OPTIMIZATION ENGINEERING

OE is a scientific approach to problem solving for executing decision making which requires the formation of mathematical economical model for decision & control problems to deal with situation arising out of risk & uncertainity

In fact decision & Control problems in any organisation are more often related to certain daily operation like inventory control, production setting, man-power planning, distribution & maintainence.

This can be modified by accn to ORSA (Operation Research society of America)

It is a tool which is concerned with the design & operation of man-machine system scientifically usually under conditions of requiring optimium allocation of limited resource.

According to Operation Research Society of Great Britain, it approaches of scientific method to complex problem arising the direction & management of large system of man, machines, material & money in industry, business & govt. (during 1948)

* The origin & development of OR can be studied under the following classification -

- Pre world war II
- Development during world war II
- post world war II
- Computer era
- Inclusion of uncertainity model.

MODELS:-

is an abstraction of reality.

Some eg. of models are:-

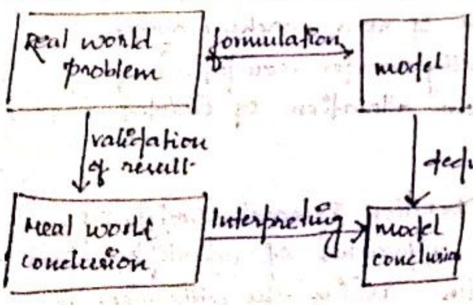
road map of the city

Short route from given source to destination

electrical network problem

Linear eqn to forecast demand of a product.

STEPS OF MODELLING:-



(i) Identify the management decision problem of real world.

(ii) Formulate model for the ~~model~~ real world problem which in the form of following steps.

→ Identify the parameters & Variables which are involved in the management decision problem. Define them verbally to make an valid conclusion among the variable & parameters.

→ Select the variable that appears to the most influential so that the model may be kept as few as possible & simple. Classify the variable in controlable & non-controlable.

→ State verbal relationship among the variables based upon Known principle

→ Constanct a model by combining

→ Perform symbolic manipulation such as solving a system of eqn interacting state or steps or making statistical analysis certain measure of performance & draw model conclusion.

→ Interpret the model Conclusion in term of real world problem characteristics.

→ Test & Validate result

→ Impliment the result

→ Revise the model as and when necessary.

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IMPORTANT TOPIC IN OE :-

1. Linear Programming – It involves linear objective function with set of linear Constraint.

2. Integer Programming – It is an extension of LP with only integer value for the decision variable of the problem.

3. Distance Related network technique :- This technique are deal with transportation problem, shortest path problem, minimum spanning the problem, travelling saleman problem.

4. Project management -
It is a technique to schedule the activity of a project.
eg- Construction of a bridge such that the total project completion time is minimum.

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5. Inventory Control -
It is a technique to optimize plan and procure or produce raw material or semifinished product such that total cost of the inventory system is minimized.

6. Dynamic Programming -
It is a systematic complete enumeration technique to solve a problem optimally in a emerging era by integrating the soln of its solve problem.

7. Queuing Theory -
It is a technique based on the probability to study the waiting behaviour of some real life queuing system.

8. Game theory -
It is a technique to deal with uncertainity situation related to management such as bedding of tender or playing a game.

9. Replacement theory -
It is a technique to determine the economic life of an asset as compared to the minm total cost.

10. Goal Programming -
The idea here is to convert multiple objective into a single goal i.e to reach a compromise for multipurs-objective models.

11. Non-linear programming -
It involves non-linear objective function with set of non-linear constraint.

12. Simulation -
It is a technique to deal with probability situation whose mathematical model fail to provide soln to real life problems.

SCOPE OF OE -

1. Defence application
2. Industrial
3. public system

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MODULE-I

LINEAR PROGRAMMING PROBLEM

Linear programming is a mathematical programming technique to optimise performance (eg - profit or cost) under a ~~eg - profits~~ set of resources constraint (m/c hours, man hours, material & money) are specified by an organisation.

