deformation textures in metals with common crystal structures

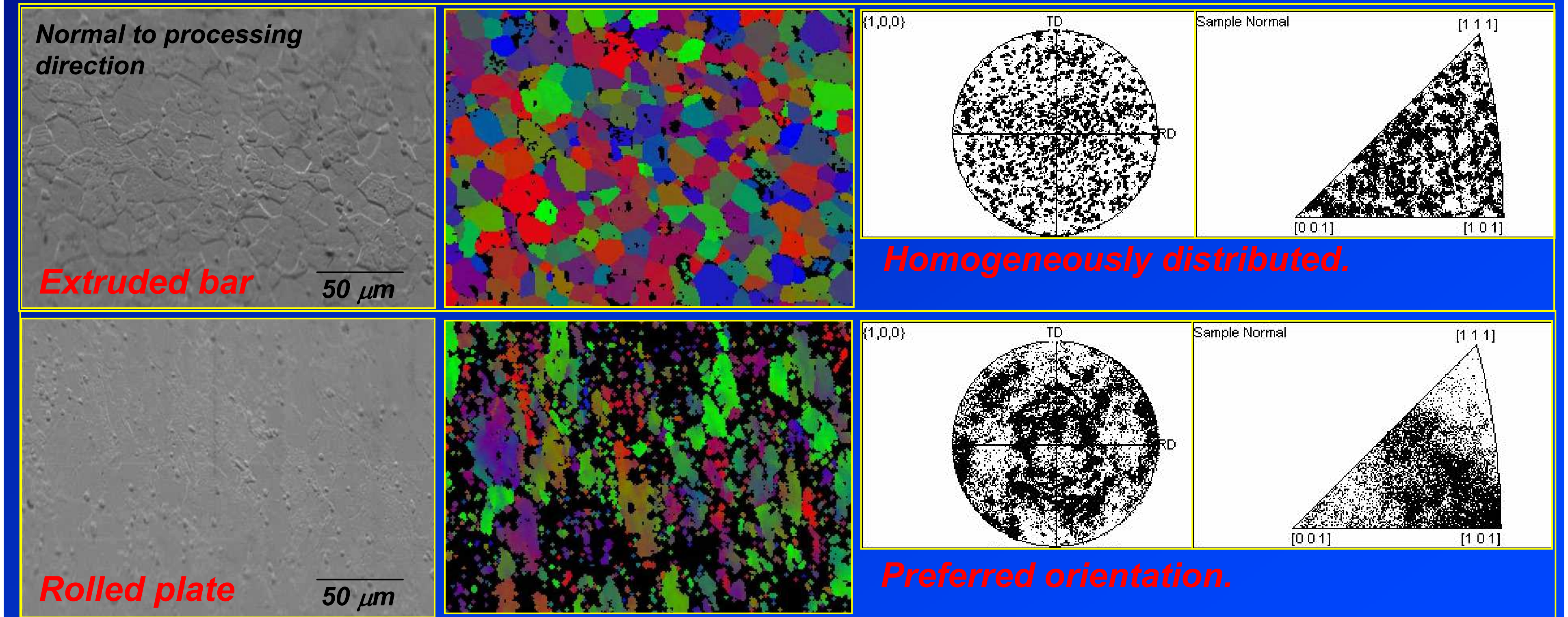
Structure	*Wire (fibre texture)*	*Sheet (rolling texture)*
bcc	[1 1 0]	{1 1 2} ⟨1 $\bar{1}$ 0⟩ to {1 0 0} ⟨0 1 1⟩
fcc	[1 1 1], [1 0 0] double fibre	{1 1 0} ⟨1 1 2⟩ to {3 $\bar{5}$ 1} ⟨1 1 2⟩
cph	[2 1 0]	{0 0 0 1} ⟨1 0 0 0⟩

Note: the deformation texture cannot in general be eliminated by an annealing operations

Grain orientation by EBSD analysis



• EBSD analysis employs ***back scattered electrons*** to give grain orientation information.



SEM micrograph ***Orientation map*** ***Pole figure*** ***Inverse pole figure***

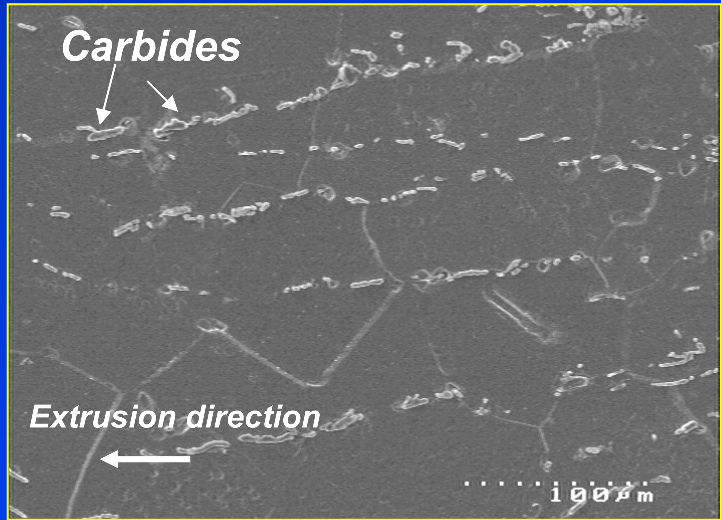
Mechanical fibering (fibrous texture)

• ***Fibrous texture*** *is produced along the maximum stress direction acting on the materials.*

• Inclusions, cavities and second phase constituents are aligned in the main direction of mechanical working.

• The ***geometry of the flow*** and the ***amount of the deformation*** are the most important variables.

• Mechanical fibering increases ***mechanical properties*** along the working (fibre) direction, with the transverse direction having inferior properties. → ***anisotropic properties.***



Alignment of carbides along the extrusion direction in β-Ti alloy.

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Fracture

Subjects of interest

- *Introduction/ objectives*
- *Types of fracture in metals*
- *Theoretical cohesive strength of metals*
- *The development in theories of brittle fracture*
- *Fractographic observation in brittle fracture*
- *Ductile fracture*
- *Ductile to brittle transition behaviour*
- *Intergranular fracture*
- *Factors affecting modes of fracture*
- *Concept of the fracture curve*

Objectives

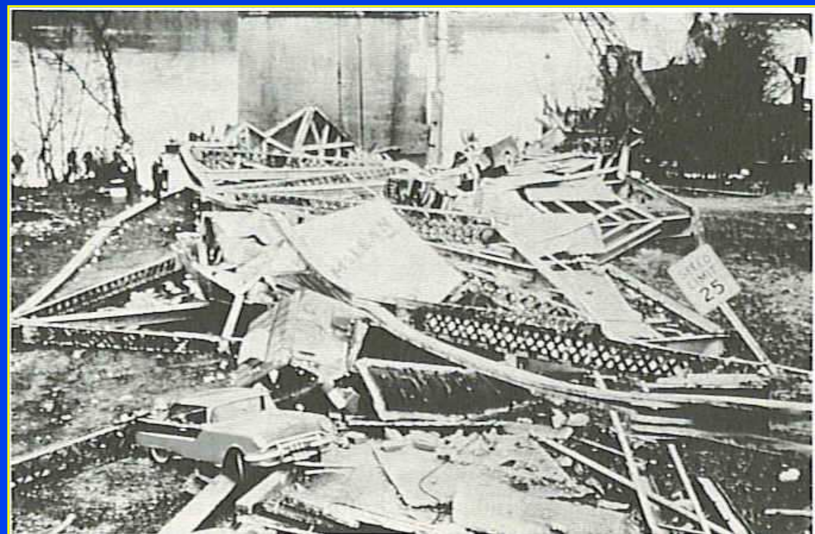
- This chapter provides the development in the theories of brittle fractures together with mechanisms of fracture that might occur in metallic materials.
- Factors affecting different types of fracture processes such as brittle cleavage fracture, ductile failure or intergranular fracture will be discussed.

Introduction

Failure in structures leads to lost of properties and sometimes lost of human lives unfortunately.

Failed fuselage of the Aloha 737 aircraft in 1988.

Failure of Liberty Ships during services in World War II.



Collapse of Point Pleasant suspension bridge, West Virginia, 1967.

Types of fracture in metals

- *The concept of material strength and fractures has long been studied to overcome failures.*
- *The introduction of malleable irons during the revolution of material construction led to the perception of brittle and ductile fractures as well as fatigue failure in metals.*

Failure in metallic materials can be divided into two main categories;

Ductile failure

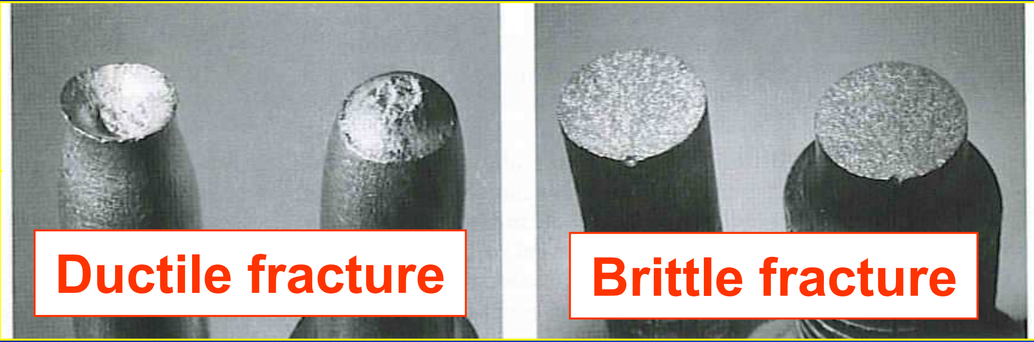
Ductile fracture involves a large amount of plastic deformation and can be detected beforehand.

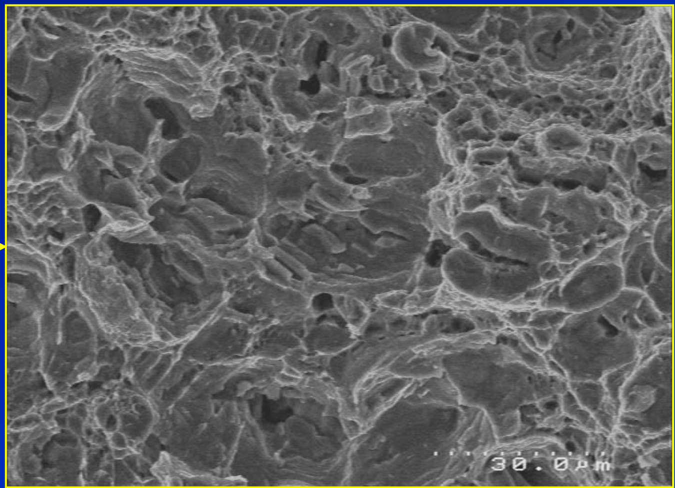
Brittle failure

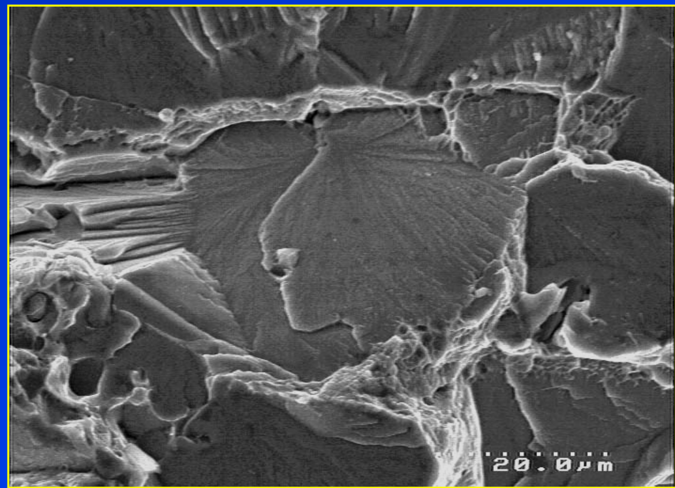
Brittle fracture is more catastrophic and has been intensively studied.

Theories of brittle fracture

Failure modes







- ***High energy*** *is absorbed by microvoid coalescence during ductile failure (high energy fracture mode)*

Less catastrophic

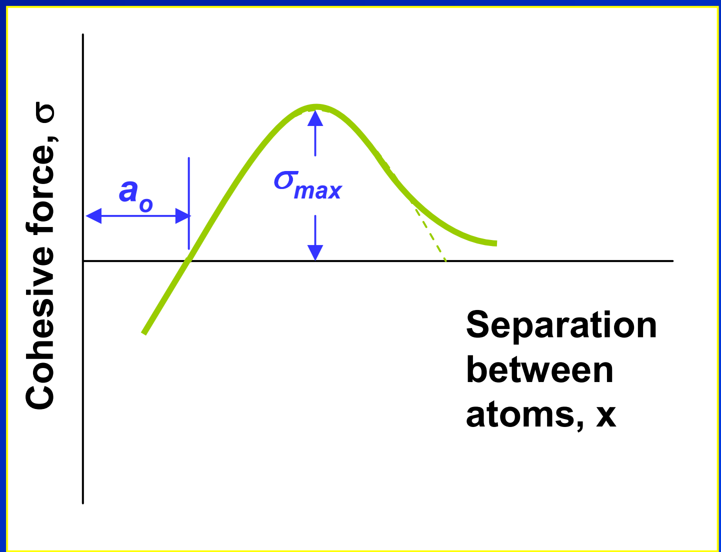
- ***Low energy*** *is absorbed during transgranular cleavage fracture (low energy fracture mode)*

More catastrophic

Theoretical cohesive strength of metal

• In the most basic term, strength is due to the ***cohesive forces between atoms***.

• The ***attractive*** and ***repulsive*** force acting on the two atoms lead to ***cohesive force*** between two atoms which varies in relation to the separation between these atoms, see *fig*.



Cohesive force as a function of the separation between atoms.

The ***theoretical cohesive strength*** σ_{max} can be obtained in relation to the sine curve and become.

$$\sigma_{max} = \left(\frac{E\gamma_s}{a_o} \right)^{1/2} \qquad \text{...Eq. 1}$$

Where

γ_s is the surface energy

a_o is the unstrained interatomic spacing.

*Note: Convenient estimates of σ_{max} ~ **E/10**.*

Fracture in single crystals

The ***brittle fracture*** of single crystals is related to the resolved normal stress on the cleavage plane.

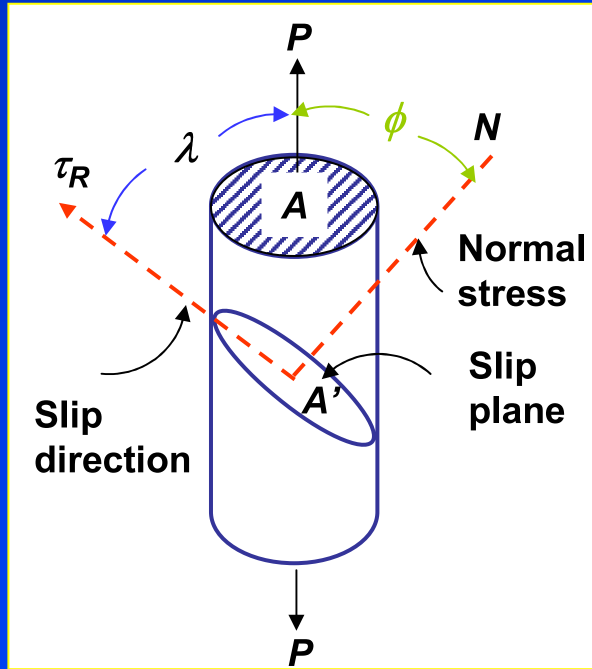
> ***Sohncke's law*** *states that fracture occurs when the* ***resolved normal stress*** *reaches a critical value.*

From the critical resolved shear stress τ_R for slip

$$\tau_R = \frac{P\cos\lambda}{A/\cos\phi} = \frac{P}{A}\cos\phi\cos\lambda \qquad \text{...Eq. 2}$$

The critical normal stress σ_c for brittle fracture

$$\sigma_c = \frac{P\cos\phi}{A/\cos\phi} = \frac{P}{A}\cos^2\phi \qquad \text{...Eq. 3}$$



Note: shear stress → slip
tensile stress → crack propagation → fracture.

Example: *Determine the cohesive strength of a silica fibre, if **E** = 95 GPa, γ_s = 1 J.m^{-2}, and **a**$_o$ = 0.16 nm.*

$$\sigma_{max} = \left(\frac{E\gamma_s}{a_o}\right)^{1/2} = \left(\frac{95\times10^9\times1}{0.16\times10^{-9}}\right)^{1/2} = 24.4 \;\; GPa$$

- This ***theoretical cohesive strength*** is exceptionally higher than the fracture strength of engineering materials.
- This difference between cohesive and fracture strength is due to ***inherent flaws or defects*** in the materials which lower the fracture strength in engineering materials.
- ***Griffith*** explained the discrepancy between the ***fracture strength*** and theoretical cohesive strength using the ***concept of energy balance***.

Theories of brittle fracture

Griffith theory of brittle fracture

The first analysis on cleavage fracture was initiated by ***Griffith*** using ***the concept of energy balance*** in order to explain discrepancy between the theoretical cohesive strength and observed fracture strength of ideally brittle material.

The development in cleavage fracture models

- ***Modified Griffith theory*** *by Irwin and Orowan.*
- ***Zener's model*** *of microcrack formation at a pile-up of edge dislocations.*
- ***Stroh's model*** *of cleavage crack formation by dislocation pile-up.*
- ***Cottrell's model*** *of cleavage crack initiation in BCC metals*
- ***Smith's model*** *of microcrack formation in grain boundary carbide film.*

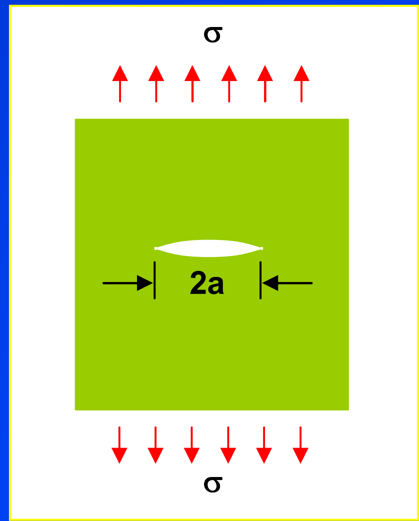
Griffith theory of brittle fracture

Observed fracture strength is always lower than theoretical cohesive strength

Griffith explained that the discrepancy is due to the ***inherent defects*** in brittle materials leading to stress concentration. → lower the fracture strength of the materials

Crack propagation criterion:

Consider a through thickness crack of length ***2a,*** subjected to a uniform tensile stress σ, at infinity.



Griffith crack model

Crack propagation occurs when the released elastic strain energy is at least equal to the energy required to generate new crack surface.

- The stress required to create the new crack surface is given as follows;

$$\sigma = \left(\frac{2E\gamma_s}{\pi a} \right)^{1/2} \quad \text{...Eq. 4}$$

- In plane strain condition, *Eq.4* becomes

$$\sigma = \left(\frac{2E\gamma_s}{(1-\upsilon^2)\pi a} \right)^{1/2} \quad \text{...Eq. 5}$$

The Griffith's equation

Modified Griffith equation

- The ***Griffith equation*** is strongly dependent on the crack size ***a***, and satisfies only ***ideally brittle materials*** like glass.

- However, ***metals*** are not ideally brittle and normally fail with certain amounts of plastic deformation, the fracture stress is increased due to ***blunting of the crack tip***.

- ***Irwin*** and ***Orowan*** suggested ***Griffith's equation*** can be applied to ***brittle materials*** undergone plastic deformation before fracture by including the plastic work, γ_p, into the total elastic surface energy required to extend the crack wall, giving the ***modified Griffith's equation*** as follows

$$\sigma_f = \left[\frac{2E(\gamma_s + \gamma_p)}{\pi(1-\upsilon^2)a}\right]^{1/2} \approx \left(\frac{E\gamma_p}{(1-\upsilon^2)a}\right)^{1/2}, when\ \gamma_p >> \gamma_2$$

...Eq. 6

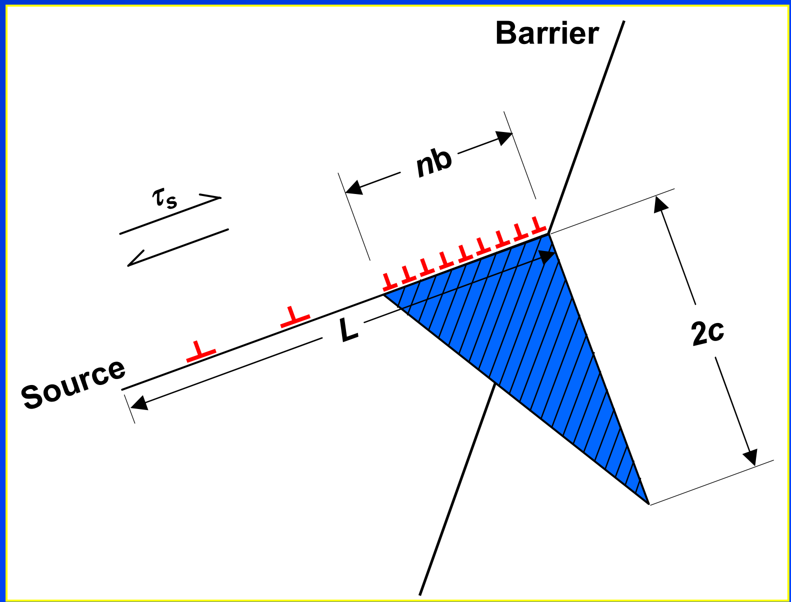
Zener's model of microcrack formation at a pile-up of edge dislocations

The ***Griffith theory*** only indicated the stress required for crack propagation of an existing crack of length ***2a*** but did not explain the ***nucleation of the crack***.

Zener and ***Stroh*** showed that the crack nucleation of length ***2c*** occurs when the shear stress τ_s created by ***pile-up*** of ***n*** dislocations of ***Burgers vector*** **b** at a grain boundary reaches the value of

$$\tau_s \approx \tau_i + \left(\frac{2\gamma_s}{nb}\right) \qquad \text{...Eq. 7}$$

Where τ_i is the lattice friction stress in the slip plane.



Dislocation pile-ups at barrier.