Some application of LPP are listed below—

(i) Product mix problems
(ii) Diet planning "
(iii) Cargo loading "
(iv) Capital budgeting "
(v). Man-power planning "

Concept Of Linear Programming Model—

The model of any Linear programming problem contains objective funct^n, set of resource constraint & non-negative restriction.

Each component of the LPP depends upon the following factors.

(i) Decision Variables
(ii) Objective funct^n Co-efficient
(iii) Technological co-efficient
(iv) Availability of resource.

PRODUCT MIX PROBLEM —

These problem is determined the level of production activity to be carried out during a pre-specified time frame so as to gain maxm. profit.

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Example—

A company manufactures 2 different type of product and P_1 & P_2. Each product requires processing on milling machine & drilling m/c. But each type of m/c has limited hr available for week. The net profit per unit of products resource requirement of the product & available resource are given below.

M/c type	Processing time product (P_1)	Processing time product (P_2)	m/c hrs available process week
milling m/c	2	5	200
drilling m/c	4	2	240
profit/unit (Rs)	250	400	

Develop LP model to determine optimal production value of each of the production such that total profit is maximised subject to the availability of m/c hrs.

Soln

Let n be the no. of products to be manufactured and m be the no. of different m/c types used.

Decision Variable —

A decision Variable is used to represent the level of achievement of a particular course of action

The soln of LPP will provide the optimal value for each & every decision variable of the model.

A General format of decision variable are as follows:-

Let $x_1, x_2 \ldots x_n$ be the production volume value of $P_1, P_2 \ldots P_n$ respectively.

from our problem let x_1 & x_2 be the production volume of the product P_1 & P_2.

Objective function Coefficient -

It is a constant representing profit per unit or cost per unit of carrying out an activity.

Let $c_1, c_2 \ldots c_n$ be the profit/unit of product $P_1, P_2 \ldots P_n$ respectively.

from our problem let c_1 be the profit per unit of the product which is equal to $P_1 = ₹250$ and $P_2 = ₹400$.

Objective function :-

It is an expression representing total profit/cost carrying out of a set of activity of some level.

The objective functn will be either maximization type or minimization type.

The benefits related comes under maximization type and the cost related comes under minimization type.

General form of objective function which is of the form

$\boxed{\text{Maximize } Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n}$

from our problem

Maximize,

$Z = c_1x_1 + c_2x_2 = 250x_1 + 400x_2$

Technological Coefficient-

The technological coefficient (a_{ij}) is the amt. of resource i for the activity j where i varies 1 to n and j varies $1 \ldots m$

A General format of technological co-efficient

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

from Our problem

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 4 & 2 \end{bmatrix}.$$

Availability Of Resource (or) Resource availability :-

The constant b_i is the amt. of resource available during the planning period.

General form of b_i = $\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$

from our problems,

$$b_i = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 200 \\ 240 \end{bmatrix}$$

Set Of Constraints —

A constraints is a kind of striction on the total amt no. of particular resource required to carry out activity to various levels.

General format of constraints can be represented in following manner —

$$a_{11}x_1 + a_{12}x_2 + \dots\dots + a_{1n}x_n \le \text{ or } = \text{ or } \ge b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots\dots + a_{2n}x_n \le \text{ or } = \text{ or } \ge b_2$$
$$a_{m1}x_1 + a_{m2}x_2 + \dots\dots + a_{mn}x_n \le \text{ or } = \text{ or } \ge b_m$$

benefit related ($\le$).
Cost related ($\ge$)

from our problem

$$a_{11}x_1 + a_{12}x_2 \le b_1 .$$
$$a_{21}x_1 + a_{22}x_2 \le b_2$$

$$2x_1 + 5x_2 \le 200$$
$$4x_1 + 2x_2 \le 240 .$$

Non-negative restrictions-

Each & every decision Variable of the LPP problem are non-negative (i.e +ve)

$x_1, x_2, \dots \quad x_n \ge 0 .$

from our problem

$x_1, x_2 \ge 0 .$

Complete form Of LPP model —

General form of LPP model can be represented as follows —

maximise $Z = 250x_1 + 400x_2$.

subjected to $2x_1 + 5x_2 \le 200$
$4x_1 + 2x_2 \le 240$
$x_1, x_2 \ge 0$.

Assumption Of LPP Problem —

There are 4 assumption of solving LPP —

1. Linearity
2. Divisibility
3. Non-negativity
4. Additivity

Linearity —

The amt. of resource reqd to for a given activity level is directly proportional to the level of that activity.

Divisibility —

The fractional value of decision Variables are permitted.

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Non-negativity -

Decision variable are permitted which to have only the value which are ≥ 0.

Additivity -

This means the total o/p for a given combination of activity label is the algebraic sum of each individual process.

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Feasible Solution :-

If all the constraints of the given LP model are satisfied by the soln of the model, then that soln are called Feasible solution.

Optimal Solution -

If there are no other superior solution of the solutions obtained by the given LPP then that solution is known as optimal soln.

Unbounded soln :-

If for some LP model the objective value can increased or decreased infinitely without any limitation then that soln is known as Unbounded soln.

Infeasible solution -

If there is no combination of values of decision variable satisfying all the constraints of LP model then the soln is called Infeasible soln.

Degenerate Solution -

In LP model intersection of 2 constraints will define a corner pt of the feasible region but if more than 2 constraint pass through any one of the corner pt. of the feasible region then excess constraints will not satisfy our purpose, then the common feasible region degenerate the other constraint.

Alternate Optimum Solution -

For some LP model there may be more than one combinational values of the decision variable getting best objective value of decision variable are called Alternate optimum soln.

Example

A company manufacture 2 type of products P_1 & P_2. Each product uses lathe & milling machine. The processing time per unit of P_1 on the lathe 5 hrs and milling m/c is 4 hr. The processing time per unit of P_2 on the is 10 hr and on milling m/c 4 hrs. The max^m no. of hrs available for week on the lathe and milling m/c is 60 hrs & 40 hrs resp. and also profit per unit of saling P_1 & P_2 are ₹ 6 and ₹ 8 resp. Formulate a LP model determine the production volume for each of the such that the total profit is maximize.

M/c type	processing time		limit on resource hrs
	P_1	P_2	
Lathe	5	10	60
milling m/c	4	4	40
profit/unit	6	8	

Let x_1 and x_2 be the production volume of the product P_1 and P_2 resp.

Maximize $Z = 6x_1 + 8x_2$

subjected to $5x_1 + 10x_2 \leq 60$

$4x_1 + 4x_2 \leq 40$

$x_1 \geq 0$, $x_2 \geq 0$

A nutrition schemes for baby is prepared by a committee of doctor baby can be given 2 type of food I & II which are available with standard size packet 50gm. The cost of the packet of the food 2 & 3. The vitamin availablity for each type of food per packet & min. vitamin required for each type of vitamin are given below. Develop LP model determine the optimal combination of food type with the min cost such that min requirement of V&t in type are given.

Vitamin	Food I	Food II	min req^n vitamin
I	1	1	6
II	7	1	14
cost/packet	2	3	

Let x_1 and x_2 be the packet of vitamins of Food I and Food II resp. is suggested for baby.

Minimize $Z = 2x_1 + 3x_2$.

subjected to $x_1 + x_2 \geq 6$

$7x_1 + x_2 \geq 14$

s.t $x_1, x_2 \geq 0$.

Solutions Of LPP model –

a. Graphical soln
b. Simplex method.

Max $Z = 6x_1 + 8x_2$

$5x_1 + 10x_2 \le 60$.

$4x_1 + 4x_2 \le 40$

$x_1, x_2 \ge 0$.