Stroh's model of cleavage crack formation by dislocation pile-up

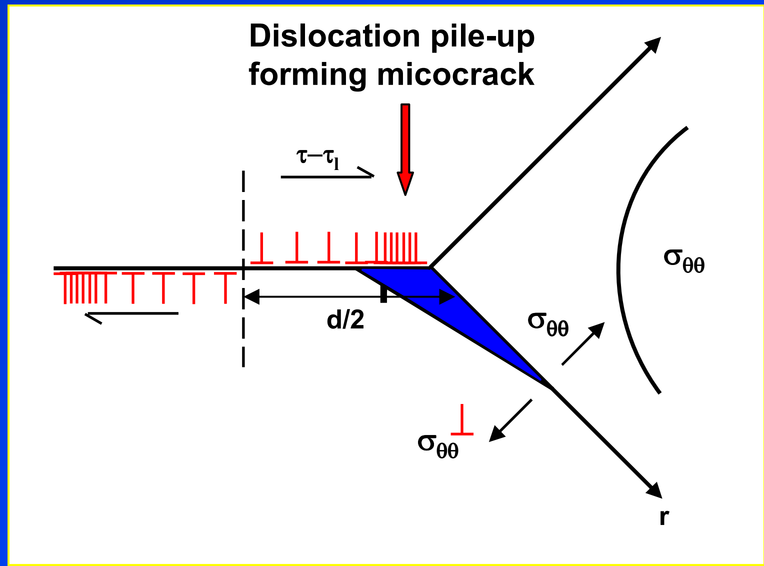
Stroh included the effect of the grain size ***d*** in a model, suggesting the condition of the shear stress created by ***dislocation pile-up*** of the length ***d/2*** to nucleate a ***microcrack*** as follows

$$\tau_{eff} = \tau_y - \tau_i \sqrt{\frac{E\pi\gamma}{4\left(1-\upsilon^2\right)d}} \qquad \text{...Eq. 8}$$

where

τ_{eff} is the effective shear stress

τ_y is the yield stress



Stroh's model of cleavage crack formation by dislocation pile-up.

<u>Note:</u> This model indicates that the fracture of the material should depend only on the ***shear stress*** acting on the slip band.

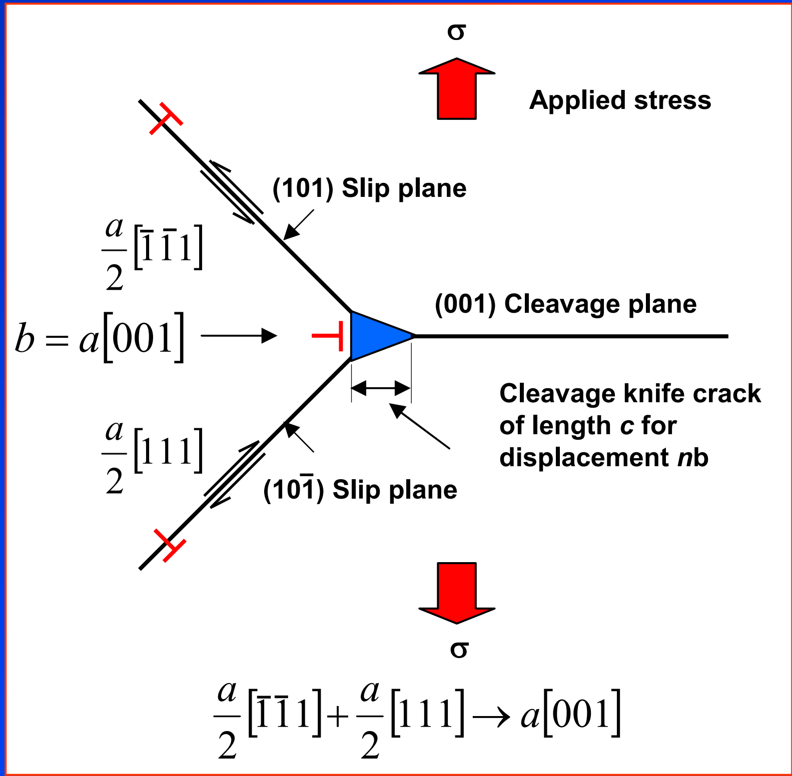
Cottrell's model of cleavage crack initiation in BCC metals

Cottrell later suggested that the fracture process should be controlled by ***the critical crack growth stage*** under the applied tensile stress, which required higher stress than the ***crack nucleation*** itself as suggested by ***Stroh***.

Cottrell also showed that the ***crack nucleation stress*** can be small if the microcrack is initiated by ***intersecting of two low energy slip planes*** to provide a preferable cleavage plane.

$$\frac{a}{2}[\bar{1}\bar{1}1]+\frac{a}{2}[111]\rightarrow a[001] \qquad \text{...Eq. 9}$$

This results in a wedge cleavage crack on the (001) plane. Further propagation of this crack is then controlled by the applied tensile stress.



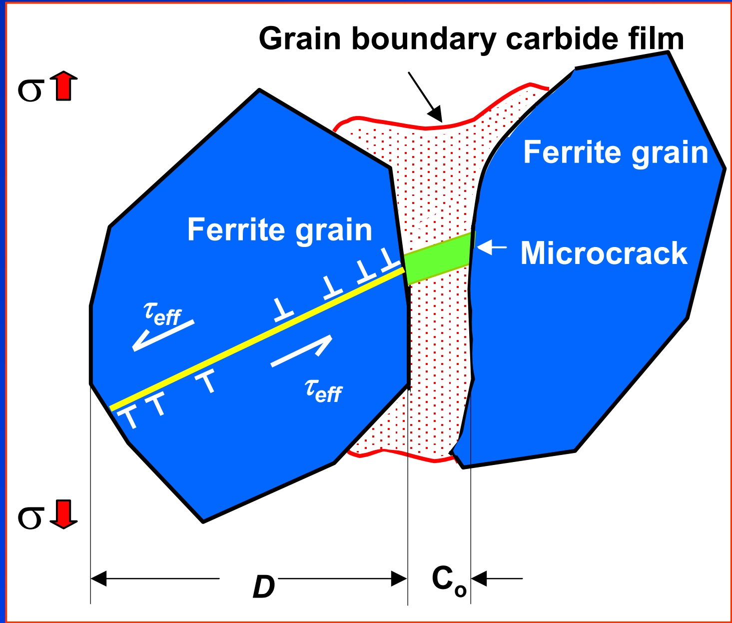
Cottrell's model of cleavage crack initiation in BCC metals

Smith's model of microcrack formation in grain boundary carbide film

Models proposed by ***Stroh*** and ***Cottrell*** involve ***crack initiation by dislocation pile-up*** of length ***D/2,*** but exclude the effect of ***second phase particles***.

Smith then proposed ***a model for cleavage fracture in mild steel*** concerning microcracking of grain boundary carbide by dislocation pile-up of length equal to half of the grain diameter ***D/2***.

Microcrack is initiated when sufficiently high applied stress causes local plastic strain within the ***ferrite grains*** to nucleate microcrack in brittle grain boundary carbide of thickness C_o.



Smith's model of microcrack formation in grain boundary carbide film

...Eq. 10

$$\sigma_f^2\left(\frac{c_o}{d}\right)+\tau_{eff}^2\left\{1+\frac{4}{\pi}\left(\frac{C_o}{d}\right)^{1/2}\frac{\tau_i}{\tau_{eff}}\right\}^2 \geq \frac{4E\gamma_p}{\left(1-\upsilon^2\right)\pi d}$$

*Note: Further propagation of the GB carbide crack follows the **Griffith theory**.*

Fractographic observation in brittle fracture

The process of cleavage fracture consists of three steps:

1) Plastic deformation to produce dislocation pile-ups.

2) Crack initiation.

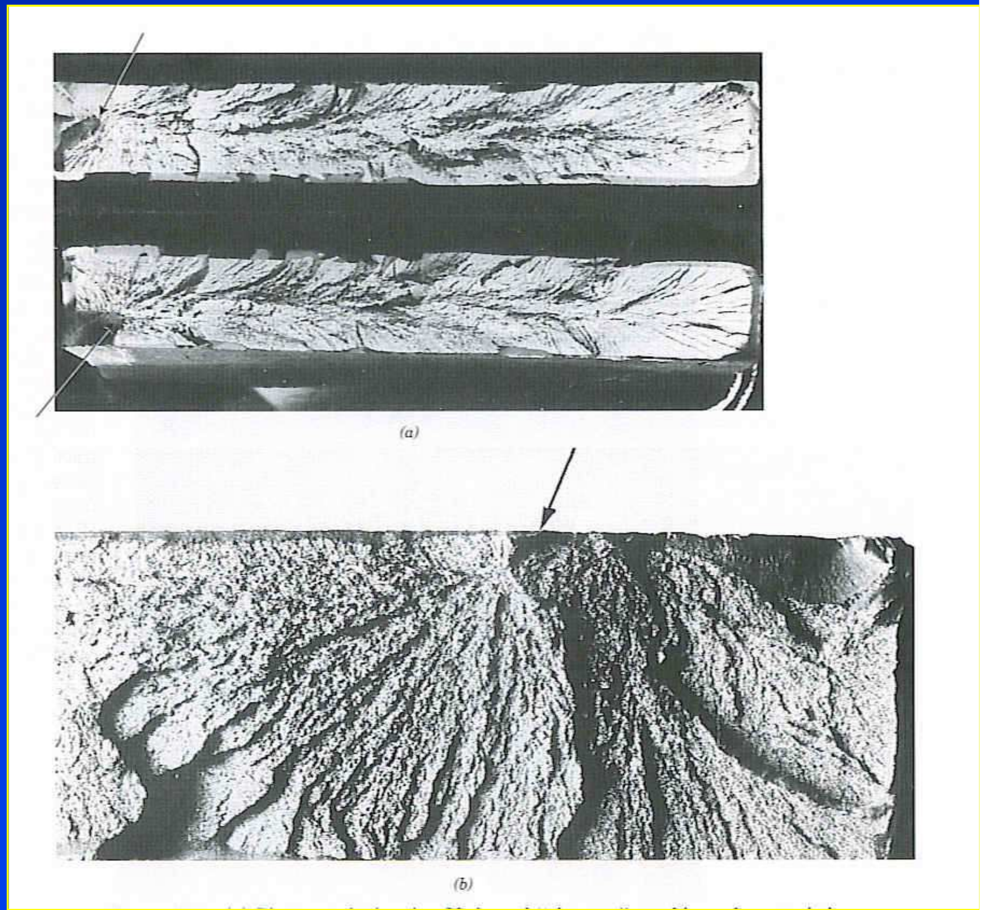
3) Crack propagation to failure.

Distinct characteristics of brittle fracture surfaces:

1) The absence of gross plastic deformation.

2) Grainy or Faceted texture.

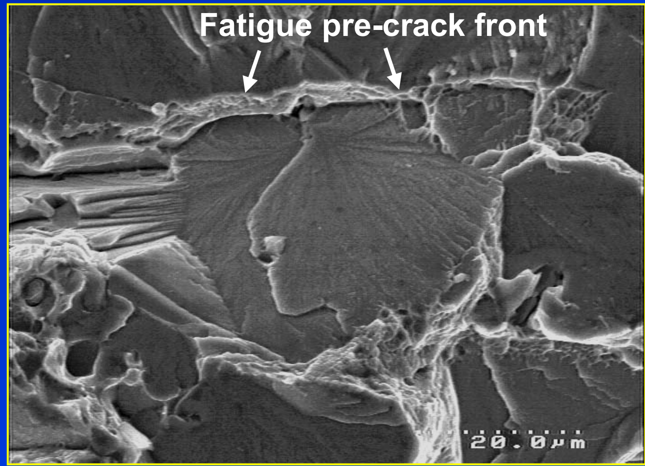
3) River marking or stress lines.



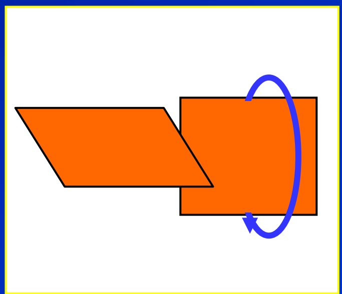
Brittle fracture indicating the origin of the crack and crack propagation path

Brittle fracture surface

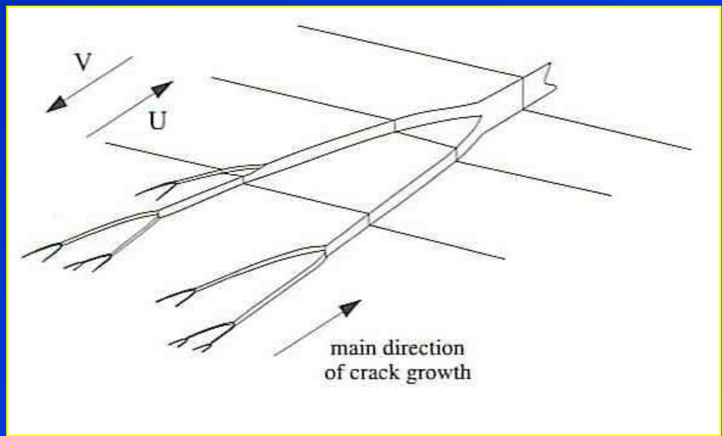
- ***Cleavage fracture surface*** is characterised by flat facets (with its size normally similar to the grain size).
- ***River lines*** or the ***stress lines*** are steps between cleavage on parallel planes and always converge in the direction of local crack propagation.



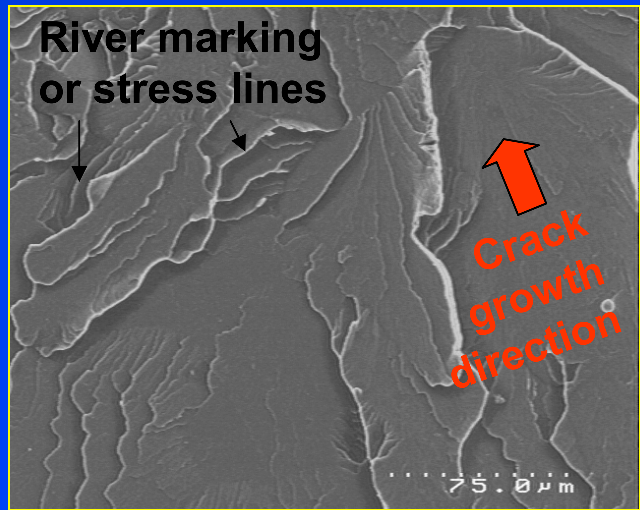
Cleavage facet



Twist boundary



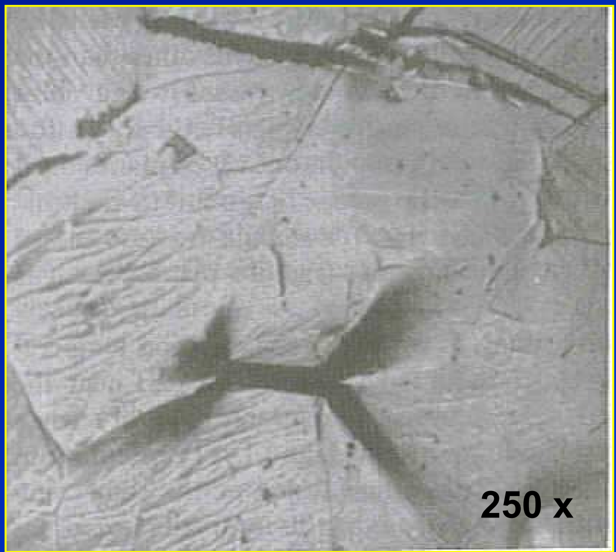
Schematic of river-line pattern.



Brittle cleavage facet

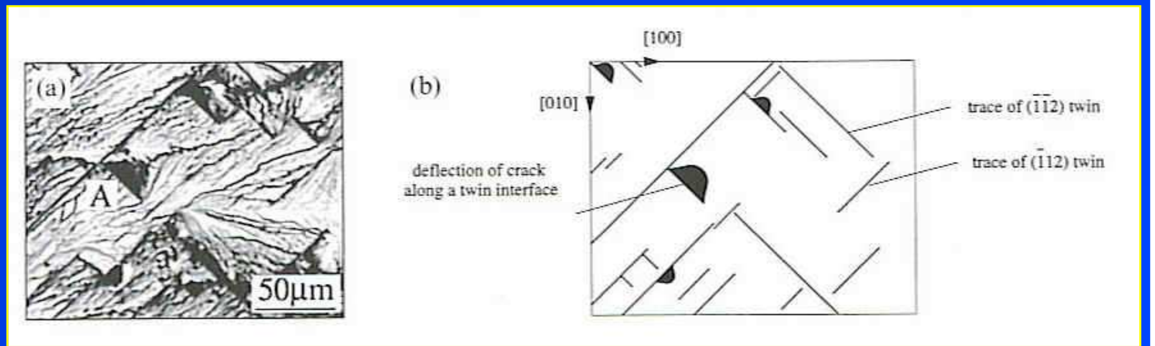
Initiation of microcracks from deformation and twins

- ***Microcracks*** can be produced by the deformation process, see *fig*.



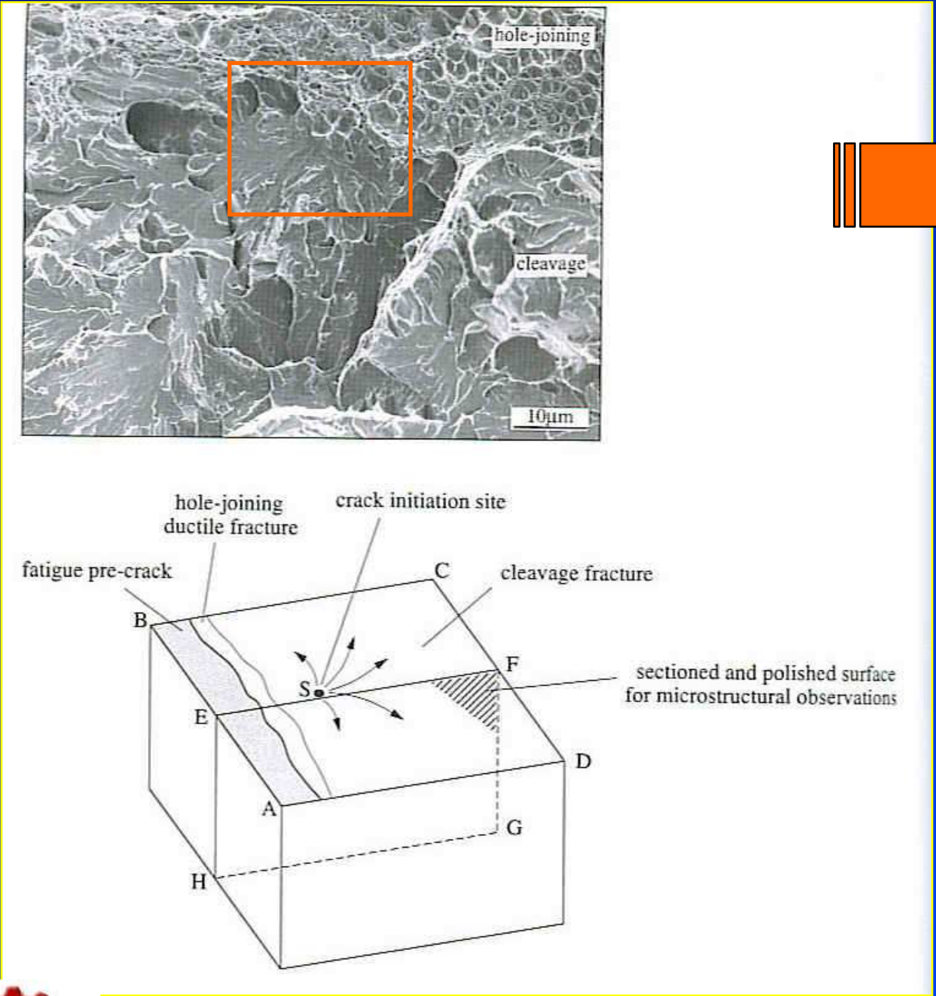
Microcracks produced in iron by tensile deformation at 133 K.

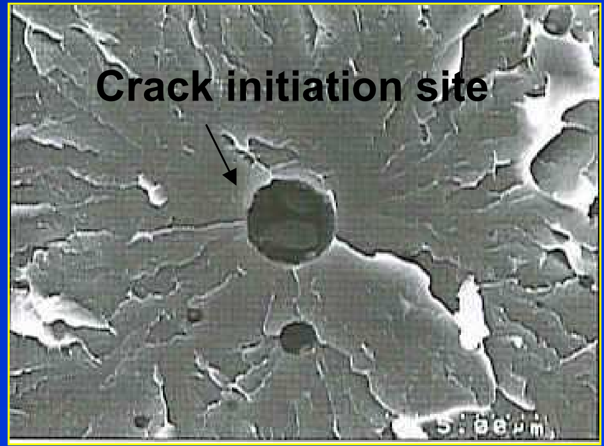
- ***Microcracks*** can also be initiated at ***mechanical twins***, especially in large grained bcc metals at low temperature.
- Crack initiation sites are due to the intersections of twins with other twins or intersection of twins with grain boundaries.



Cleavage along twin-matrix interfaces.

Crack initiation from particles in cleavage fracture

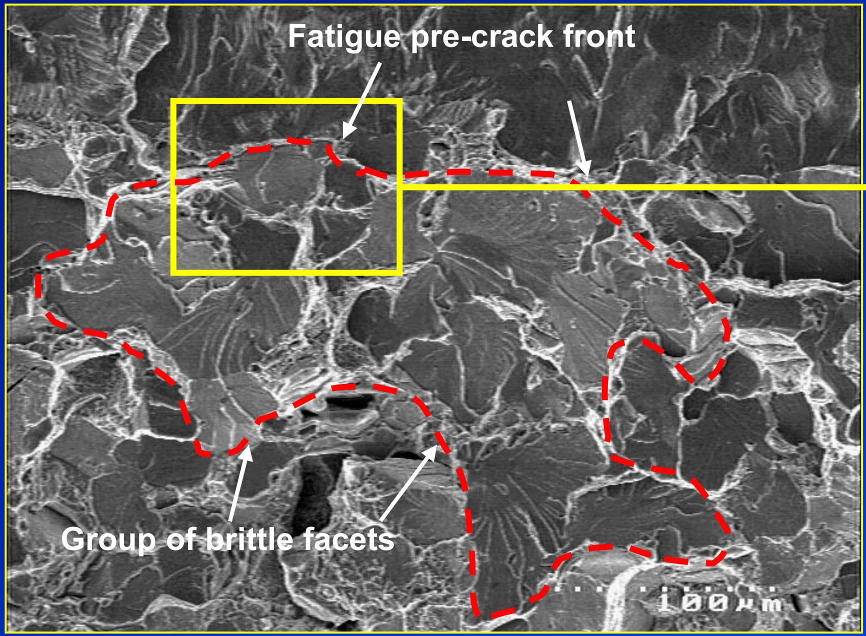


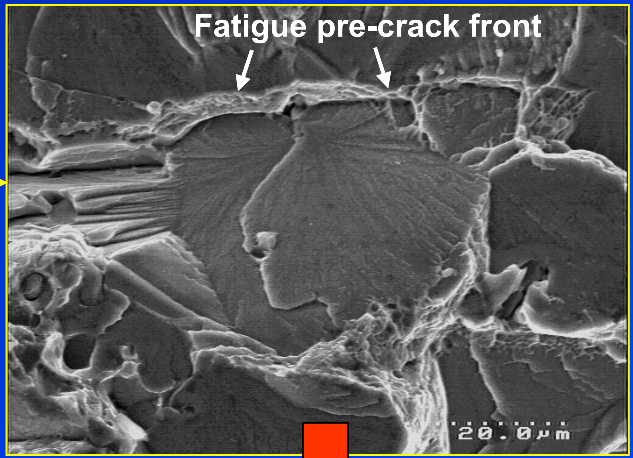


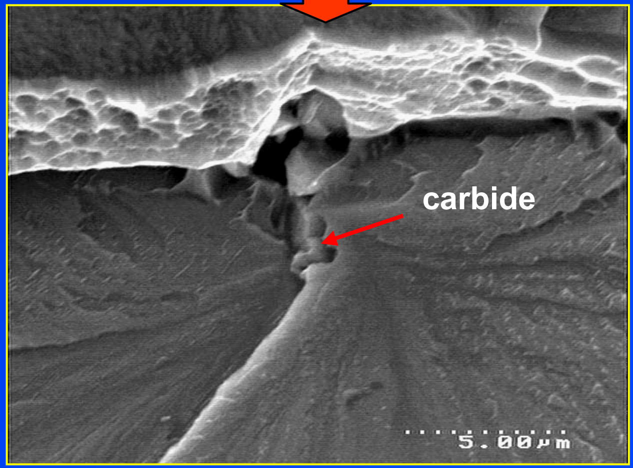
- Inclusions, porosity, second-phase particles or precipitates are preferential sites (***stress raiser***) for cleavage initiation.
- Fracture occurs along the ***crystallographic planes***.
- The direction of the river pattern represents the ***direction of the crack propagation***.

Example: ***Crack initiation from carbide particles observed in β-Ti alloy.***

Titanium carbides act as ***stress raiser*** which are preferential site for transgranular cleavage fracture.



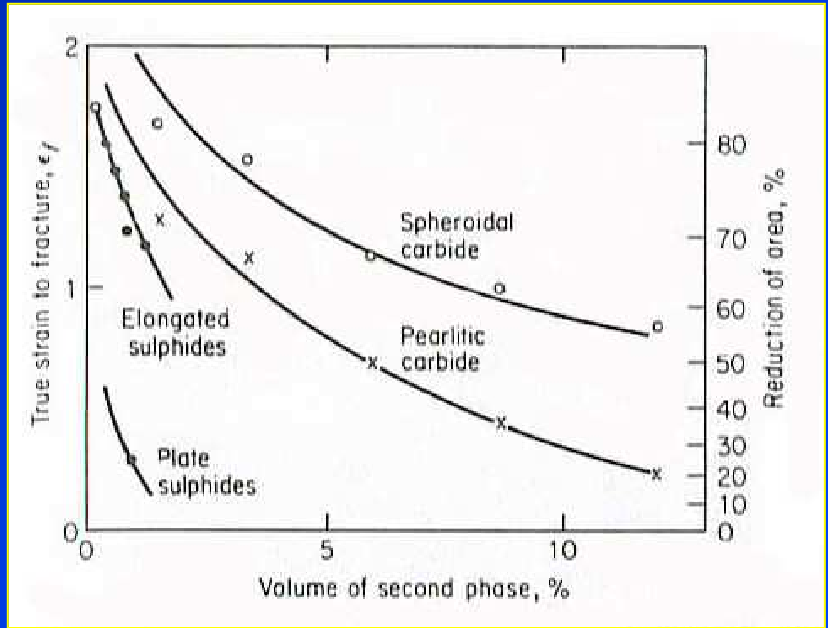




High local tensile stresses raised by dislocation pile-ups ahead of the carbide cause micro-cracking of carbide, which further propagate to cause global failure.

Effects of second phase particles on tensile ductility

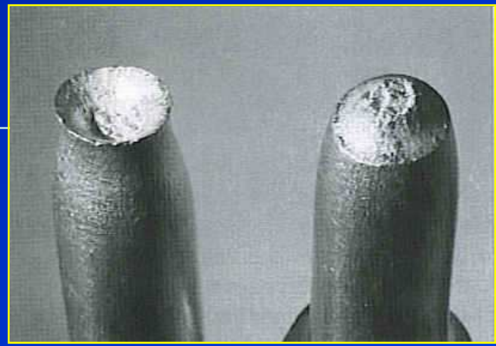
- Second-phase particles which are ***readily cut by dislocation*** produce ***planar slips***, producing large dislocation pile-ups which are susceptible for brittle fracture.



- Second-phase particles which are ***impenetrable by dislocations***, greatly ***reduce the slip distance*** → the number of dislocations is sustained → reduce the pile-up.

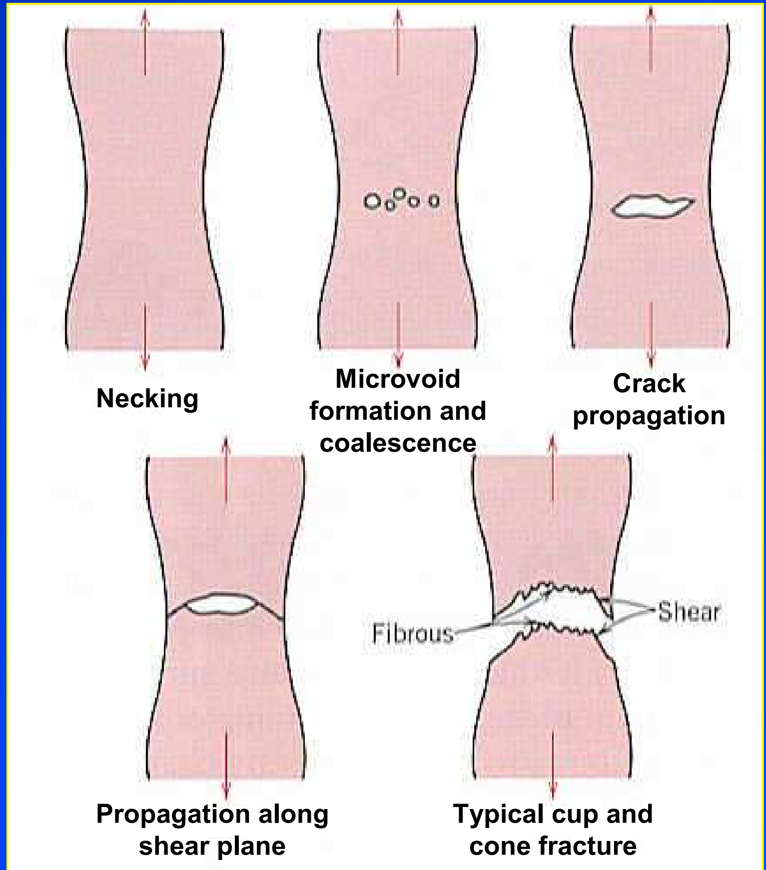
- Small spherical particles ***(r<1 μm)*** are more resistant to cracking.
- A soft ductile phase can also impart ductility to a brittle matrix.

Ductile fracture



Ductile fracture is a much less serious problem in engineering materials since failure can be detected beforehand due to observable ***plastic deformation*** *prior to failure.*

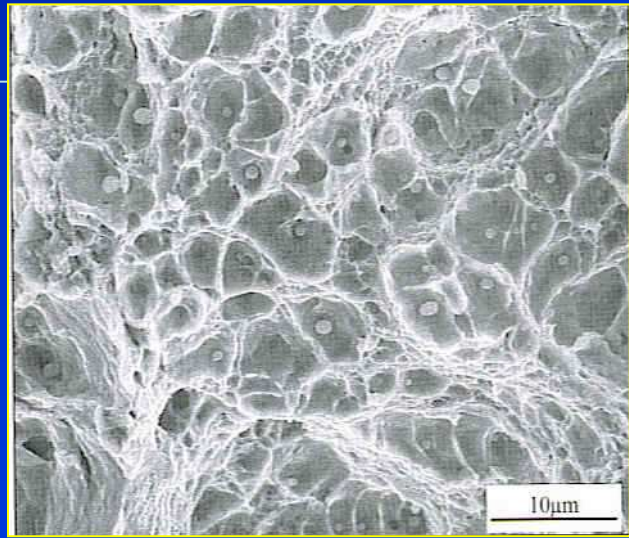
- Under uniaxial tensile force, after necking, ***microvoids*** form and coalesce to form crack, which then propagate in the direction normal to the tensile axis.
- The crack then rapidly propagate through the periphery along the shear plane at 45°, leaving the ***cub and cone fracture***.



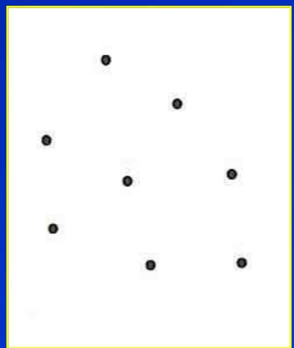
Stages in cup and cone fracture

Microvoid formation, growth and coalescence

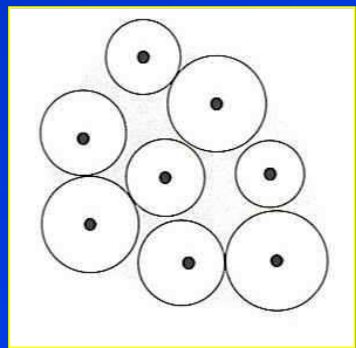
- ***Microoids*** are easily formed at inclusions, intermetallic or second-phase particles and grain boundaries.
- ***Growth*** and ***coalescence*** of microvoids progress as the local applied load increases.



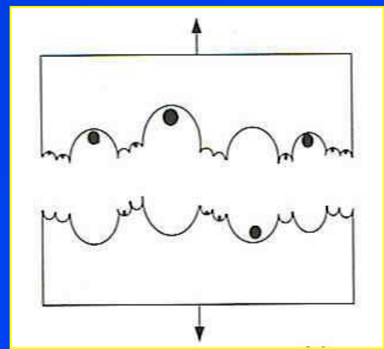
Ductile dimples centred on spherical particles



a) Random planar array of particles acting as void initiators.



b) Growth of voids to join each other as the applied stress increases.

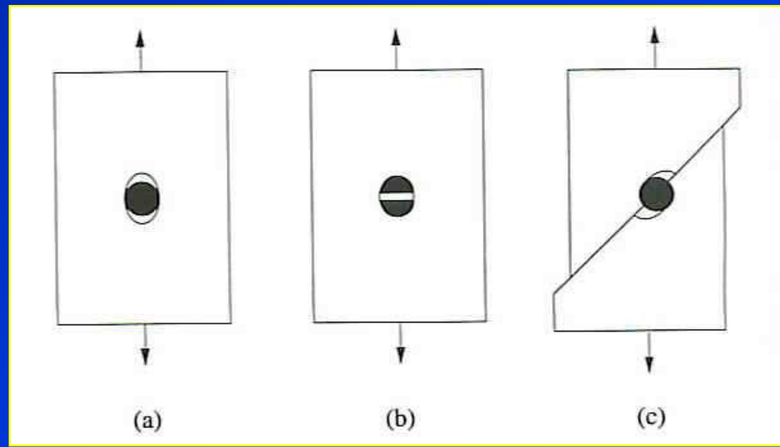


c) Linkage or coalescence of these voids to form free fracture surface.

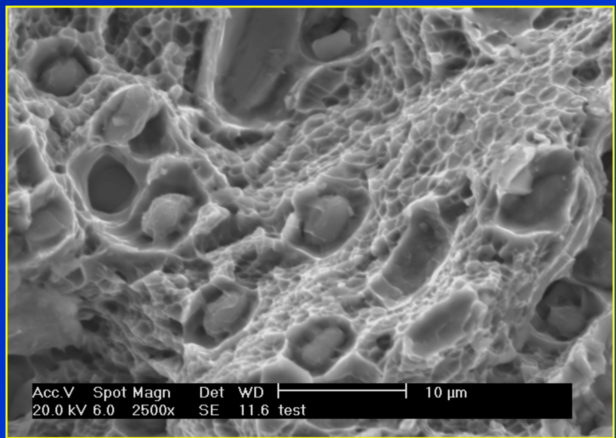
Formation of microvoids from second phase particles

Microvoids are formed by

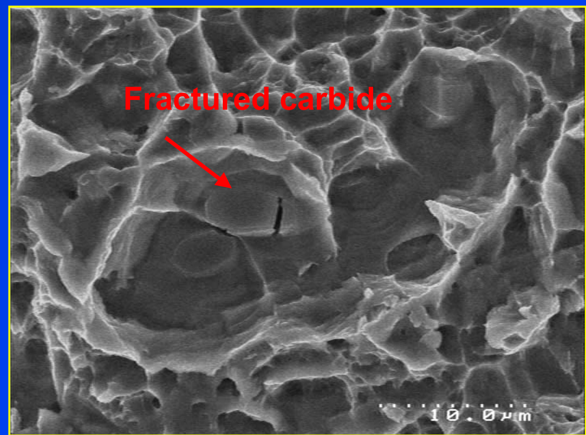
1) Decohesion at particle-matrix interface.
2) Fracture of brittle particle
3) Decohesion of an interface associated with shear deformation or grain boundary sliding.



Mechanisms of microvoid formation



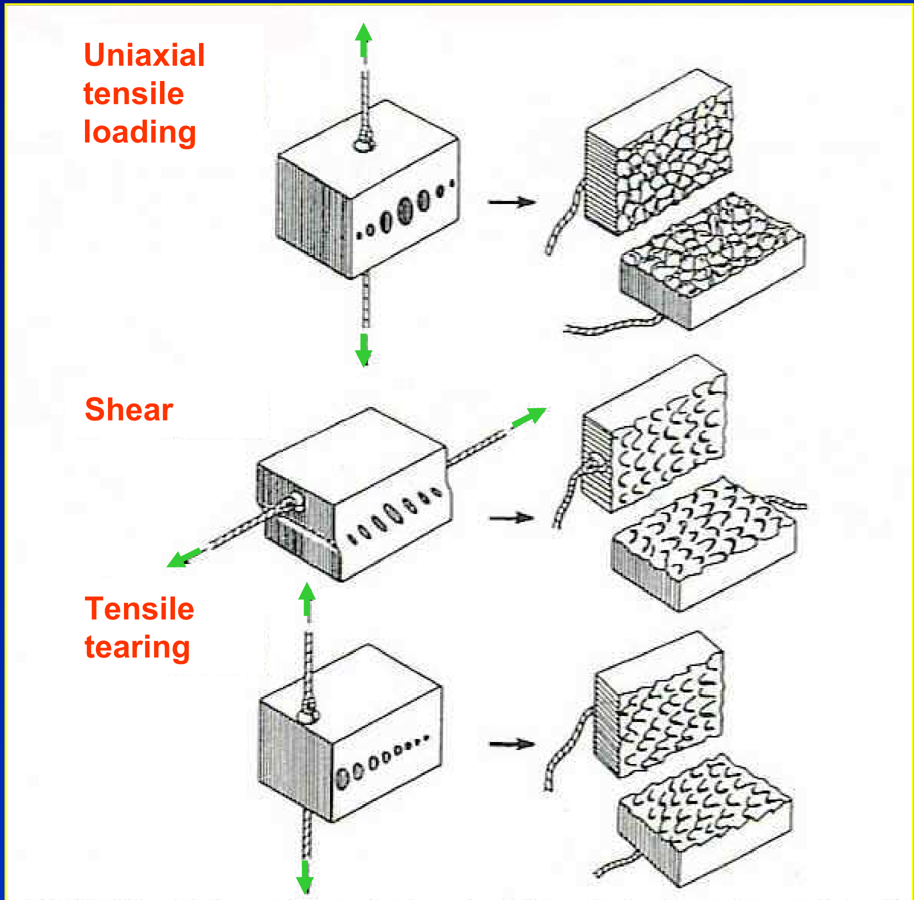
Decohesion of carbide particles from Ti matrix.



Fractured carbides aiding microvoid formation.

Microvoid shape

Microvoid shape is strongly influenced by the type of loading.



Formation of microvoids or dimples owing to uniaxial tensile loading, shear and tensile tearing

Uniaxial tensile loading
→ Equiaxed dimples.

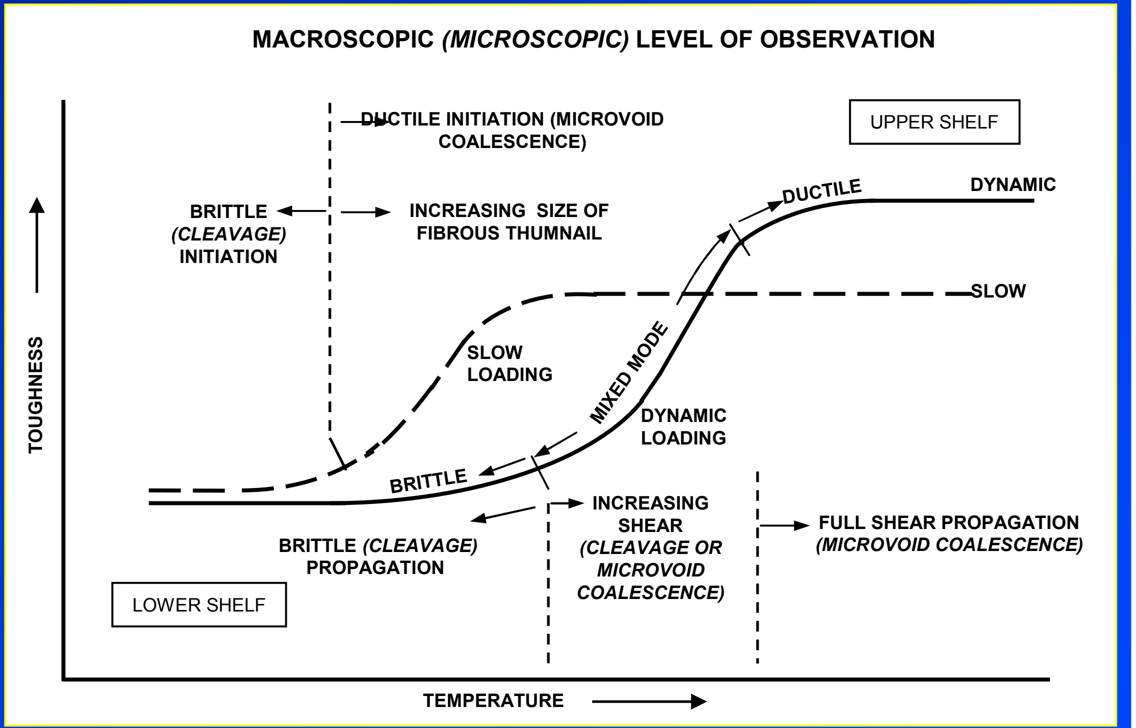
Shear loading
→ Elongated and parabolic dimples pointing in the opposite directions on matching fracture surfaces.

Tensile tearing
→ Elongated dimples pointing in the same direction on matching fracture surface.

Ductile to brittle transition behaviour

BCC structure metals *experience* ***ductile-to-brittle transition behaviour*** *when subjected to decreasing temperature, resulting from a strong yield stress dependent on temperature.*

- ***BCC*** metals possess ***limited slip systems*** available at low temperature, ***minimising the plastic deformation*** during the fracture process.
- ***Increasing temperature*** allows more slip systems to operate, yielding ***general plastic deformation*** to occur prior to failure.



Low temperature	→	**Brittle cleavage fracture**
High temperature	→	**Ductile fracture**

Theory of the ductile to brittle transition

The *criterion* for a material to change its fracture behaviour from ***ductile to brittle mode*** is when the ***yield stress*** at the observed temperature is larger than the ***stress necessary for the growth of the microcrack*** indicated in the ***Griffith theory***.

Cottrell studied the role of parameters, which influence the ***ductile-to-brittle transition*** as follows;

$$\left(\tau_i D^{1/2} + k'\right)k' = G\gamma_s\beta \qquad \text{...Eq. 11}$$

The criterion for ductile to brittle transition is when the term on the left hand side is greater than the right hand side.

where

τ_i is the lattice resistance to dislocation movement

k' is a parameter related to the release of dislocation into a pile-up

D is the grain diameter (associated with slip length).

G is the shear modulus

β is a constant depending on the stress system.

Factors affecting ductile to brittle transition

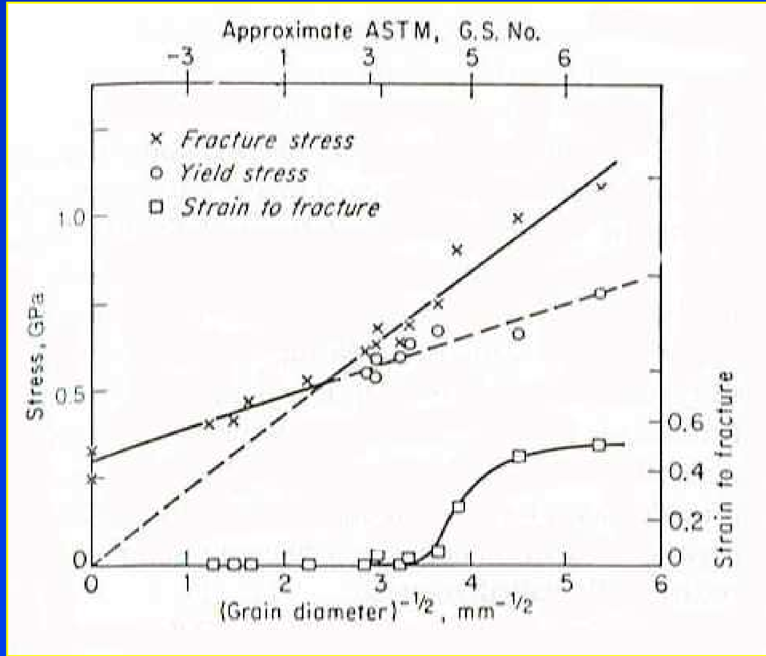
*From equation, materials having high lattice resistance τ_i, grain size **D** and **k'** has a high tendency to become brittle with decreasing temperature.*

$$\left(\tau_i D^{1/2} + k'\right)k' = G\gamma_s\beta$$

- The τ_i in **BCC** material is strongly dependent on temperature.
- Materials with high ***k'*** i.e., ***Fe*** and ***Mo*** are more susceptible for brittle fracture.
- Smaller grain sized metals can withstand brittle behaviour better.