$5x_1 + 10x_2 = 60$.

x_1	0	12
x_2	6	0

$4x_1 + 4x_2 = 40$.

x_1	0	10
x_2	10	0

$$\begin{array}{r} 400 \\ 240 \\ \hline 160 \end{array}$$

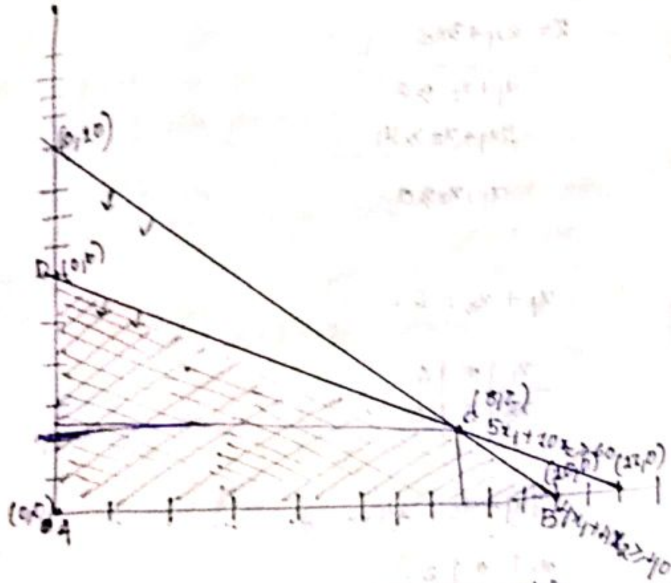
$(5x_1 + 10x_2 = 60) \times 4 \quad = \quad 20x_1 + 40x_2 = 240$

$(4x_1 + 4x_2 = 40) \times 10 \quad\quad 40x_1 + 40x_2 = 400$

$-20x_1 = -160$

$\boxed{x_1 = 8}$

$\boxed{x_2 = 2}$



ABCD – feasible region of the graph.

The objective function value at each of the corner pt of the closed polygon is computed by substituting in the co-ordinate as follows –

Z	$6x_1 + 8x_2$
(0,0)	0
(10,0)	60
(8,2)	64 → maximum value.
(0,6)	48

(ii)

$Z = 2x_1 + 3x_2$

$x_1 + x_2 \geq 6$

$7x_1 + x_2 \geq 14$.

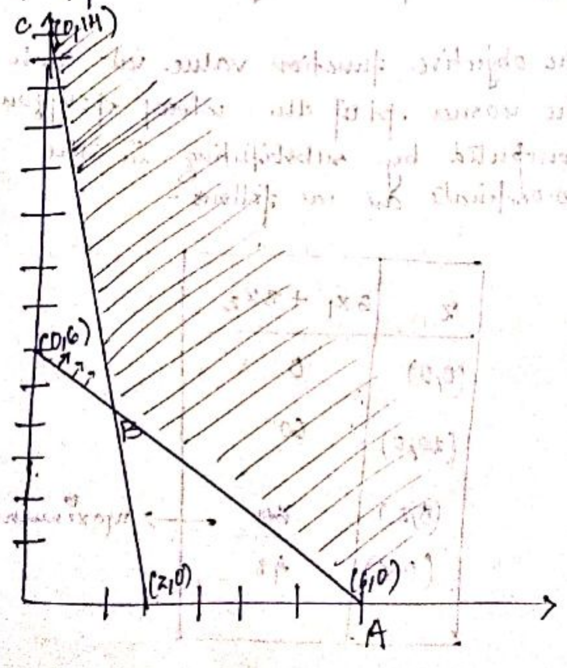
$x_1, x_2 \geq 0$.

$x_1 + x_2 = 6$.

x_1	0	6
x_2	6	0

$7x_1 + x_2 = 14$.

x_1	0	2
x_2	14	0



$x_1 + x_2 = 6$

$7x_1 + x_2 = 14$

$-6x_1 = -8$

$x_1 = 8/6 = 4/3 = 1.3$.

$x_2 = 6 - 1.3 = 4.7$.

Z	$2x_1 + 3x_2$.
A (6,0)	12 → min
B (1.3, 4.7)	16.7.
C (0,14)	42.

Special Case Of LPP -

- Infeasible soln
- ~~Unbounded soln~~
- unbounded soln finite soln
- alternate soln
- Degenerate soln

(1) Infeasible soln

Max $Z = 10x_1 + 3x_2$.