Note: Alloy chemistry and microstructure also affect the ductile to brittle transition behaviour.

In mild steel Ni lowers DBTT
C, P, N, S, Mo raise DBTT

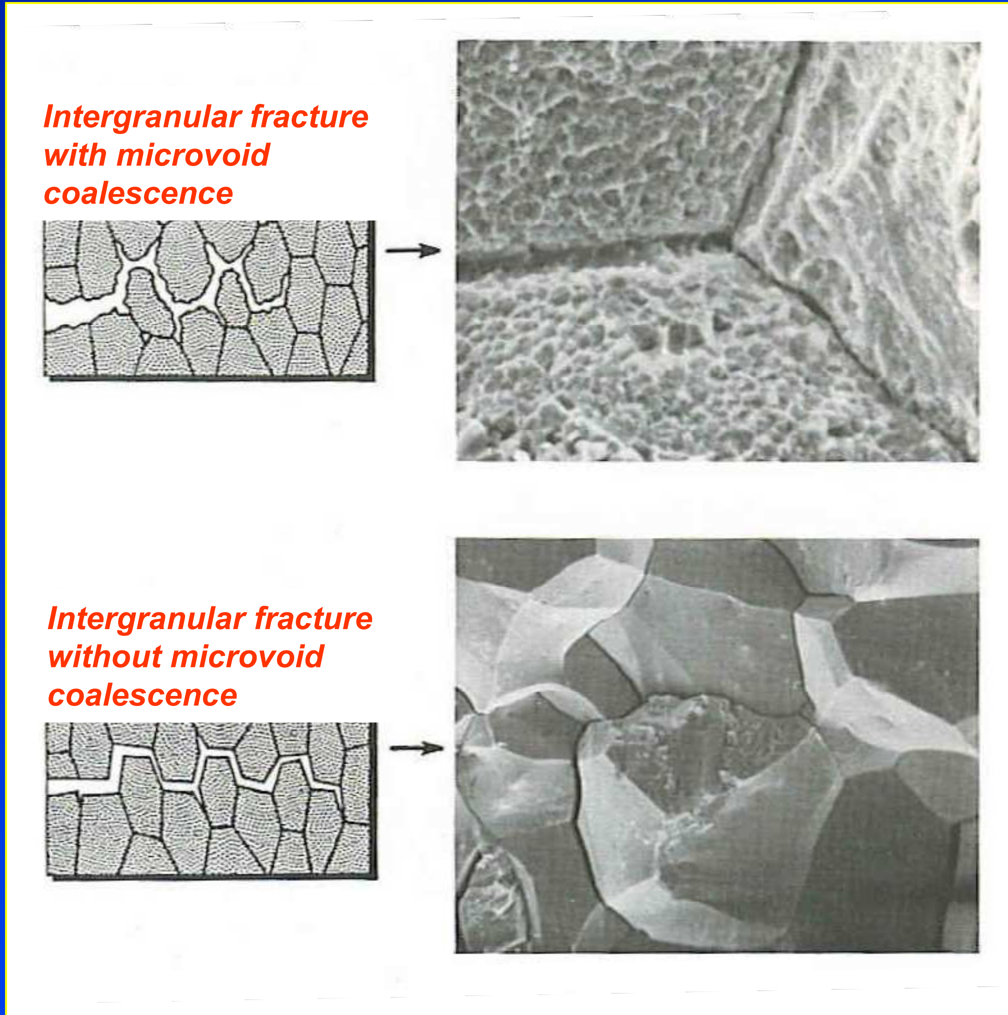


Effect of grain size on the yield and fracture stresses for a low-carbon steel tested in tension at -196°C.

Intergranular fracture

- ***Intergranular failure*** is a moderate to low energy brittle fracture mode resulting from ***grain boundary separation*** or segregation of embrittling particles or precipitates.

- ***Embrittling grain boundary particles*** are weakly bonded with the matrix, → high free energy and unstable, which leads to ***preferential crack propagation path***.



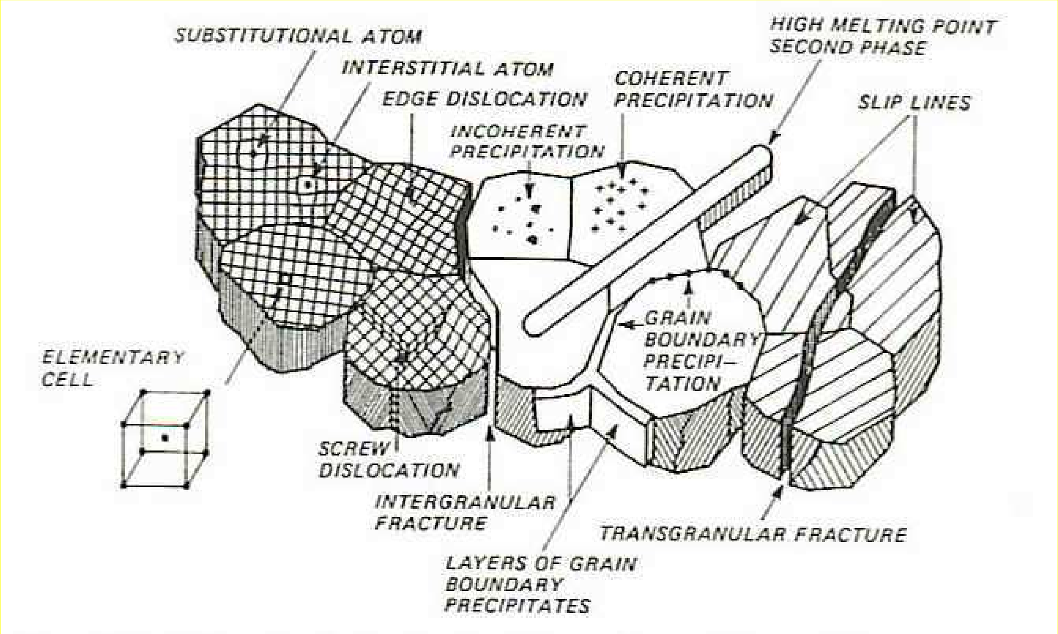
Intergranular fracture with and without microvoid coalescence.

Factors affecting modes of fracture

	Brittle fracture	Ductile fracture
Metallurgical aspect	Large grained materials with GB particles.	Fine grained material without GB particles.
Temperature	Low temperature	High temperature
State of stresses (notch effect)	Triaxial state of stresses (notch effect)	Absence of the notch
Strain rate	High strain rate	Low strain rate
Loading condition		Hydrostatic pressure (suppress crack initiation)

Metallurgical aspect of fracture

- Microstructure in metallic materials are highly complex.
- Various microstructural features affect how the materials fracture.



Microstructural features in metallic materials

There are microstructural features that can play a role in determining the fracture path, the most important are;

Second phase

Particles and precipitates

Grain size

Fibering and texturing

- ***High strength materials*** usually possess several microstructural features in order to optimise mechanical properties by ***influencing deformation behaviour / fracture paths***.

State of stresses (notch effect)

The difference in the state of stresses in the presence of a sharp crack or notch affects fracture in materials.

A notch or a sharp crack increases the ***tendency for brittle fracture*** in four important ways;

1) **Producing high local stresses**
2) **Introducing a triaxial state of stresses**
3) **Producing high local strain hardening and cracking**
4) **Producing a local magnification to the strain rate.**

*Note: the notch also raises the **plastic-constraint factor q,** which does not exceed the value of 2.75*

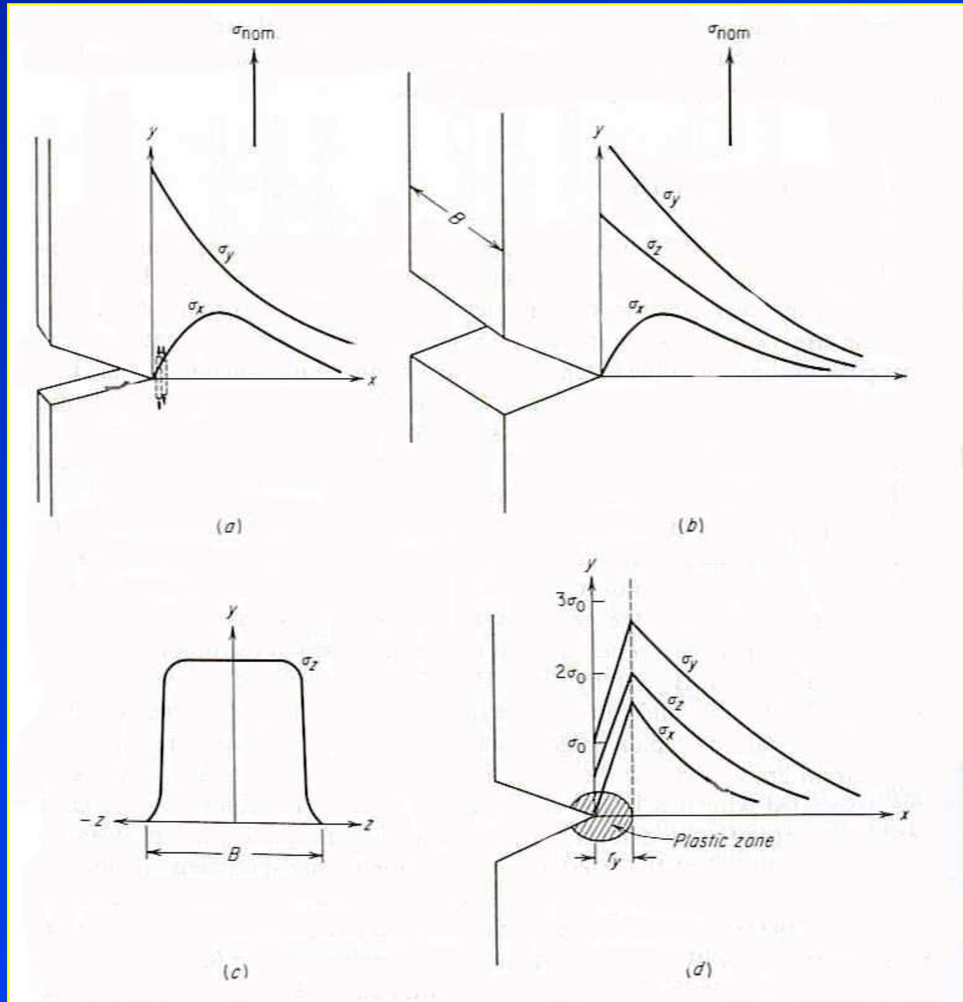
notch effect

The presence of the notch alters stress distribution

- In a thin plate, stress in the *z* (thickness) direction is absence, the specimen is not constrained.

- In thicker plate, σ_y (in the tensile direction) is constrained due to the reaction of σ_z and σ_x, leading to triaxial state of stresses.

- ***Triaxial stresses*** limit plastic deformation ahead of the crack tip → raising the general yield → material prone to brittle fracture



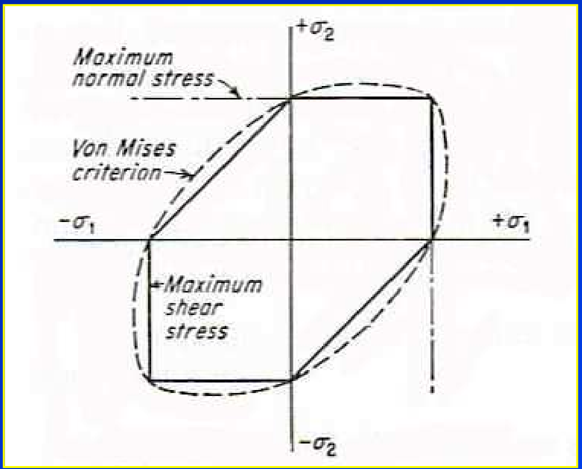
Elastic stresses beneath a notch in thin and thick plates

Effects of combined stress and hydrostatic pressure on fracture

Combined stress

• Yielding under complex states of stress is difficult to predict.

• Available data on ductile metals, i.e., ***Al*** and ***Mg*** alloys and ***steel*** indicate that the ***maximum-shear stress criterion*** for fracture are in the best agreement.

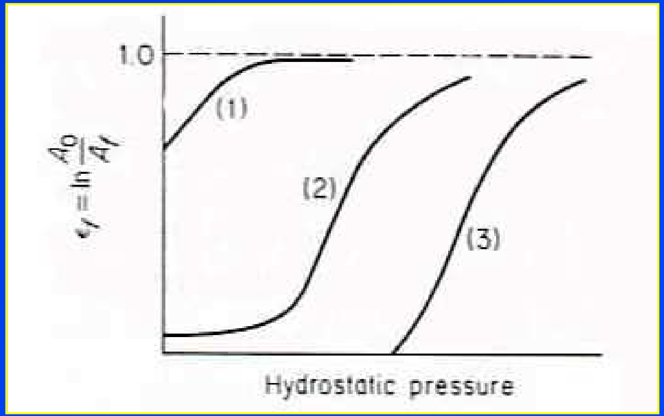


Proposed fracture criteria for biaxial state of stress in ductile metal

Hydrostatic pressure

• ***hydrostatic pressure*** is triaxial compressive stress resist fracture and increase ductility.

• ***Hydrostatic pressure*** exerts no shear stress, it therefore does not influence ***crack initiation*** but affects ***crack propagation***.



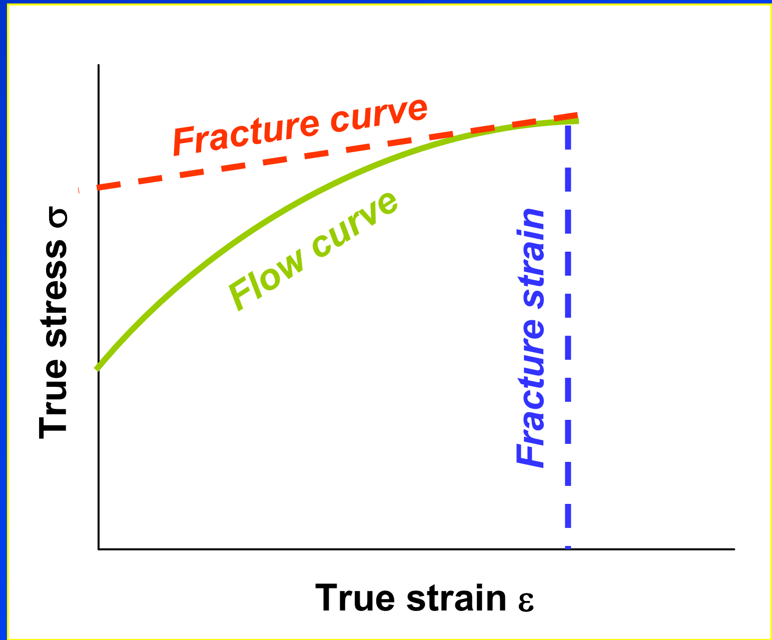
Effect of hydrostatic pressure on ductility in tension

Concept of the fracture curve

Ludwik proposed that a metal has a fracture stress curve in addition to a flow curve (true stress - true strain curve) and that ***fracture occurs when the flow curve intersects the fracture curve***.

- The plastic deformation is inhibited when strain hardening, triaxial stress, or high strain rate, causing sufficiently high stress to break the material.

- ***Fracture stress*** is difficult to measure since most metals exhibit small plastic deformation prior to failure even in the presence of the notch and at very low temperature.

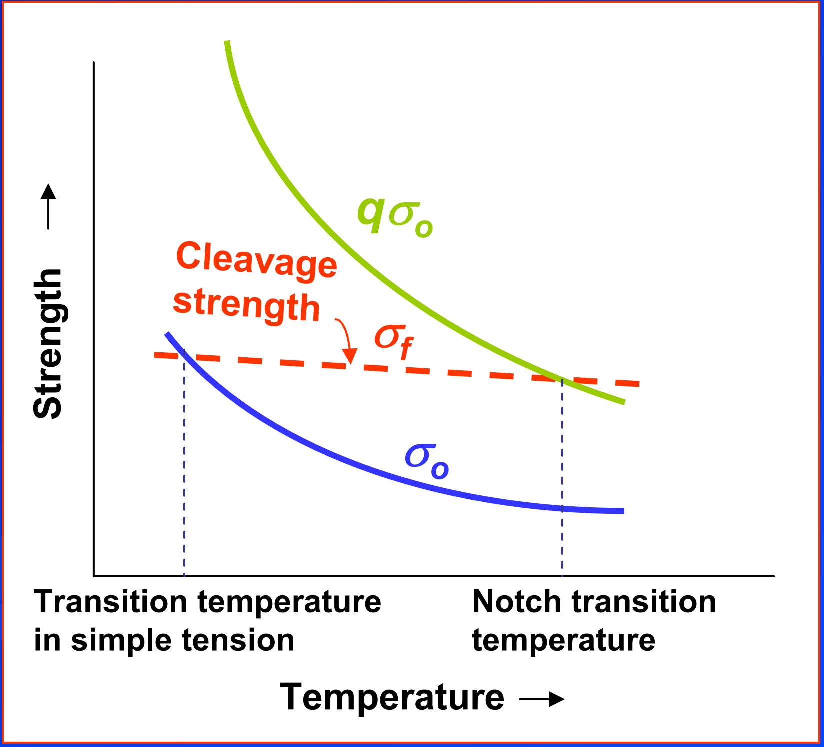


Intersection of flow curve and fracture curve.

Notch effect on transition temperature

The *fracture stress* σ_f is much less temperature sensitive than the *flow stress* σ_o.

- The σ_o of the *unnotched specimen* is lower than σ_f at temperatures above the transition temperature.
- The metal therefore deforms plastically before fracture. Below the transition temperature $\sigma_o > \sigma_f$, metal fails without plastic deformation.
- The *presence of the notch* raises the σ_o by the *plastic-constraint factor q*. This shifts the transition temperature to the right hand side.



Description of transition temperature

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Tension test

Subjects of interest

- *Introduction/Objectives*
- *Engineering stress-strain curve*
- *True stress-true strain curve*
- *Instability in tension*
- *Stress distribution at the neck*
- *Ductility measurement in tension tests*
- *Effect of strain rate on flow properties*
- *Effect of temperature on flow properties*

Tension test

Subjects of interest

- *Influence of testing machine on flow properties*
- *Thermally activated deformation*
- *Notch tensile test*
- *Tensile properties of steel*
- *Anisotropy of tensile properties*

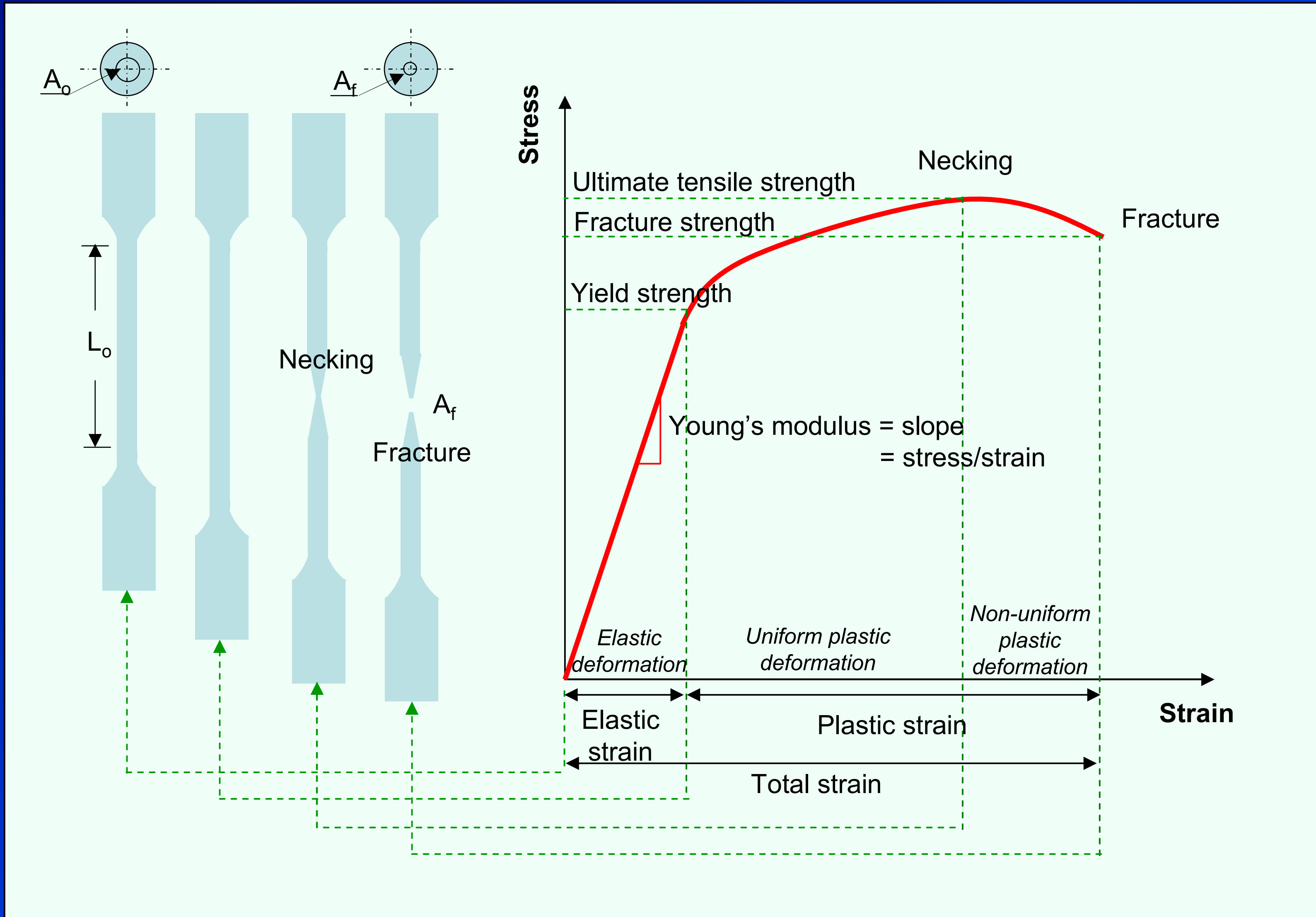
Objectives

• This chapter provides fundamental backgrounds of tension tests where appropriate material parameters can be used for material selection.