$2x_1 + 3x_2 \leq 18$

$8x_1 + 5x_2 \geq 60$.

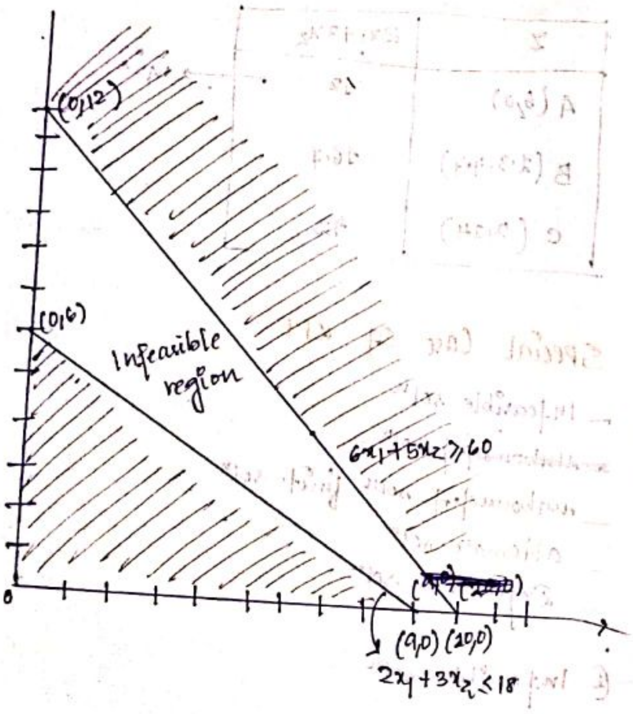
$x_1, x_2 \geq 0$.

$2x_1 + 3x_2 = 18$

x_1	0	9
x_2	6	0

$6x_1 + 5x_2 = 60$

x_1	0	10
x_2	12	0



If the set of ~~values~~ basic variable contain atleast one artificial variable then the given variable is said to have infeasible soln.

(2) Unbounded soln —

Q. $\max z = 12x_1 + 25x_2$

subject to $12x_1 + 3x_2 \geq 36$

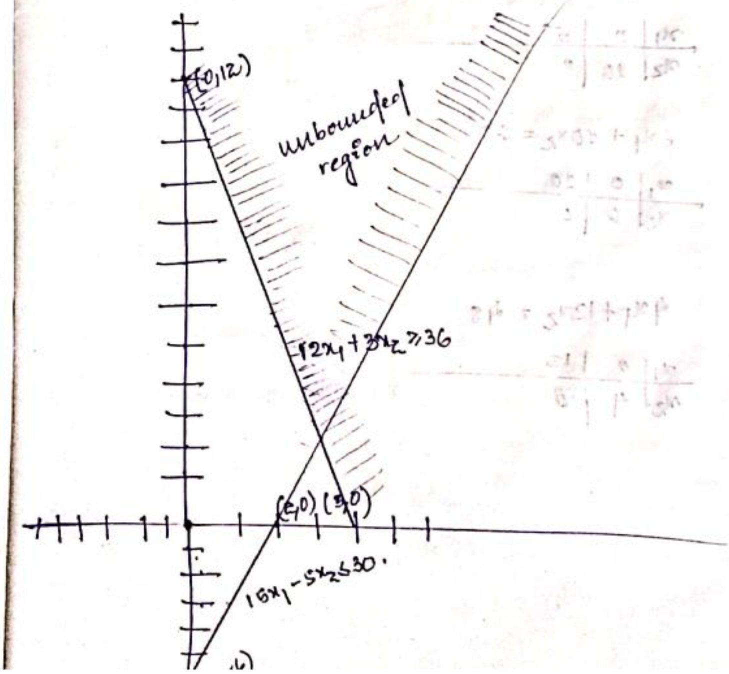
$15x_1 - 5x_2 \leq 30$.

$12x_1 + 3x_2 = 36$.

x_1	0	3
x_2	12	0

$15x_1 - 5x_2 = 30$.

x_1	0	2.
x_2	-6	0



If the constraint co-efficients of the entering variable either less than or equal to zero then the soln space is bounded and has no finite optimum soln.

Alternate soln :-

max $z = 20x_1 + 10x_2$

subject to $20x_1 + 5x_2 \leq 50$

$6x_1 + 10x_2 \leq 60$

$4x_1 + 12x_2 \leq 48$

$x_1, x_2 \geq 0$.

$10x_1 + 5x_2 = 50$

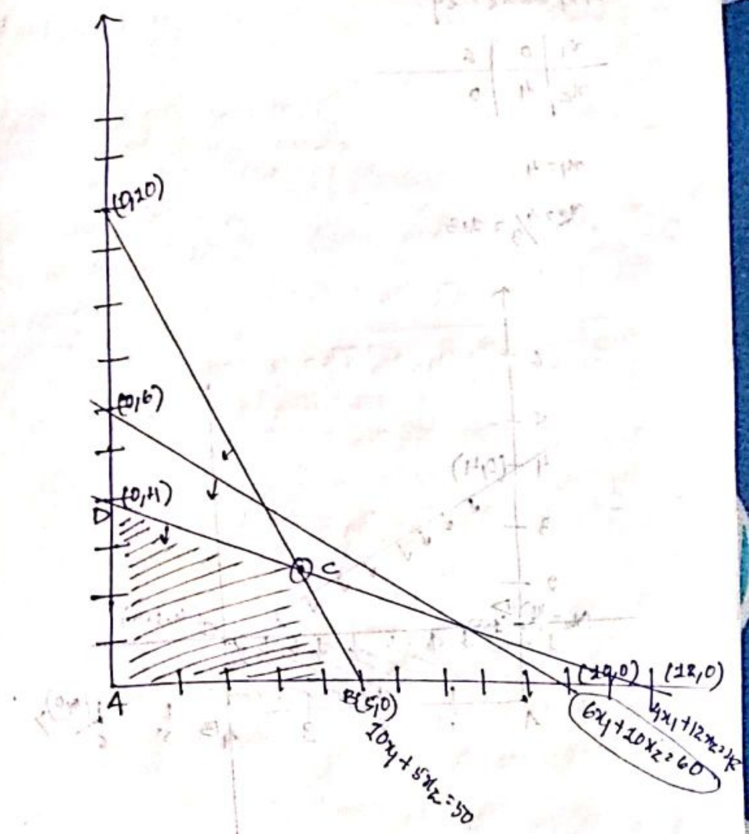
x_1	0	5
x_2	10	0

$6x_1 + 10x_2 = 60$

x_1	0	10
x_2	6	0

$4x_1 + 12x_2 = 48$

x_1	0	12
x_2	4	0



Degeneraty soln-

$z = 100x_1 + 50x_2$

Subject to $4x_1 + 6x_2 \leq 24$

$x_1 \leq 4$

$x_2 \leq 4/3$

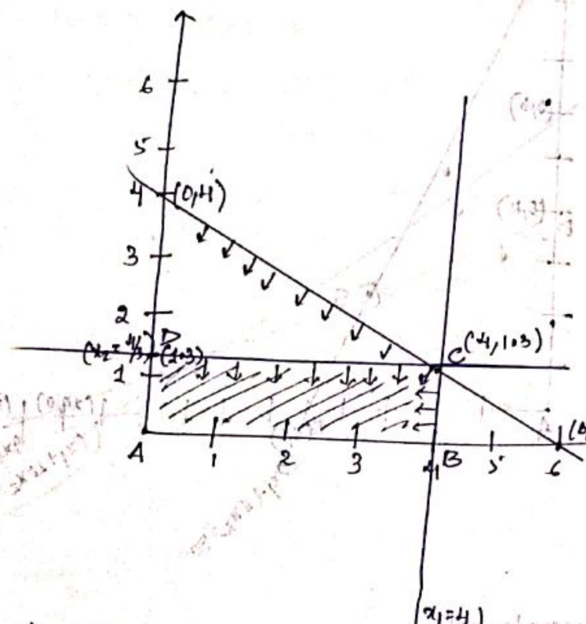
$x_1, x_2 \geq 0$.