• Differences between engineering stress-strain curve and true stress – true strain curve will be clearly understood.

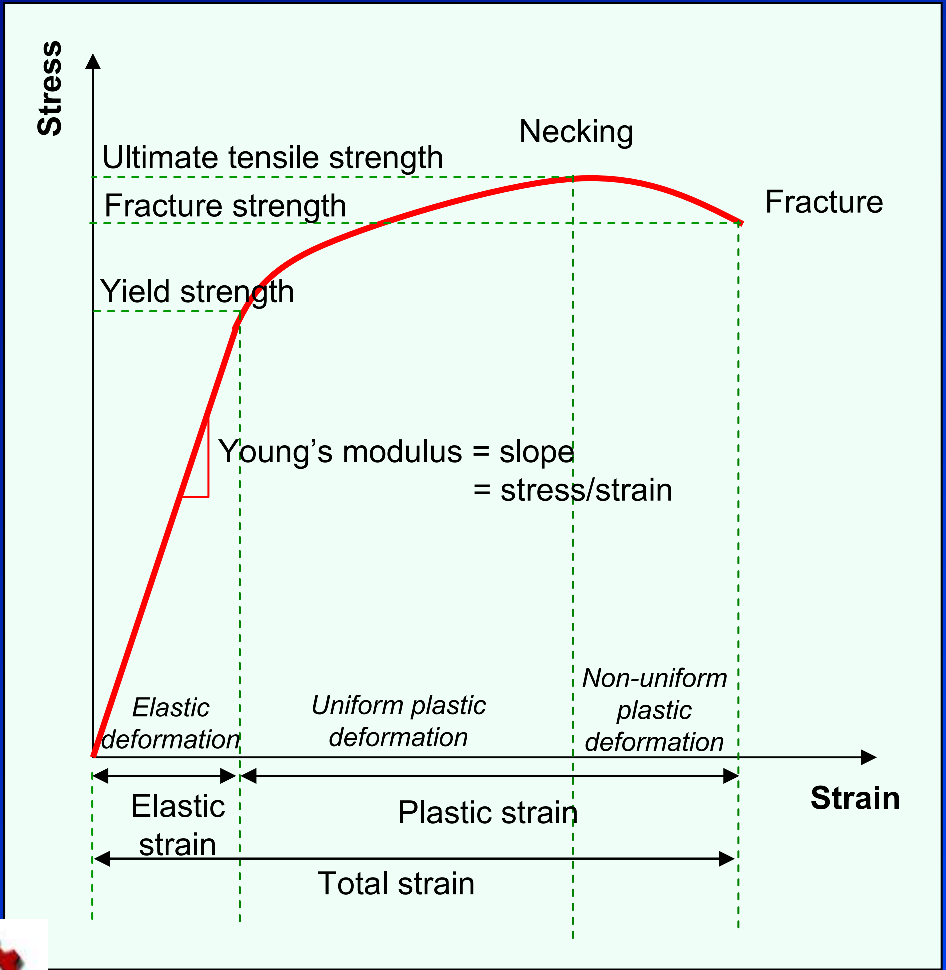
• Effects of strain rate, test temperature, testing machine as well as notch and anisotropy on tensile properties will be highlighted.

Engineering stress-strain curve



Engineering stress-strain curve

- Basic design information on the strength of materials.
- An acceptance test for the specification of materials.



Average longitudinal tensile stress

$$s = \frac{P}{A_o} \qquad \text{Eq.1}$$

Average linear strain

$$e = \frac{\delta}{L_o} = \frac{\Delta L}{L_o} = \frac{L - L_o}{L_o} \qquad \text{Eq.2}$$

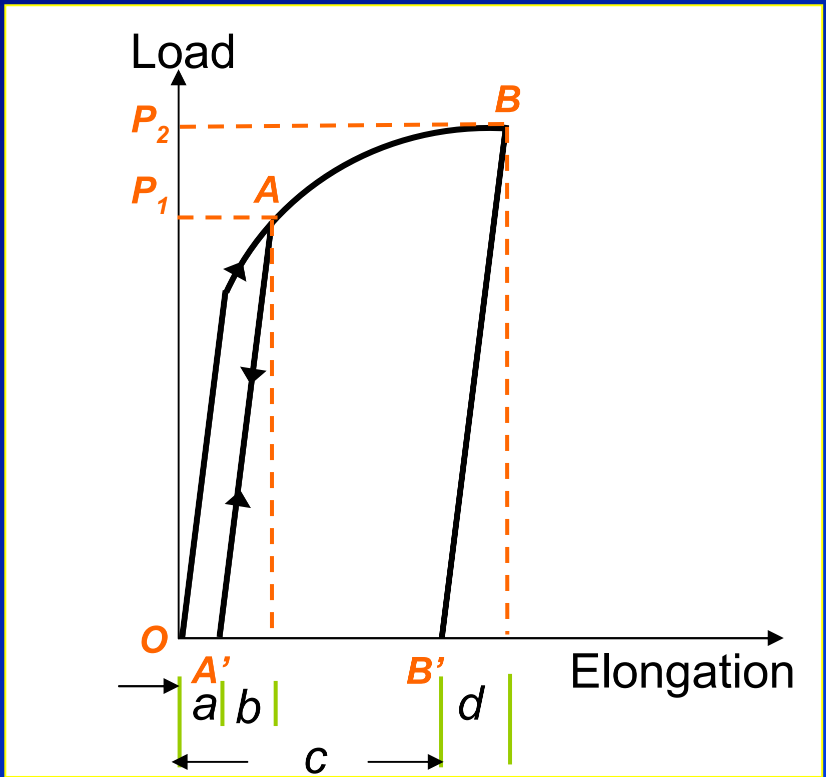
Factors affecting shape and magnitude of stress-strain curve

- *Composition*
- *Heat treatment*
- *Prior history of plastic deformation*
- *Strain rate*
- *Temperature*
- *State of stress*

Metallurgical factors (Composition, Heat treatment, Prior history of plastic deformation)

Test conditions (Strain rate, Temperature, State of stress)

Recoverable elastic strain and plastic strain



- Loading of tensile sample beyond yield point to ***A*** and then unloading give the unloading curve ***AA'*** with its slope parallel to the elastic Young's modulus.
- ***Recoverable elastic strain b*** on unloading is given by

$$b = \frac{\sigma_1}{E} = \frac{P_1 / A_o}{E}$$

Eq.3

- ***Permanent plastic strain a***
- Loading and unloading following ***OABB'*** gives ***plastic deformation c*** whereas ***elastic deformation*** under loading is ***d***.

Tensile strength

***Tensile strength** or ultimate tensile strength (UTS) **s_u** is the maximum load **P_{max}** divided by the original cross-sectional area **A_o** of the specimen.*

$$s_u = \frac{P_{\max}}{A_o} \qquad \textit{Eq.4}$$

- Tensile strength is the most value quoted from tensile test results.
- Useful for ***specifications***, ***quality control*** of a product.
- In engineering design, ***safety factor*** should be applied.

Note: yield stress is more practical for ductile materials. But it has little relation to ***complex conditions of stress***.

Yielding

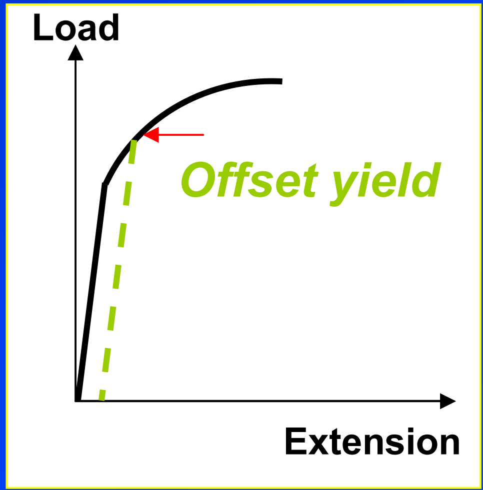
Various criteria for the ***initiation of yielding*** *are used depending on the sensitivity of the strain measurements and the intended use of the data.*

1) ***True elastic limit***: based on microstrain measurement at strains on order of 2×10^{-6}. Very low value and is related to the motion of a few hundred dislocations.

2) ***Proportional limit:*** the highest stress at which stress is directly proportional to strain.

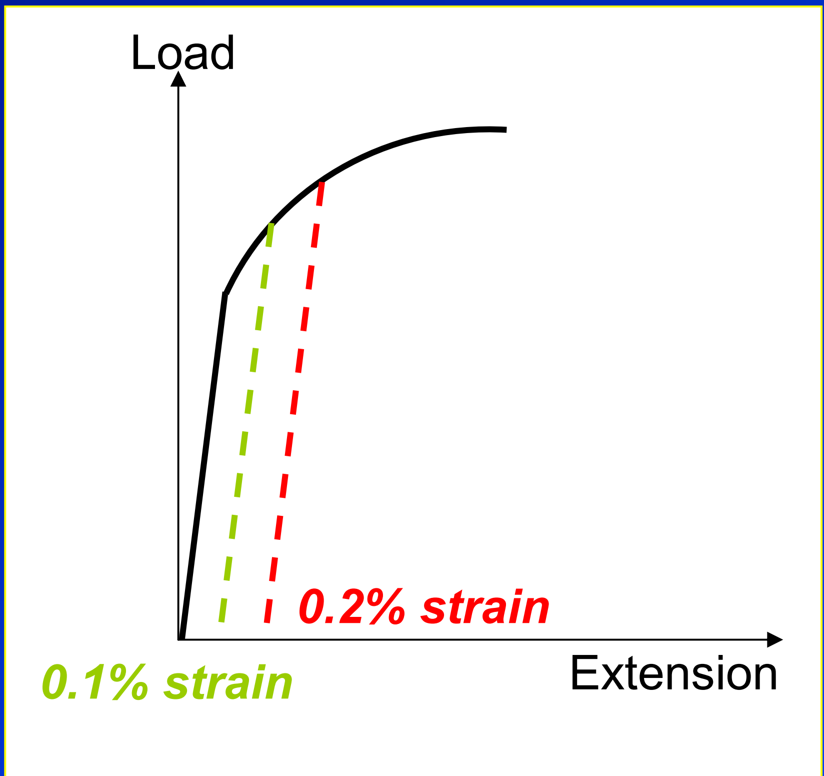
3) ***Elastic limit:*** is the greatest stress the material can withstand without any measurable permanent strain after unloading. ***Elastic limit > proportional limit***.

4) ***Yield strength*** is the stress required to produce a small specific amount of deformation.



Yield strength of materials

The offset yield strength can be determined by the stress corresponding to the intersection of the stress-strain curve and a line parallel to the elastic line offset by a strain of 0.2 or 0.1%. (e = 0.002 or 0.001)



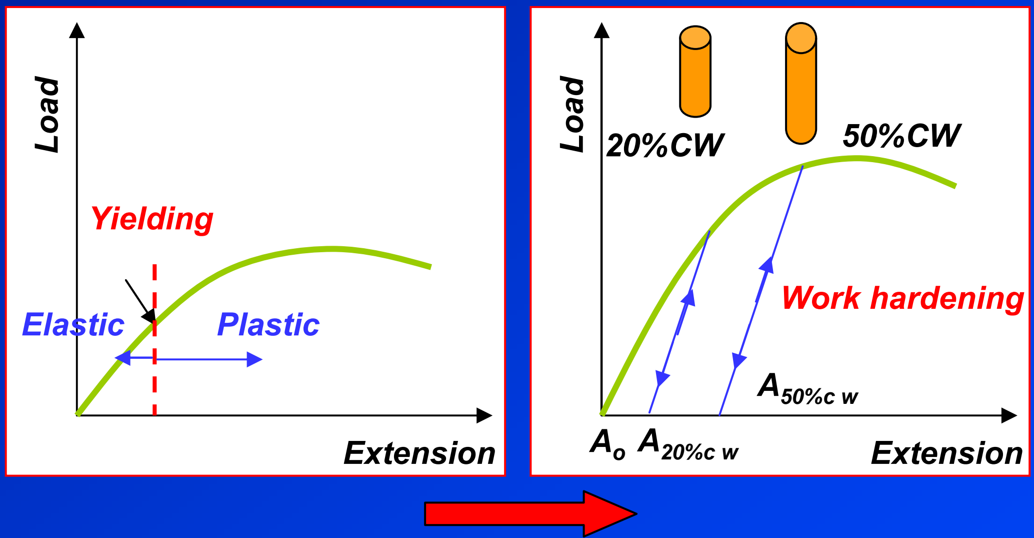
$$s_o = \frac{P_{(strain\ \ offset=0.002)}}{A_o}$$ *Eq.5*

In Great Britain, the offset yield stress is referred to ***proof stress*** either at 0.1 or 0.5%strain.

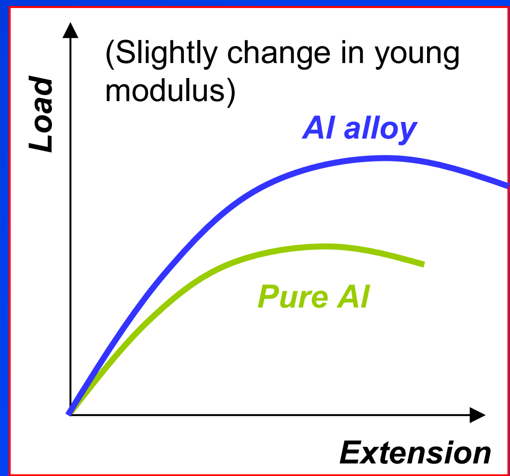
Used for design and specification purposes to avoid the practical difficulties of measuring the elastic limit or proportional limit.

Yield strength of materials

- *FCC* lattice materials (*Al, Cu*) have no definite yield point. The yield strength is therefore defined by the *offset of yielding*.
- *Yield strength* can be improved by *work hardening* (cold working). → up to 300:1 stronger than original.
- Alloying of *Al* can improve *elastic limit* 1.5-2 times.



Improvement of yielding by cold working

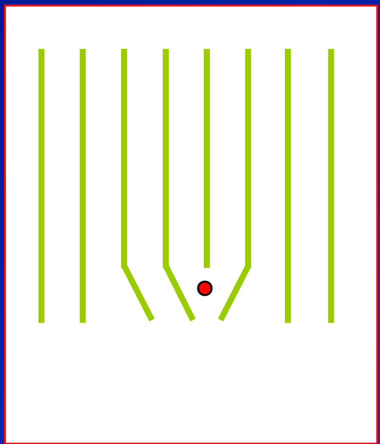


Improvement in elastic limit by alloying

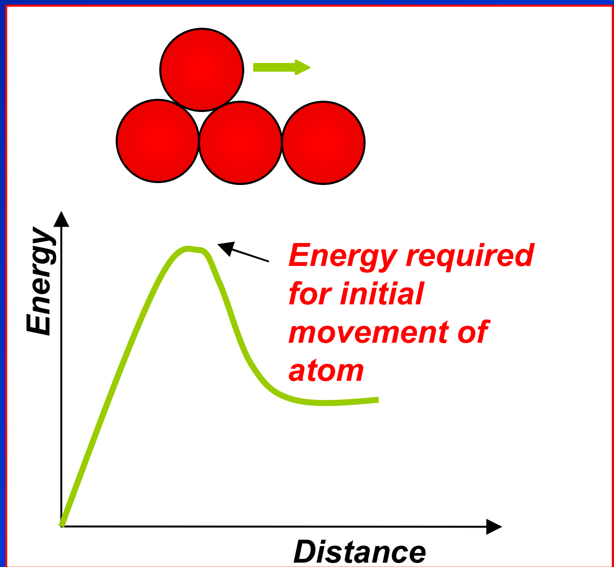
Yield strength of materials

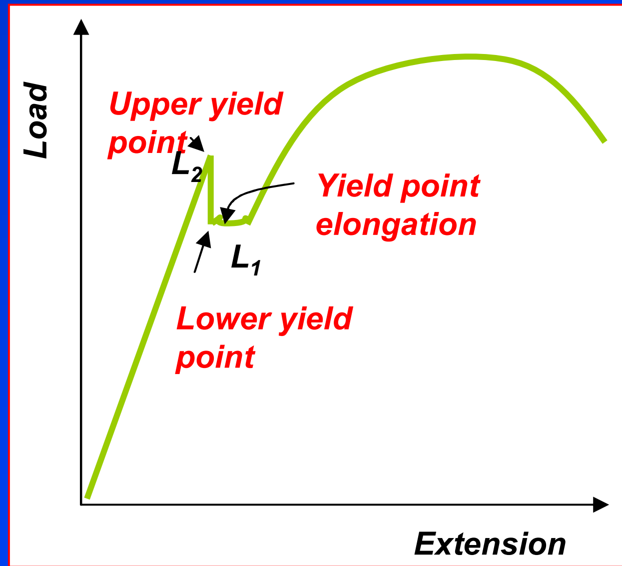
- **BCC** lattice materials (**Fe**) show a yield point phenomenon → ***Upper and lower yield points*** (depending on testing machine).
- ***Condition:*** Polycrystalline & small amounts of interstitial solute atoms.

Upper yield point $\frac{L_2}{A_o}$ **Lower yield point** $\frac{L_1}{A_o}$



Interstitial solute atom





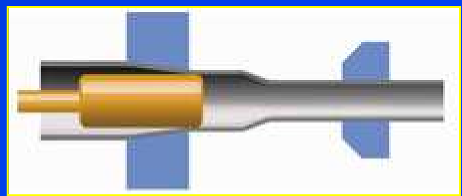
At yield point, localised internal friction requires more energy for interstitial atom to move dislocation, after that dislocation are free from interstitial atom (carbon, nitrogen).

Ductility

Ductility is a qualitative, subjective property of a material.

In general, ductility is of interest in three different ways

1) ***For metal working operation*** : indicating amount of deformation can be applied without failure.
2) ***For stress calculation or the prediction of severe load*** : indicating the ability of the metal to flow plastically before failure.
3) ***For indication of any changes in heat treatments or processing conditions in metal***.



Measures of ductility

Elongation

$$e_f = \frac{L_f - L_o}{L_o}$$ *Eq.6*

Reduction of area, *q*

$$q = \frac{A_o - A_f}{A_o}$$ *Eq.7*

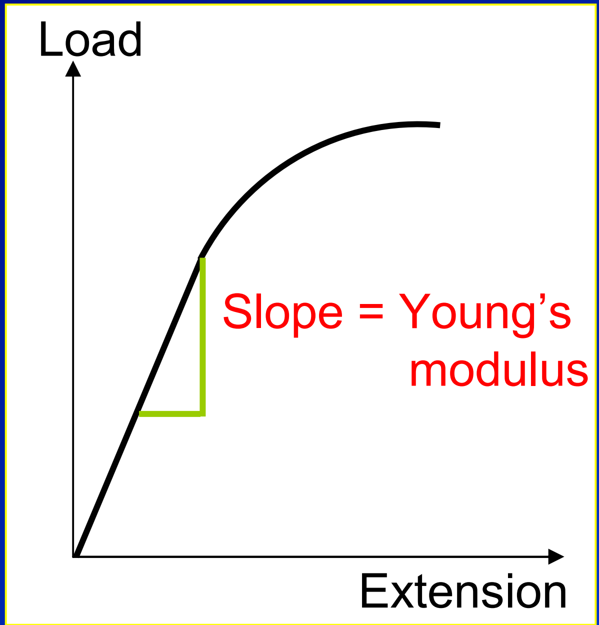
These ***parameters*** are obtained after fracture by putting specimen back together and taking the measurement.

Zero-gauge length elongation

$$e_o = \frac{L - L_o}{L_o} = \frac{A_o}{A} - 1 = \frac{1}{1-q} = \frac{q}{1-q}$$ *Eq.8*

Modulus of elasticity

Modulus of elasticity or ***Young's modulus*** is a measure of material stiffness (given by the slope of the stress-strain curve).



- Modulus of elasticity is determined by the binding forces between atoms (structure insensitive property)
- Cannot change ***E,*** but can improve by forming composites.
- Only slightly affected by alloying addition, heat treatment or cold work.

Temp ↑ ***Young's modulus*** ↓

Young's modulus ↑

Stiffness ↑

Deflection ↓

Table 8-1 Typical values of modulus of elasticity at different temperatures

Material	Modulus of elasticity, GPa: Room temp.	477 K	700 K	810 K	922 K
Carbon steel	207	186	155	134	124
Austenitic stainless steel	193	176	159	155	145
Titanium alloys	114	97	74	70	
Aluminum alloys	72	66	54		

Example: *A 13 mm diameter tensile specimen has a 50 mm gauge length. The load corresponding to the 0.2% offset is 6800 kg and the maximum load is 8400 kg. Fracture occurs at 7300 kg. The diameter after fracture is 8 mm and the gauge length at fracture is 65 mm. Calculate the standard properties of the material from the tension test.*

$$A_o = \frac{\pi}{4}(13)^2 = 132.7 \ mm^2 = 132.7\times10^{-6} \ m^2$$

$$A_f = \frac{\pi}{4}(8)^2 = 50.3 \ mm^2 = 50.3\times10^{-6} \ m^2$$

$$e_f = \frac{L-L_o}{L_o} = \frac{65-50}{50} = 30\%$$

$$q = \frac{A_o - A_f}{A_o} = \frac{132.7-50.3}{132.7} = 62\%$$

$$s_u = \frac{P_{\max}}{A_o} = \frac{8400\times9.8}{132.7\times10^{-6}} = 620 \ MPa$$

$$s_o = \frac{P_y}{A_o} = \frac{6800\times9.8}{132.7\times10^{-6}} = 502 \ MPa$$

$$s_f = \frac{P_f}{A_o} = \frac{7300\times9.8}{132.7\times10^{-6}} = 539 \ MPa$$

If ***E = 207 GPa***, the elastic recoverable strain at maximum load is

$$e_E = \frac{P_{\max}/A_o}{E} = \frac{620\times10^6}{207\times10^9} = 0.0030$$

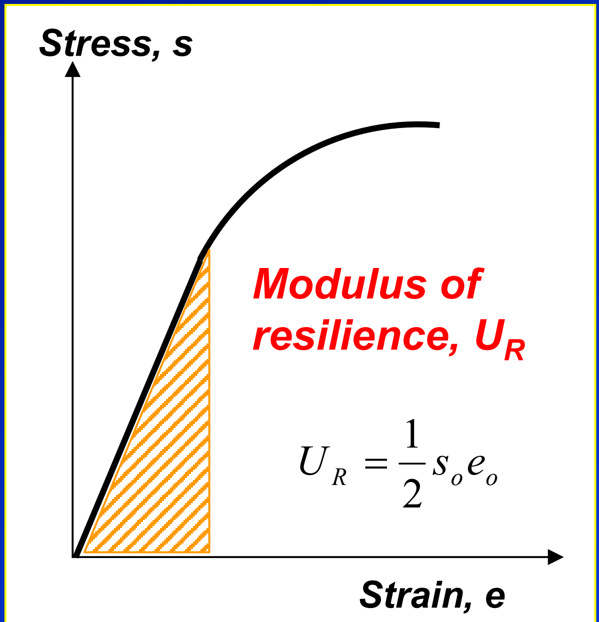
If the elongation at maximum load (the uniform elongation) is 22%, what is the plastic strain at maximum load?

$$e_p = e_{total} - e_E = 0.2200 - 0.0030 = 0.2170$$

Resilience

• ***Resilience*** *is an ability of a material to* ***absorb energy when elastically deformed*** *and to return it when unloaded.*

• *Usually measured by* ***modulus of resilience*** *(strain energy per unit volume required to stress the material from zero to the yield stress,* σ_o*.*



$$U_o = \frac{1}{2}\sigma_x e_x = U_R = \frac{1}{2} s_o e_o = \frac{s_o^2}{2E}$$ *Eq.9*

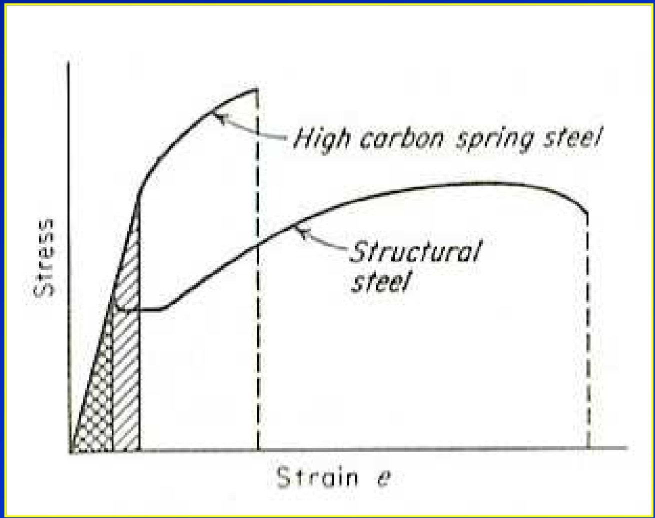
Table 8-2 Modulus of resilience for various materials

Material	E, GPa	s_0, MPa	Modulus of resilience, U_R, kPa
Medium-carbon steel	207	310	232
High-carbon spring steel	207	965	2250
Duralumin	72	124	107
Copper	110	28	3.5
Rubber	0.0010	2.1	2140
Acrylic polymer	3.4	14	28

Note: *for* ***mechanical springs*** *→ high yield stress and low modulus of elasticity.*

Toughness

- **Toughness** is an ability to **absorb energy in the plastic range**.
- *Or the ability to withstand occasional stresses above the yield stress without fracture.*
- *Can be simply defined by the* ***area under the stress-strain curve*** *(amount of work per unit volume that the material can withstand without failure.)*



Comparison of stress-strain curves for high and low-toughness materials.

- The structural steel although has a lower yield point but more ductile than high carbon spring steel. → ***Structural steel is therefore tougher***.
- ***Toughness = strength + ductility***

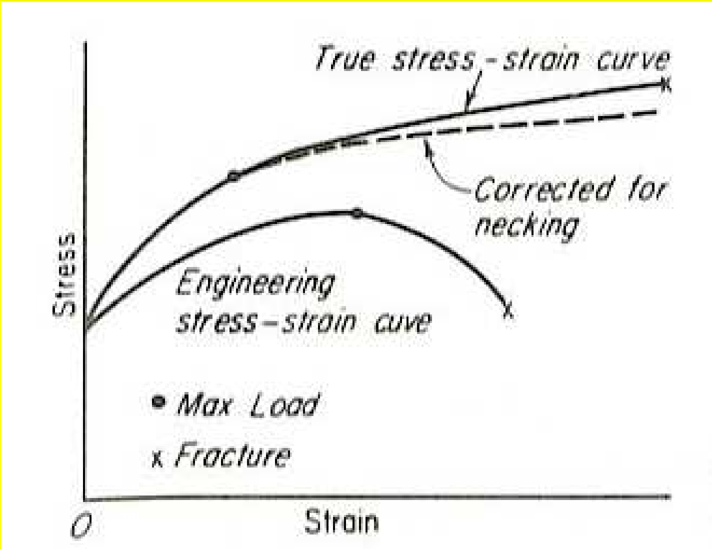
Ductile materials	**Brittle materials**
$U_T \approx s_u e_f$ $U_T \approx \frac{s_o + s_u}{2} e_f$ *Eq.10*	$U_T \approx \frac{2}{3} s_u e_f$ *Eq.11*

(only approximation)

True-stress-true-strain curve

• ***True stress-strain curve*** gives a true indication of deformation characteristics because it is ***based on the instantaneous dimension*** of the specimen.

• The ***true stress-strain curve*** is also known as the ***flow curve***.



Comparison of engineering and the true stress-strain curves

• In ***engineering stress-strain curve***, stress drops down after necking since it is based on the original area.

• In ***true stress-strain curve***, the stress however increases after necking since the cross-sectional area of the specimen decreases rapidly after necking.

True stress

$$\sigma = \frac{P}{A_o}(e+1) = s(e+1)$$

Eq.12

True strain

$$\varepsilon = \ln(e+1)$$

Eq.13

Note: these equations are used for data upto the onset of necking. Beyond necking, use the actual measurements of load, cross-sectional area, diameter.

True stress at maximum load

- ***True stress at maximum load*** corresponds to the ***true tensile strength***.

The ultimate tensile strength

$$s_u = \frac{P_{max}}{A_o}$$

The true stress at maximum load

$$\sigma_u = \frac{P_{max}}{A_u}$$

And true strain at maximum load

$$\varepsilon_u = \ln\frac{A_o}{A_u}$$

Eliminating P_{max} gives

$$\sigma_u = s_u \frac{A_o}{A_u} = s_u e^{\varepsilon_u}$$

Eq.14

Where

σ_u	true stress at maximum load
ε_u	true strain at maximum load
A_u	cross-sectional area of the specimen at maximum load

True fracture stress

• The true fracture stress σ_f is the load at fracture $P_{fracture}$ divided by the cross sectional area at fracture A_f.

$$\sigma_f = \frac{P_{fracture}}{A_{fracture}}$$

Eq.15

Note: Need to be corrected for the triaxial state of stress existing in the tensile specimen at fracture. → Often error.

True fracture strain

• The true fracture strain ε_f is based on the original area A_o and the area after fracture A_f.

$$\varepsilon_f = \ln \frac{A_o}{A_f}$$

Eq.16

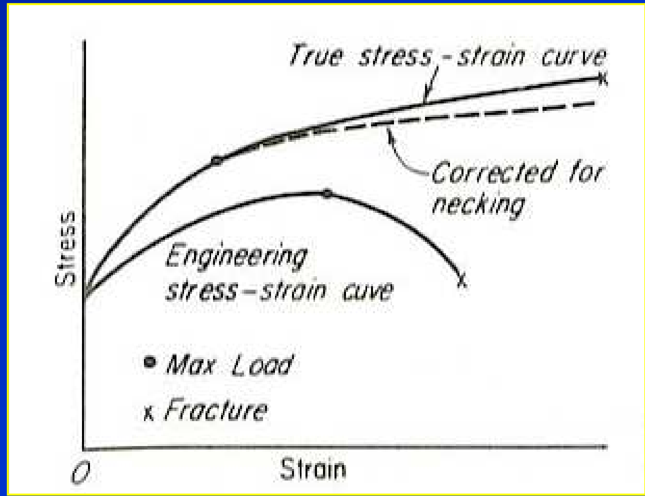
• After necking, the true fracture strain can be related to the area of reduction q.

$$\varepsilon_f = \ln \frac{1}{1-q}$$

Eq.17

True uniform strain

- The ***true uniform strain*** ε_u is the true strain based only on the strain up to the maximum load.
- Can either be measured from A_u or L_u at maximum load.
- The uniform strain is often used in ***estimating the formability of metals*** from the result of a tension test.



Engineering and true stress-strain curves

$$\varepsilon_u = \ln \frac{A_o}{A_u} \qquad \text{Eq.18}$$

True local necking strain

- The true local necking strain is the strain required to deform the specimen from the maximum load to fracture.

$$\varepsilon_n = \ln \frac{A_u}{A_f} \qquad \text{Eq.19}$$

Power-law flow curve

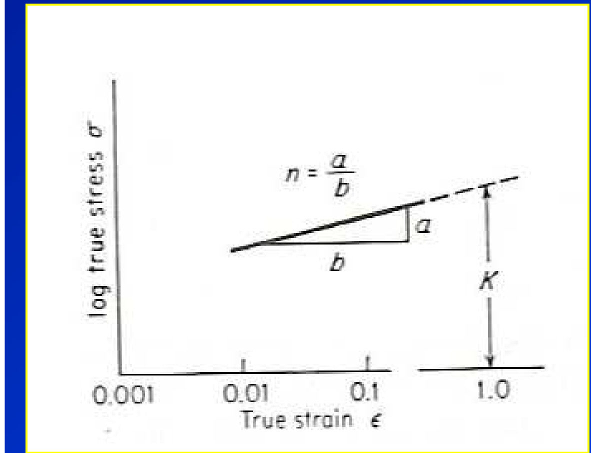
• The flow curve of many metals in the region of uniform plastic deformation can be expressed by the ***simple power law***.

$$\sigma = K\varepsilon^n$$

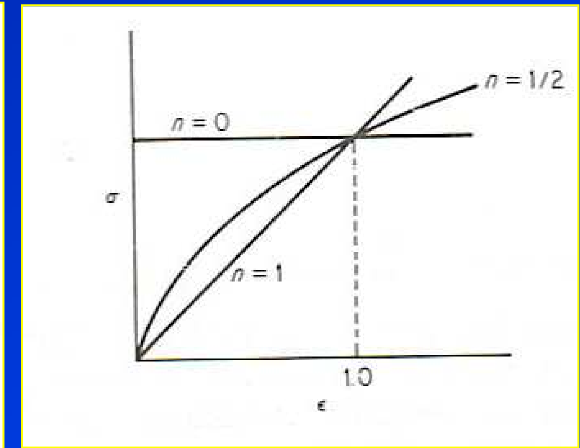
Eq.20

Where ***n*** is the strain hardening exponent
K is the strength coefficient

• ***Log-log plot of true stress-strain curve*** from yield point up to the maximum load will result in a straight line where ***n*** is the slope and ***K*** is the true stress at ε = 1.0.



Log-log plot of true stress-strain curve



Different forms of power curve $\sigma = K\varepsilon^n$

n = 0 perfectly plastic solid
n = 1 elastic solid
For most metals, 0.1< ***n*** < 0.5

Variations of the power-law flow curve

Datsko showed that ε_o is considered as the amount of strain hardening of the material obtained prior to tension test.

$$\sigma = K(\varepsilon_o + \varepsilon)^n \qquad \textit{Eq.21}$$

Ludwik equation relates the yield stress to the power law

$$\sigma = \sigma_o + K\varepsilon^n \qquad \textit{Eq.22}$$

True stress-strain curve of ***austenistic stainless steel*** at low strain can be expressed by

$$\sigma = K\varepsilon^n + e^{K_1} e^{n_1 \varepsilon} \qquad \textit{Eq.23}$$

Where e^{K_1} ~ proportional limit
n is the slope of the curve

Example: *In the tension test of a metal fracture occurs at maximum load. The conditions at fracture were:* $A_f = 100\ mm^2$ *and* $L_f = 60\ mm$*. The initial values were:* $A_o = 150\ mm^2$ *and* $L_o = 40\ mm$*. Determine the true strain to fracture using changes in both length and area.*

$$\varepsilon_f = \ln\left(\frac{L_f}{L_o}\right) = \ln\left(\frac{60}{40}\right) = 0.405$$

$$\varepsilon_f = \ln\left(\frac{A_o}{A_f}\right) = \ln\left(\frac{150}{100}\right) = 0.405$$

At the maximum load, both area and gauge length can be used for a strain calculation.

If a more ductile metal is tested such that necking occurs and the final gauge length is 83 mm and the final diameter is 8 mm, while $L_o = 40\ mm$ *and* $D_o = 12.8\ mm$*.*

$$\varepsilon_f = \ln\left(\frac{L_f}{L_o}\right) = \ln\left(\frac{83}{40}\right) = 0.730$$

$$\varepsilon_f = \ln\left(\frac{D_o}{D_f}\right)^2 = 2\ln\left(\frac{12.8}{8}\right) = 0.940$$

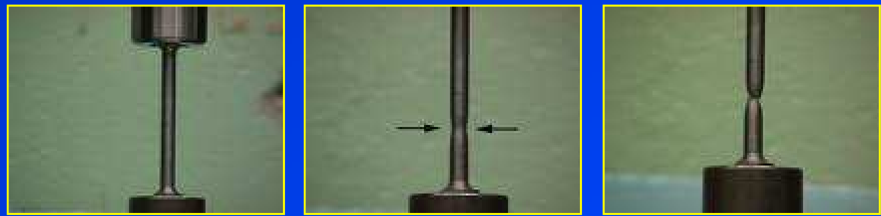
After necking, gauge length gives error but area of reduction can still be used for the calculation of true strain at fracture.

Instability in tension

Ideal plastic material → Undergo ***necking*** after yielding with no strain hardening

Most metal → ***Necking*** begins at maximum load with strain hardening → increasing load-carrying capacity

Necking *or localised deformation starts at the maximum load, which is opposed by a decrease in cross-sectional area of the specimen as it elongates.*



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Instability occurs when

An increase in stress due to reduced cross-sectional area	>	The increase in load-carrying capability due to strain hardening

Instability in tension

The condition of instability, which leads to localised deformation is defined by

$$dP = 0, P = \sigma A$$

$$dP = \sigma dA + A d\sigma = 0$$

Because the volume is constant

$$\frac{dL}{L} = -\frac{dA}{A} = d\varepsilon$$

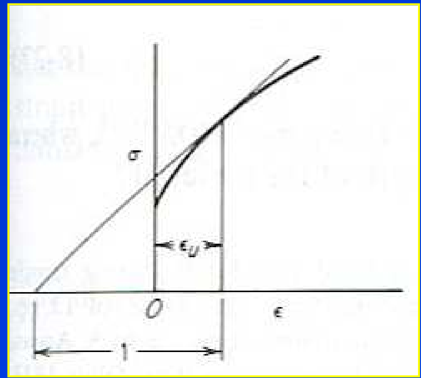
From the instability condition

$$-\frac{dA}{A} = \frac{d\sigma}{\sigma}$$

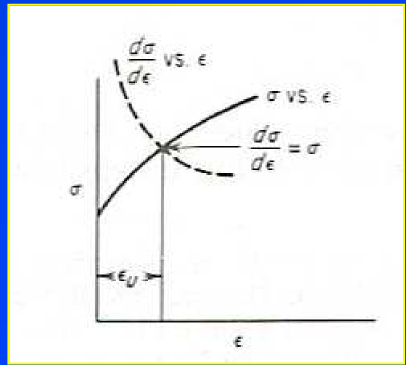
So that at a point of tensile instability

$$\frac{d\sigma}{d\varepsilon} = \sigma \qquad \textit{Eq.24}$$

Therefore the point of necking can be obtained from the true stress-strain curve by



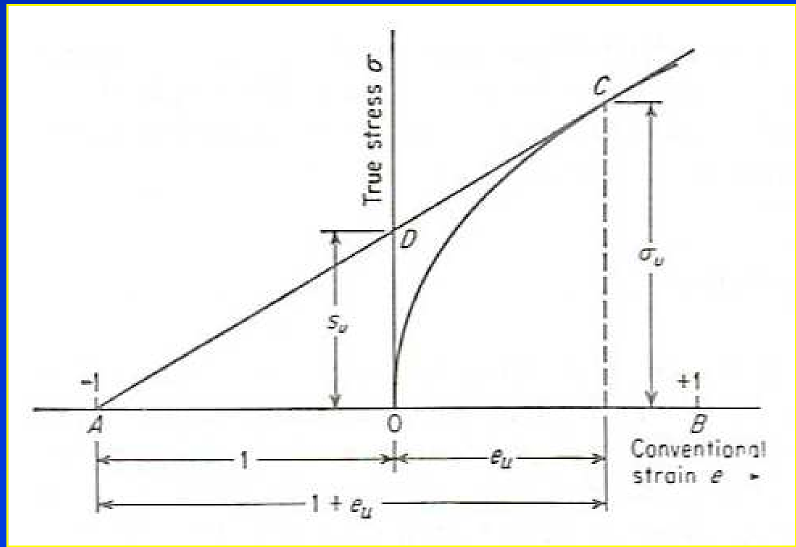
(a) Finding the point on the curve having a subtangent of unity.



(b) The point where the rate of strain hardening **$d\sigma/d\varepsilon$** equals the stress.

Considère's construction for the determination of maximum load

The ***maximum load*** can be determined from ***Considère's construction*** when the stress-strain curve is plotted in terms of true stress σ and conventional strain ***e***.



- Let point ***A*** represent a negative strain of 1.0.
- A line drawn from point ***A*** which is tangent to the stress-strain curve will give maximum load with the ***slope*** of ***σ/(1+e).***
- The strain at which necking occurs is the true uniform strain ε_u

$$\varepsilon_u = n \qquad \text{Eq.25}$$

Table 8-3 Values for n and K for metals at room temperature

Metal	Condition	n	K, MPa
0.05% C steel	Annealed	0.26	530
SAE 4340 steel	Annealed	0.15	640
0.6% C steel	Quenched and tempered 540°C	0.10	1570
0.6% C steel	Quenched and tempered 705°C	0.19	1230
Copper	Annealed	0.54	320
70/30 brass	Annealed	0.49	900

Example: *If the true stress-strain curve is given by $\sigma = 1400\varepsilon^{0.33}$, where stress is in MPa, what is the ultimate tensile strength of the material?*

The uniform elongation to maximum load is

$$\varepsilon_u = n = 0.33$$

The true stress at maximum load is

$$\sigma_u = 1400(0.33)^{0.33} = 971 \; MPa$$

From *Eq.13*

$$e_u + 1 = \exp(\varepsilon_u) = \exp(0.33) = 1.391$$

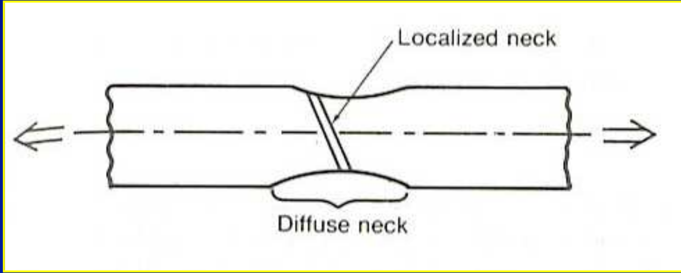
Therefore the ultimate tensile strength is

From *Eq.12*

$$s_u = \frac{971}{e^{0.33}} = \frac{971}{1.391} = 698 \; MPa$$

Flow instability (necking) in biaxial tension

Necking in a uniaxial cylindrical tensile specimen is isotropic. However in a ***sheet specimen*** *where the width of the specimen is much higher than the thickness, there are two types of flow instability:*



Diffuse and localised necking in a sheet tensile specimen.

Power law flow curve for localised necking

$$\varepsilon_u = 2n \quad \text{Eq.26}$$

1) Diffuse necking

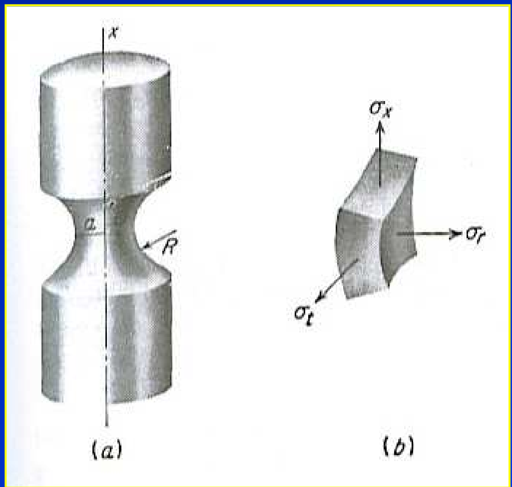
- Provide ***a large extent of necking*** on the tensile specimen similar to necking from a cylindrical specimen.
- Diffuse necking might terminate in ***fracture*** but normally followed by localised necking.

2) Localised necking

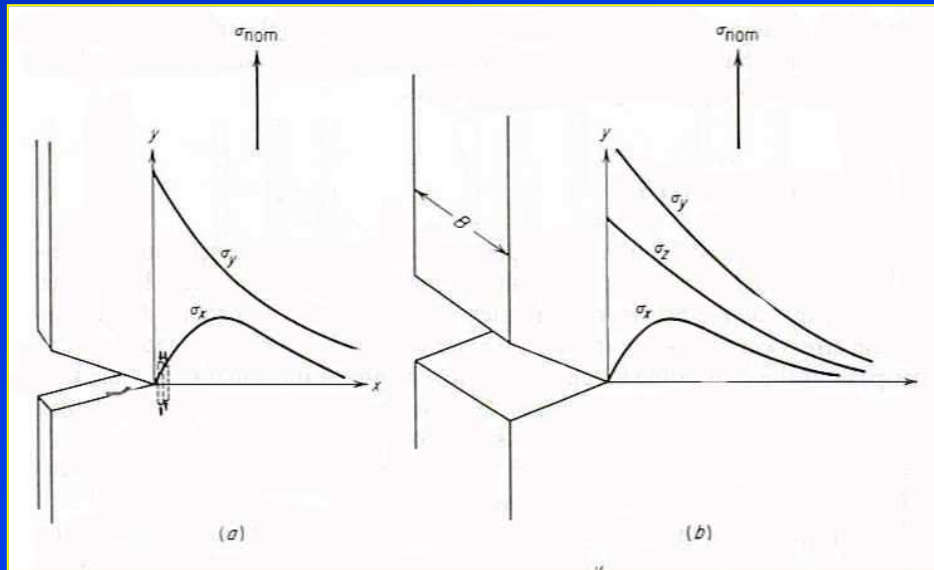
- Localised necking is a ***narrow band*** with its size ~ specimen thickness, and inclined at an angle ϕ~55°.
- Give no change in width through the localised neck → ***plain strain deformation***.

Stress distribution in necking

- ***Necking*** introduces a ***complex triaxial state of stress*** in the necked region ~ a mild notch.
- The ***average true stress at necking***, which is much higher than the stress would be required to cause a normal plastic flow due to stresses in width and thickness directions.



(a) Geometry of necked region, (b) stress acting on element at point O



Elastic stresses beneath the notch in (a) plain stress, (b) plain strain

Ductility measurement in tension test

• ***Measured elongation*** in tension specimen depends on the ***gauge length*** or ***cross-sectional area***.

Total extension

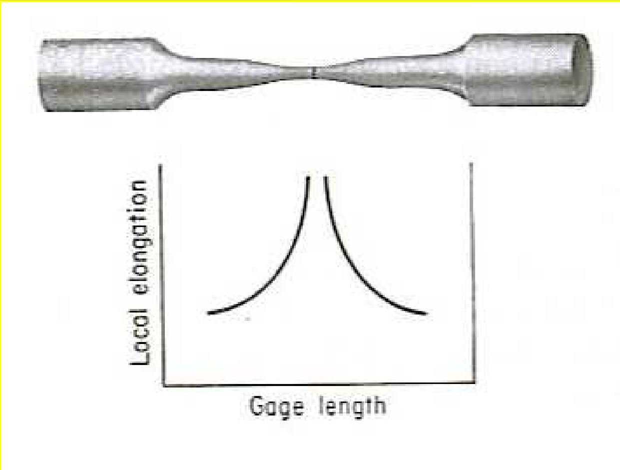
Uniform extension up to necking

Depends on

• Metallurgical condition of the material (through ***n***)

• Specimen size and shape on the development of necking

Localised extension once necking begins

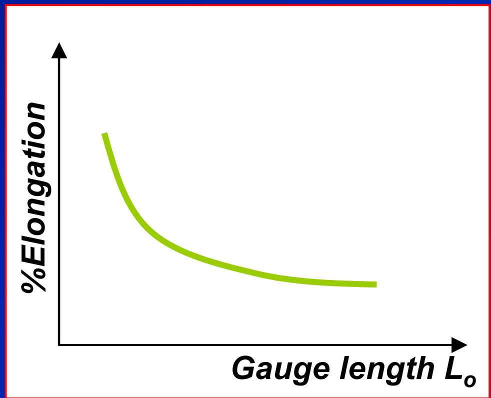


Variation of local elongation with position along gauge length of tensile specimen

The shorter the gauge length, the greater the effect of localised deformation at necking on total elongation.

Dimensional relationships of tensile specimens for sheet and round specimens

• Elongation depends on the original gauge length L_o. %elongation ↓ as L_o ↑



Example: Standard gauge length

$$L_o = 5.65\sqrt{A} \qquad \text{Eq.27}$$

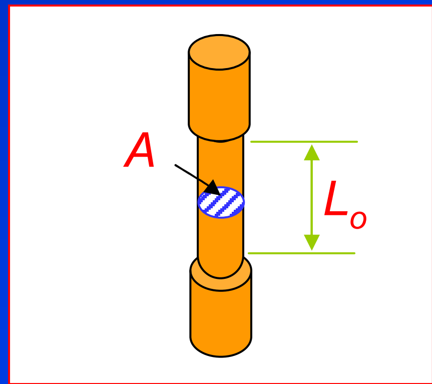


Table 8-4 Dimensional relationships of tensile specimens used in different countries

Type specimen	United States (ASTM)	Great Britain: Before 1962	Great Britain: Current	Germany
Sheet ($L_0/\sqrt{A_0}$)	4.5	4.0	5.65	11.3
Round (L_0/D_0)	4.0	3.54	5.0	10.0

Dimensional relationships for sheet and round tensile specimens used in different countries

Difference between % elongation and % reduction of area

% Elongation

• % Elongation is chiefly influenced by ***uniform elongation***, which is dependent on the ***strain-hardening capacity*** of the material.

Reduction of Area

• ***Reduction of area*** is more a measure of the deformation required to produce failure and its chief contribution results from the ***necking process***.

• Because of the ***complicated state of stress state*** in the neck, values of reduction of area are dependent on ***specimen geometry***, and ***deformation behaviour***, and they should not be taken as true material properties.

• ***RA*** is the most ***structure-sensitive ductility parameter*** and is useful in detecting ***quality changes in the materials***.

Effect of strain rate on flow properties

- Strain rate is defined as

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} \qquad \textit{Eq.28}$$

- The unit is per second, ***s^{-1}***.

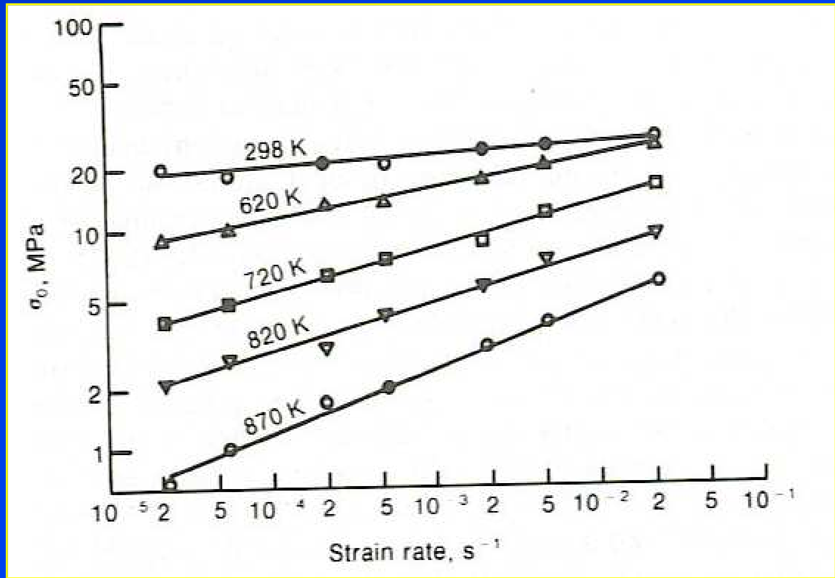
Table 8-5 Spectrum of strain rate

Range of strain rate	Condition or type test
10^{-8} to 10^{-5} s^{-1}	Creep tests at constant load or stress
10^{-5} to 10^{-1} s^{-1}	"Static" tension tests with hydraulic or screw-driven machines
10^{-1} to 10^{2} s^{-1}	Dynamic tension or compression tests
10^{2} to 10^{4} s^{-1}	High-speed testing using impact bars (must consider wave propagation effects)
10^{4} to 10^{8} s^{-1}	Hypervelocity impact using gas guns or explosively driven projectiles (shock-wave propagation)

Spectrum of strain rate

Strain rate ↑ **Flow stress** ↑

Temperature ↑ **Strain rate dependence** ↑



Flow stress dependence of strain rate and temperature

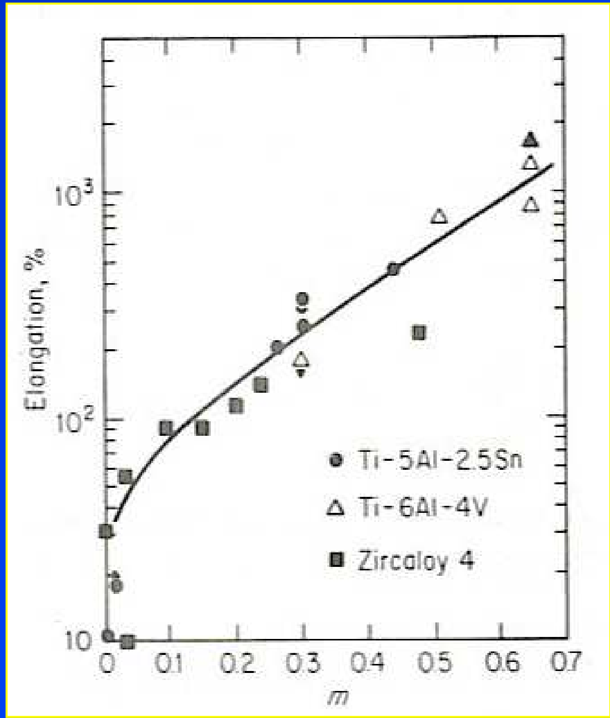
Strain rate sensitivity, m

- Strain rate sensitivity indicates any changes in ***deformation behaviour***.
- Measurement of strain rate sensitivity can be linked to ***dislocation concept*** (velocity of mobile dislocations).

- ***Strain rate sensitivity m*** can be obtained from

$$\sigma = C\left(\dot{\varepsilon}\right)^m\Big|_{\varepsilon,T} \qquad \textit{Eq.29}$$

- ***High strain rate sensitivity*** is a characteristic of ***superplastic*** metals and alloys.



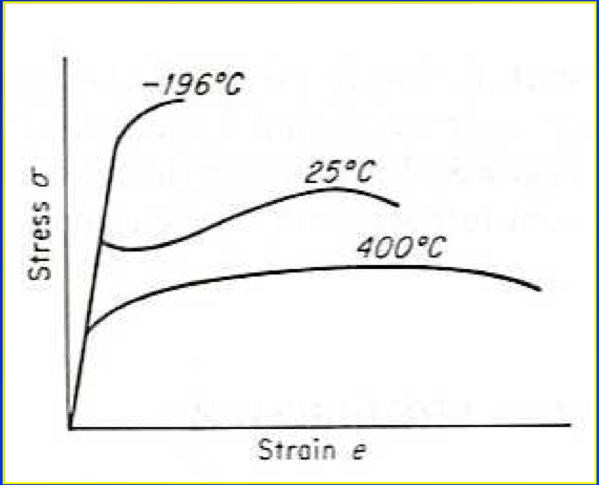
Dependence of tensile elongation on strain-rate sensitivity

Effect of temperature on flow properties

Temperature strongly affects the stress-strain curve and the flow and fracture properties.

Temperature ↑ **Strength** ↓

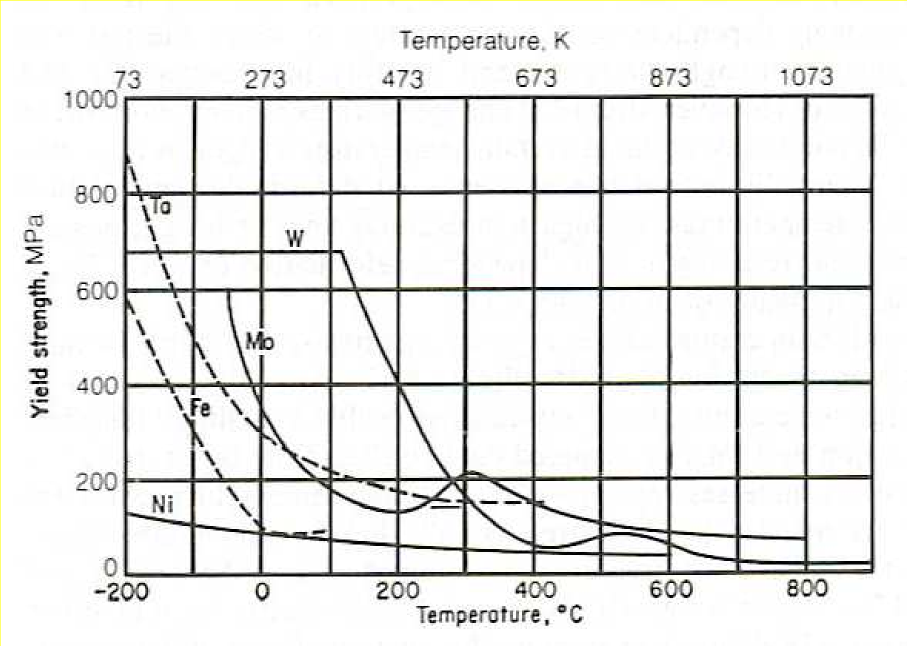
Ductility ↑



Changes in engineering stress-strain curves of mild steel with temperature

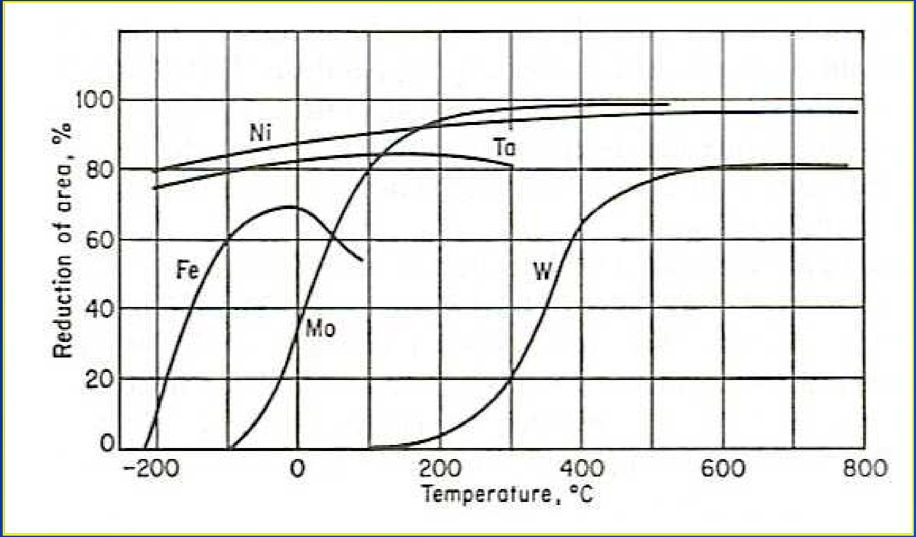
- ***Thermally activated processes*** assist deformation (dislocation motion) and reduce strength at elevated temperatures.
- ***Structural changes*** can occur at certain temperature ranges (high temp / long term exposure) to alter the general behaviour.

Effects of temperature on yield stress



• For ***BCC*** metals, the ***yield stress*** is strongly dependent on temperature where as in ***FCC*** metals, the ***yield stress*** is only slightly dependent on temperature.

Effects of temperature on ductility



• ***W*** is brittle at 100^{o}C, ***Fe*** at -225^{o}C while ***Ni*** decreases little in ***ductility*** over the entire temperature interval.

Comparison of mechanical properties of different materials at various temperature

- Mechanical properties of different materials at various temperature can be compared in terms of ***homogeneous temperature*** (the ratio of the test temperature to the melting point, expressed in degree ***kelvin)***.

$$\textbf{Homogenous temperature} = \frac{\textbf{Testing temperature}}{\textbf{Melting temperature}}$$

- And this should be compared in terms of ratios of σ / E rather than simple ratios of flow stress.

Influence of testing machine on flow properties

Load controlled machine

• The ***operator adjusts the load precisely*** and leave with whatever displacement happens to be associated with the load.

Displacement controlled machine

• ***Displacement is controlled*** and the load adjusts itself to that position. Ex: Screw driven machine.

Constant cross head velocity is the sum of

1) Elastic strain rate in specimen
2) Plastic strain rate in specimen
3) Strain rate resulting from elasticity of the machine.

Currently we can have machines which can change from load control to displacement control.

Effect of the testing machine of the shape of the stress-strain curve and fracture behavior

Hard machine

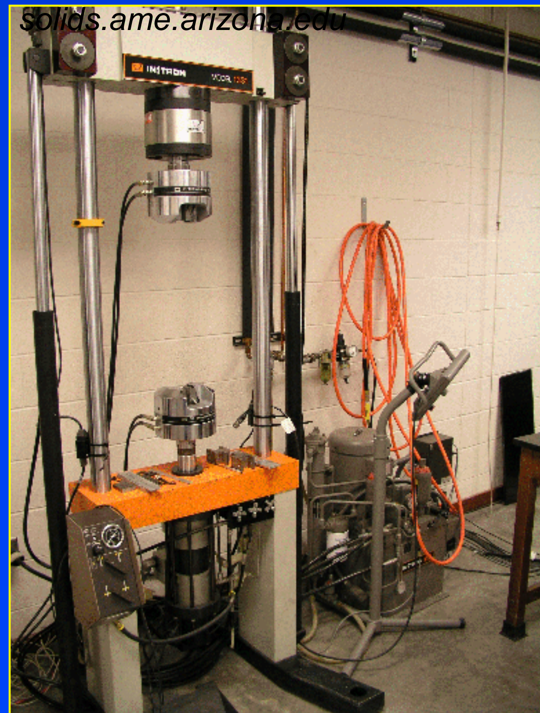
- A rigid testing machine with a high spring constant.
- ***Ex:*** Screw driven machine.
- Will reproduce faithfully the upper and the lower yield point.

Soft machine

- Hydraulic testing machine.
- The effect of upper and lower yield point will be smeared out and only the extension at constant load will be recorded.

Screw driven machine

Hydraulic testing machine

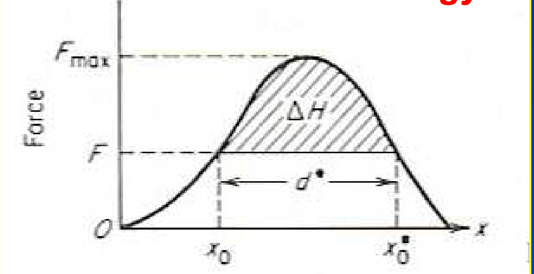


Thermally activated deformation

Plastic deformation depends on

- **Stress**
- **Temperature**
- **Deformation**
- **Stain rate,**
- **Microstructure**
- **Composition**

Thermal activation energy



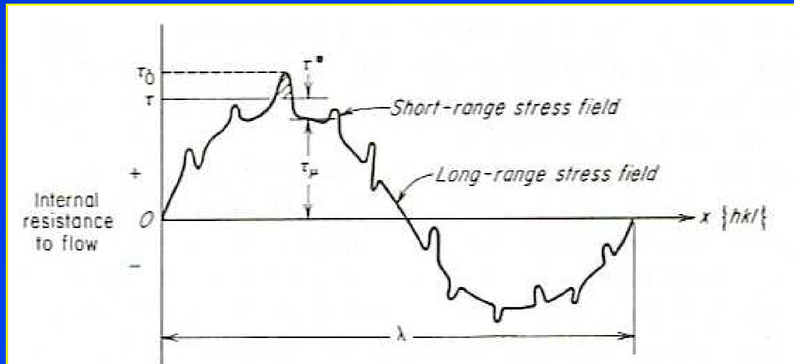
d* - distance the atom move during the process.
ΔH – Energy required to overcome the barrier.

The effective shear stress is $\tau - \tau_i$

Where τ is the applied shear stress
τ_i is the internal resisting stresses.

The τ_i can be grouped into;

1) ***Long-range obstacles*** : barriers too high and long the be surmounted by thermal fluctuation.
2) ***Short-range obstacles*** : (~10 atom diameters) thermal fluctuation can assist dislocations in surmounting these barriers. → ***thermal activation barrier***.



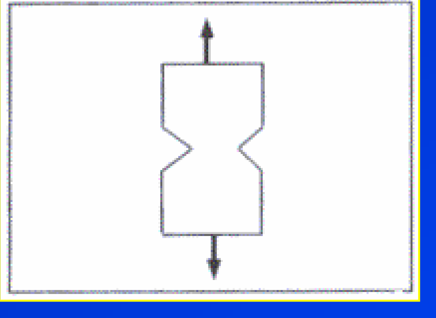
Long range and short range stress fields

Notch tensile test

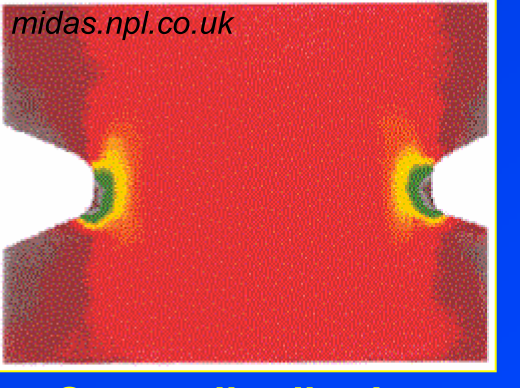
Notch tensile test *is used to evaluate* ***notch sensitivity*** *(the tendency for reduced tensile ductility in the presence of a triaxial stress field and steep stress gradient. → express metallurgical or environmental changes.*

Notch tensile specimen →

- ***60^o notch*** with a ***root radius of 0.025 mm*** or less introduced into a round (circumferential notch) or a flat (double-edge notch) tensile specimen.
- The ***cross-sectional area*** under the notch root is ***one-half*** of the unnotched area.



Notch tensile specimen.



Stress distribution around tensile notches.

Notch strength

• ***Notch strength*** is defined as the ***maximum load*** divided by the ***original cross-sectional area at the notch***.

• Due to the ***constraint*** at the notch, the ***notch strength*** is higher than the tensile strength of the unnotched specimen.

• ***Notch-strength ratio NSR*** detects notch brittleness (high notch sensitivity) from;

$$\text{NSR} = \frac{S_{net}\ \text{(for notched specimen at maximum load)}}{S_u\ \text{(tensile strength for unnotched specimen)}}$$

• If the ***NSR*** is < 1, the metal is notch brittle.

Tensile properties of steel

Ferrous materials are of commercial importance. → Great deal of work is paid to relate microstructure, composition to properties.

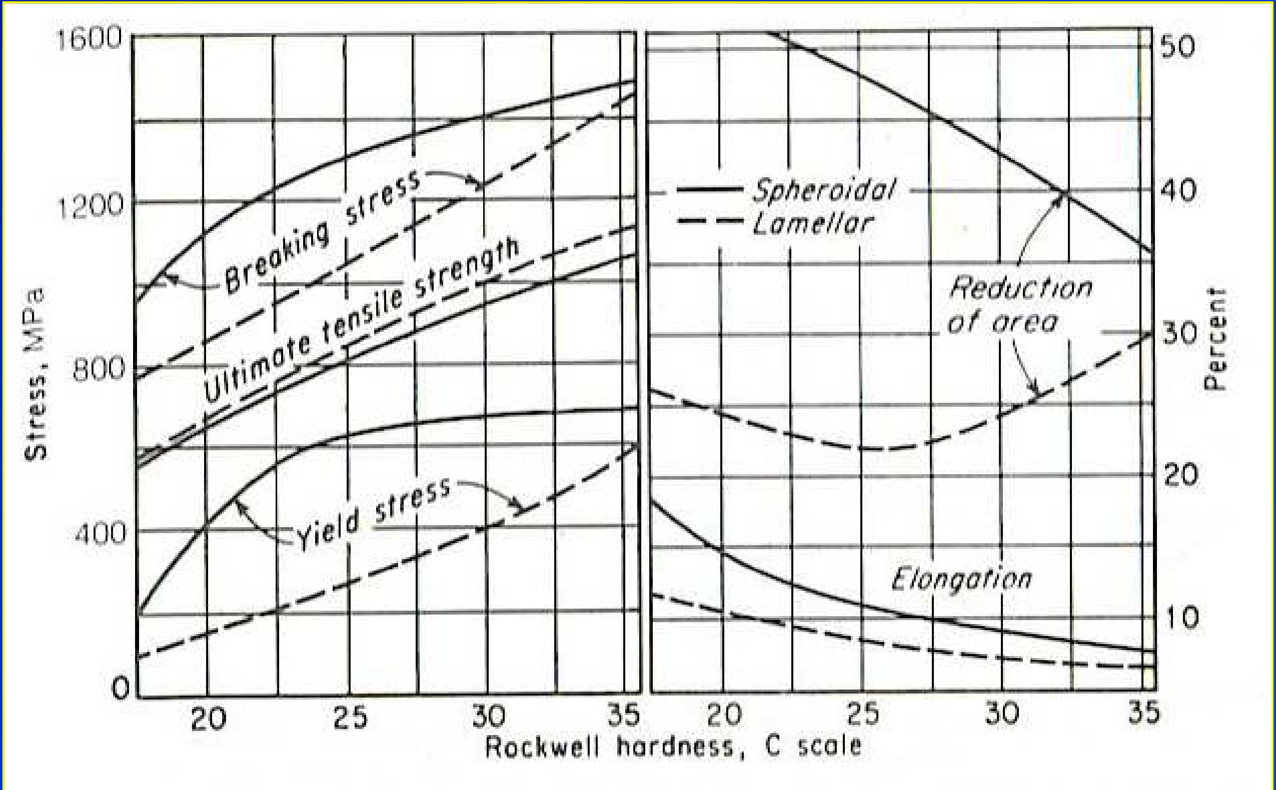
Composition and <u>more importantly microstructure</u> are the chief variables which control the properties of steel.

The tensile properties of annealed and normalised steels are controlled by

1) Flow and fracture characteristics of the ferrite (strength ~ alloying elements, grain size)
2) Amount of ferrite
3) Shape of ferrite
4) Distribution and amount of cementite (***C*** content)

Tensile properties in steels with different microstructures

• ***Normalised steel*** has higher strength than ***annealed steel*** due to more rapid rate of cooling, resulting in ***pearlite.***



Tensile properties of pearlite and spheroidite in eutectoid steel

Tensile properties in steels with different microstructures

• Strength of ***annealed steel*** can be improved by ***cold working.***

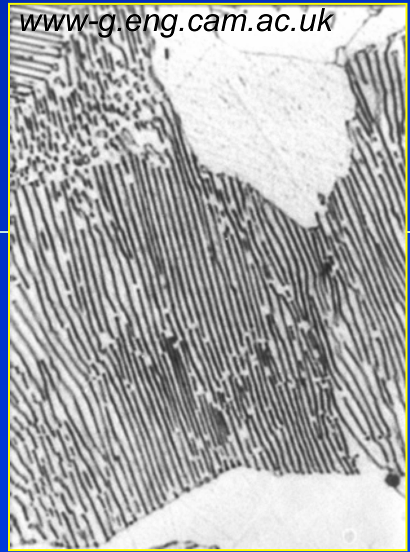
Table 8-6 Effect of cold-drawing on tensile properties of SAE 1016 steel†

Reduction of area by drawing, %	Yield strength, MPa	Tensile strength, MPa	Elongation, in 50 mm, %	Reduction of area, %
0	276	455	34	70
10	496	517	20	65
20	565	579	17	63
40	593	655	16	60
60	607	703	14	54
80	662	793	7	26

† After L. J. Ebert, "A Handbook on the Properties of Cold Worked Steels," PB 121662, Office of Technical Services, U.S. Department of Commerce, 1955.

Tensile properties in steels with different microstructures

- ***Tensile properties of pearlitic steel*** can be best controlled by ***transforming the austenite to pearlite*** at a constant temperature on continuous cooling from above the critical temperature.
- The transformation product is ***lamellar pearlite***.



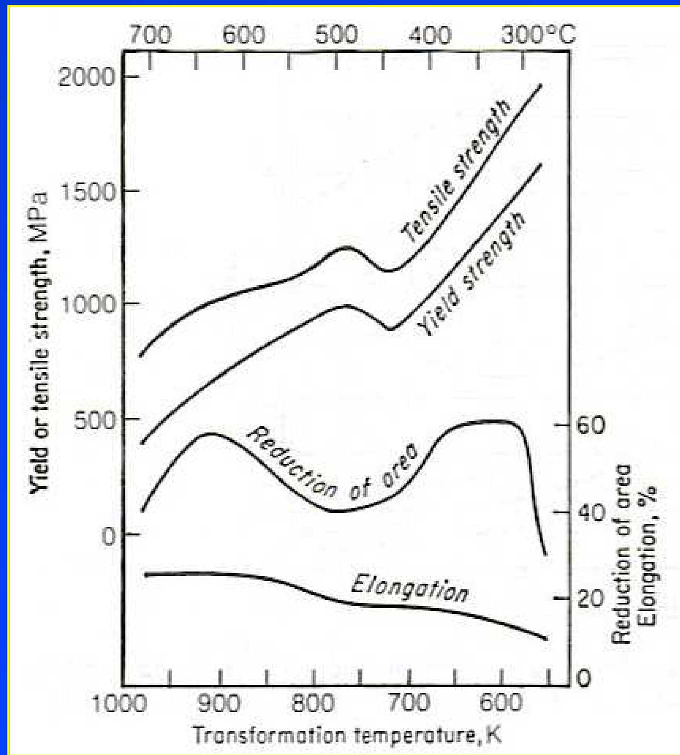
Pearlite microstructure

Transformation temperature ↑

↓

Spacing between cementite platelets ↓

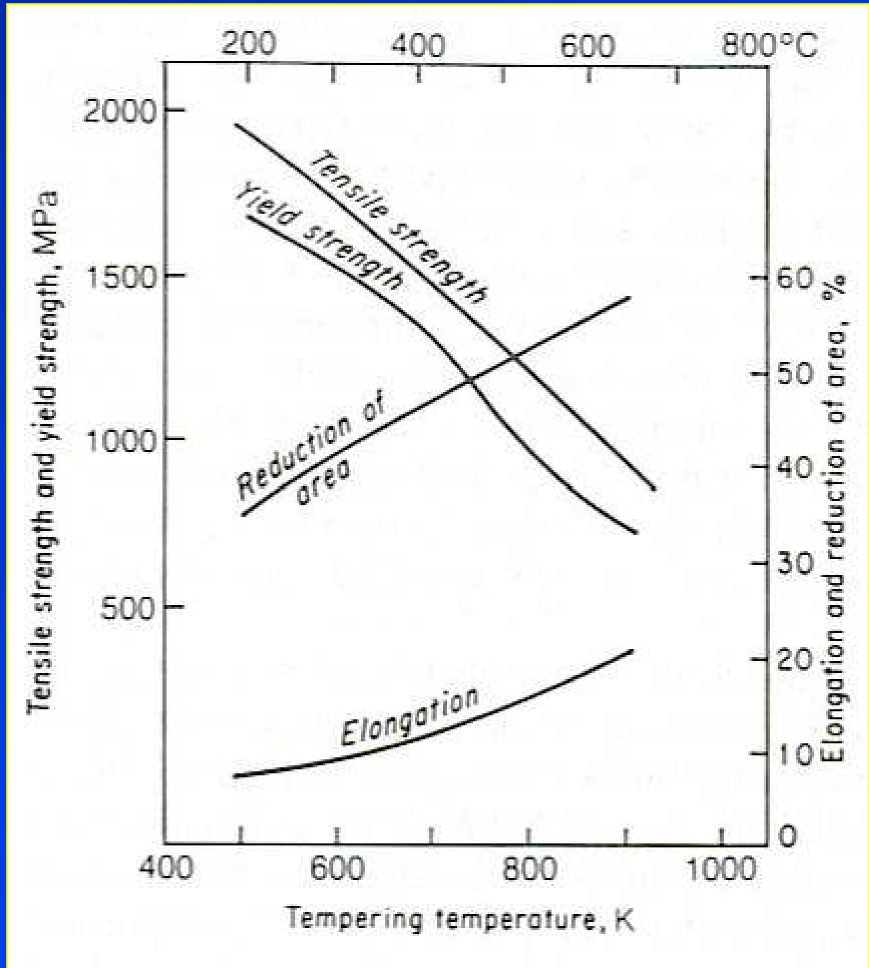
Strength ↑



Relationship of tensile properties of Ni-Cr-Mo steel to isothermal transformation temperature.

Tensile properties in quenched and tempered steels

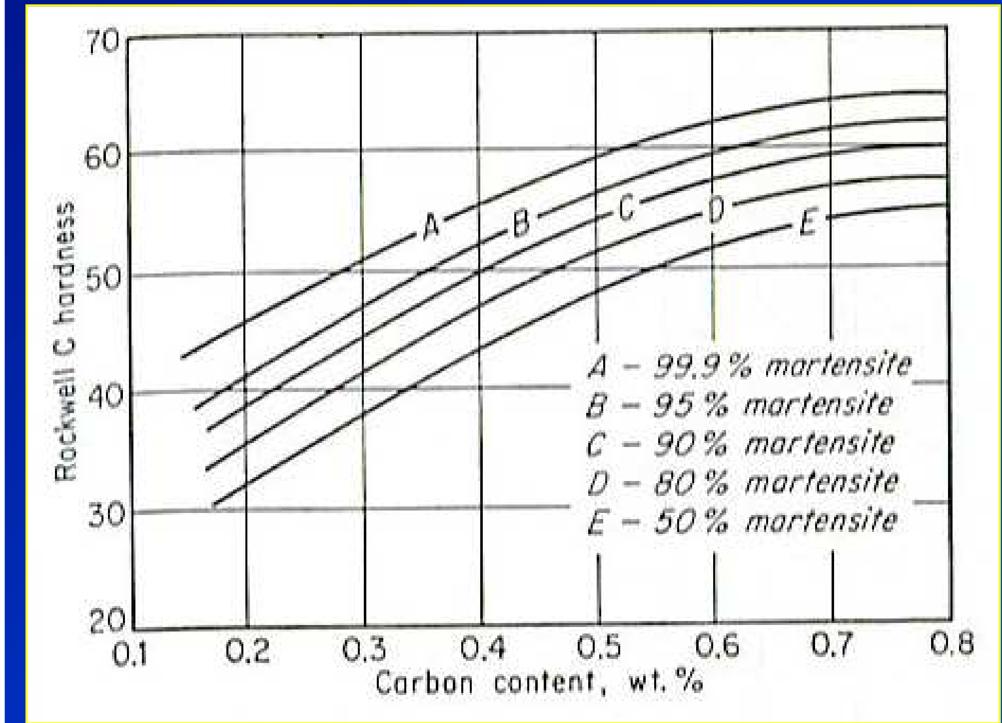
• ***The best combination of strength and ductility*** is obtained in steel which has been ***quenched*** to a fully martensitic structure and then ***tempered***.



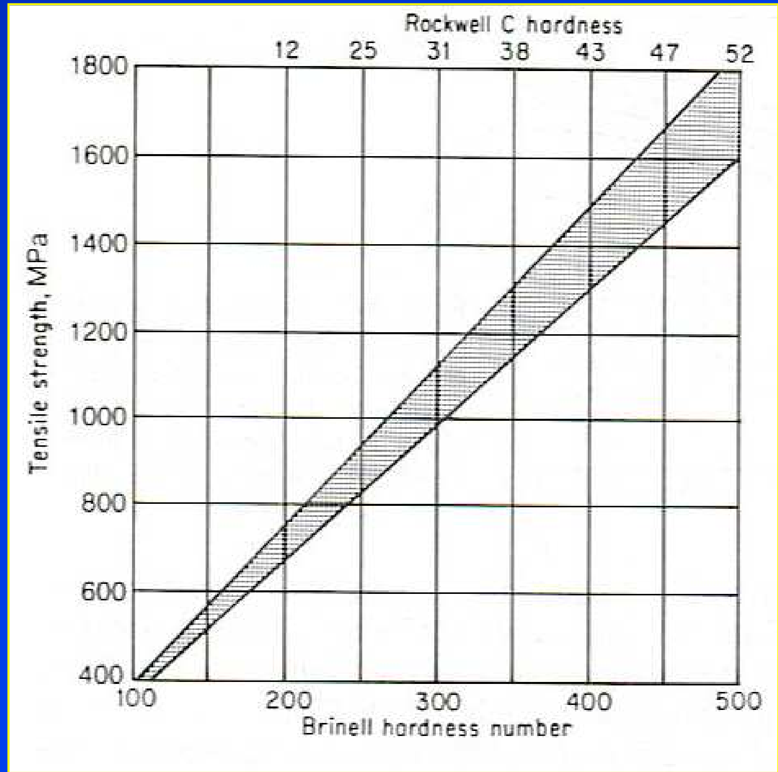
Tensile properties of quenched and tempered SAE-4340 steel as a function of tempering temperature

Tensile properties in quenched and tempered steels

- *Martensitic structure* provides ***hardness and strength.***
- Mechanical properties are changed by altering the tempering temperature.



As-quenched hardness of steel as a function of carbon content



Relationship between tensile strength and hardness for quenched and tempered, annealed and normalised steel

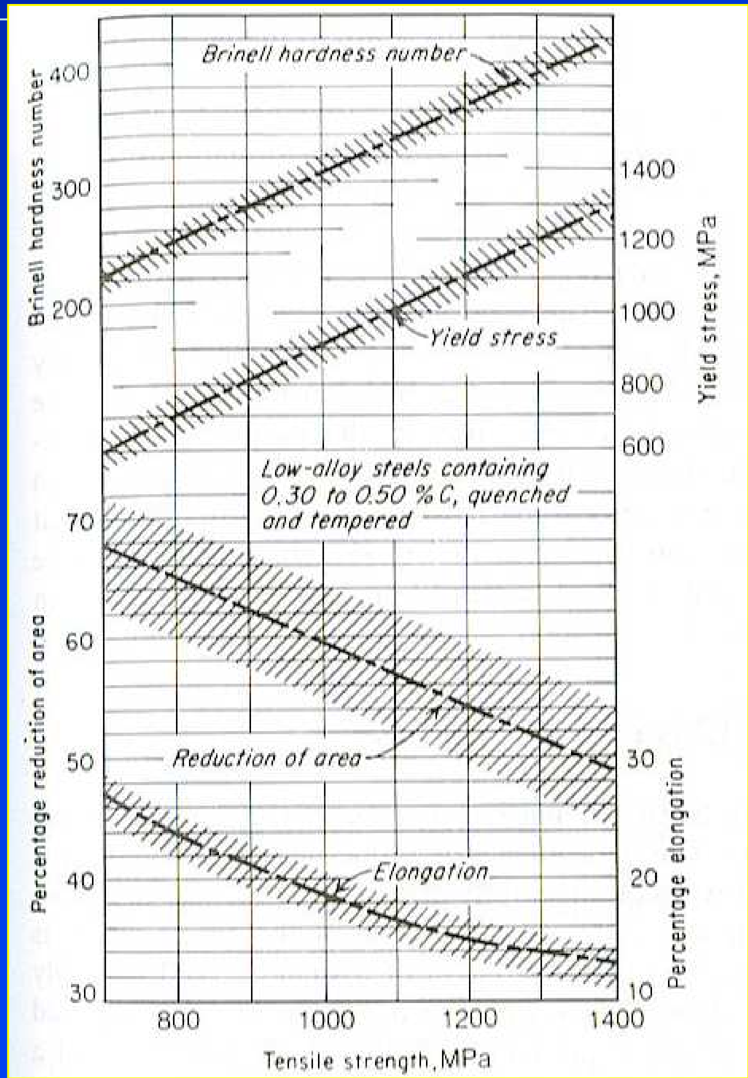
Tensile properties in low-carbon steel

• Mechanical properties of ***low-carbon steels (0.3-0.5%C)*** do not depend basically on alloy content, carbon content or tempering temperature.

• Steels quenched to essentially 100% martensite and then tempered can give ***Tensile strength*** of in the range 700 – 1400 MPa. → a wide variety of alloyed steels are used.

• A range of specific properties can be obtained as appeared in ***shaded area***.

• in large steel sections, ***slack-quenched structure*** (non-100% martensitic structure-containing ferrite, pearlite, bainite interspersed with martensite) gives ***poorer properties***.

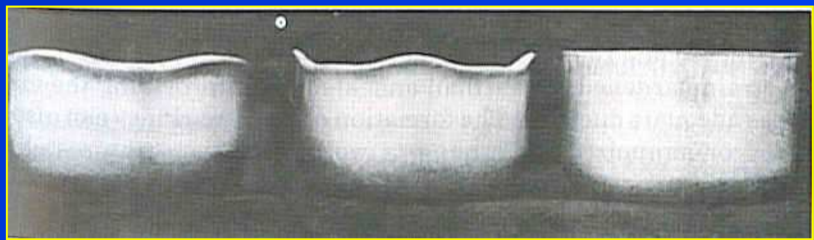


Relationships between tensile properties of quenched and tempered low-alloy steels

Anisotropy of tensile properties

Crystallographic anisotropy

- Crystallographic anisotropy results from the ***preferred orientation*** of the grains, which is produced by severe plastic deformation.
- ***Yield strength*** and ***tensile strength*** to a lesser extent, are the properties most affected.
- Crystallographic anisotropy can be eliminated by ***recrystallisation***.
- Example : ***Ears*** in deep-drawn cups.

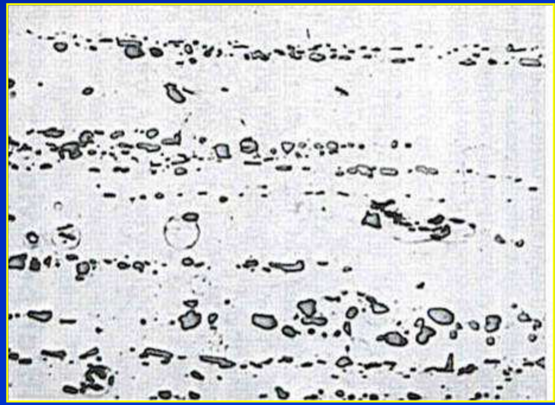


Ears in drawn cups.

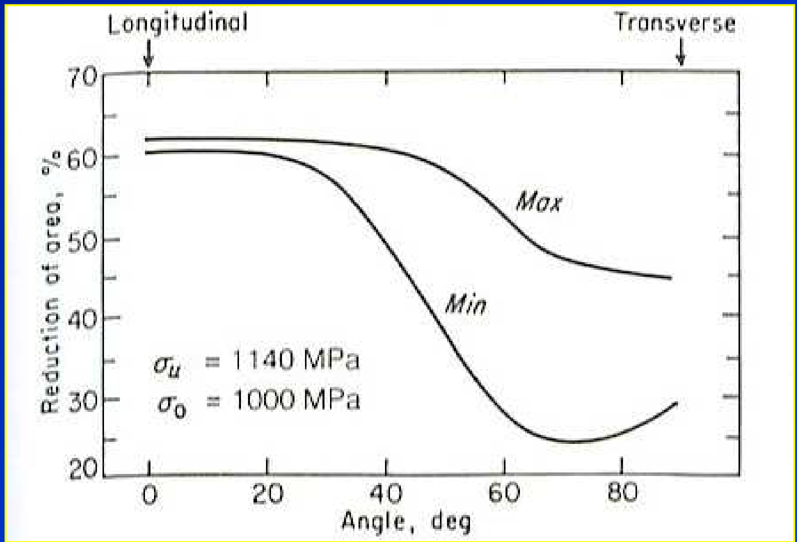
Anisotropy of tensile properties

Mechanical fibering

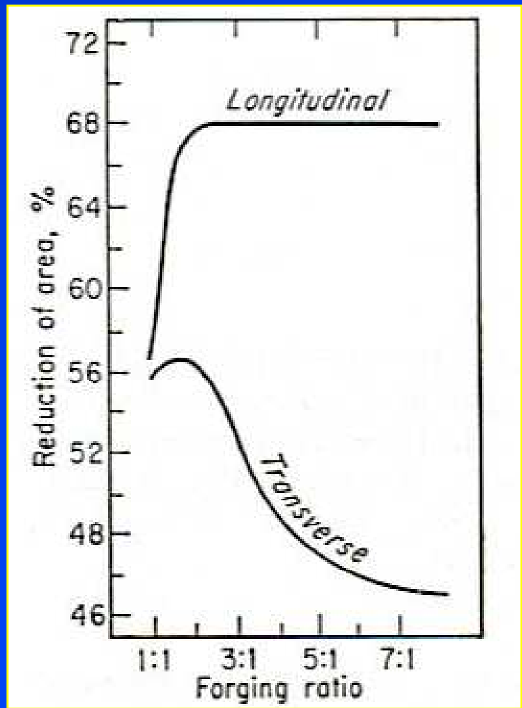
- ***Mechanical fibering*** is due to ***preferred alignment*** of inclusions, voids, seggregation, and second phase in the ***working direction***. → important in forgings and plates.
- Ductility is the most affected.



Alignment of particles or inclusions along the working direction



Reduction of area and angle between the longitudinal direction in forging and the specimen axis



Effect of forging on longitudinal and transverse reduction of area

References

- Dieter, G.E., ***Mechanical metallurgy***, 1988, SI metric edition, McGraw-Hill, ISBN 0-07-100406-8.