$4x_1 + 6x_2 = 24$

x_1	0	6
x_2	4	0

$x_1 = 4$,

$x_2 = 4/3 = 1.3$.



$z = 100 \times 4 + 50 \times 1.33$

$z = \cancel{4007}$

$= 466.5$

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SIMPLEX METHOD :-

Simplex method is the basic building block of all other method. This method is based upon to solve simultaneously linear eqn.

Assuming the existance of 'n' I.B.F.S Initial Basic feasible soln & optimal soln above LP model following steps are required -

① Check whether the objective is of maximization type or of minimization type

If $z = c_1x_1 + c_2x_2 + \dots c_nx_n$ is to minimized then to convert the problem of maximization by writing.

Minimize z = Maximize$(-z)$.

(i) check whether all b_i (RHS) are (+ve). If b_i's is -ve multiplying both sides of the constants $\times(-1)$ so as to make RHS becomes +ve

(ii) Express the problem into the standard form convert all the inequality constraints into eqn by introducing slack and or surplus variable in the constraints giving the eqn of the form

$a_{11}x_1 + a_{12}x_2 + \dots \quad a_{1n}x_n + 1s_1 + 0s_2 \dots = b_1$

② Find the I.B.F.S.

The variable is said to be basic variable if it has unit co-efficient in one of the constraint and zero co-efficients remaining ~~the~~ constants

If all the constraint are $\leq$ type then the standard form of LPP is canonical form. The canonical form is used in simplex table

key element / pivotal element

C_{Bi}	C_j	c_1	c_2	c_3				solution ~~selection~~	ratio
	Basic	x_1	x_2	x_3	s_1	s_2	s_3	b	outgoing $\rightarrow$
	s_1	a_{11}	a_{12}	a_{13}	$\boxed{1}$	0	0	b_1	
	s_2	a_{21}	a_{22}	a_{23}	0	1	0	b_2	
	s_3	a_{31}	a_{32}	a_{33}	0	0	1	b_3	
	z_j								

Body matrix ~~matrix~~

↑ incoming

In these table s_1, s_2, s_3 are basic Variables x_1, x_2 & x_3 are non-basic Variables.

Basis refers to the basic Variable

C_j denotes the co-efficient in the objective functn.

C_{Bi} denotes co-efficient of the basic varible only in the objective variable.

$$z_j = \sum_{i=1}^{m} C_{Bi}(a_{ij})$$

$$\text{New value} = \text{old value} - \frac{\text{Key row value} \times \text{key column value}}{\text{Key ~~value~~ element}}$$

Optimality -

For maximization problem If all $c_j - z_j \leq 0$ then optimality is least otherwise select the variable $c_j - z_j$ with maximum as entering ~~vale~~ variable or incoming Vector.

For minimization problem If all $c_j - z_j \geq 0$ then optimality is least otherwise select the variable with most -ve value as entering variable or incoming factors.

Feasibility Condition

To maintain the feasibility condition is each iteration following steps need to require.

(i) In each row find the ratio betn the soln column value & the value in the incoming factors or key columns.

(ii) Then select the variable with present ~~value~~ of basic variable w.r.t minimum ratio as the leaving variable. or outgoing vector.

Intersection of key row & key ~~element~~ column is key element or pival element -

Q. Maximize $Z = 6x_1 + 8x_2$

$5x_1 + 10x_2 \le 60$

$4x_1 + 4x_2 \le 40$

$x_1, x_2 \ge 0$.

Sol^n

The standard form of LPP problem can be represented by.

Max $Z = 6x_1 + 8x_2$.

$5x_1 + 10x_2 + 1s_1 + 0s_2 = 60$

$4x_1 + 4x_2 + 0s_1 + 1s_2 = 40$

$x_1, x_2, s_1, s_2 \ge 0$.

s_1, s_2 are slack variable for balancing the constraints. Tuesday 31/7/18

1st iteration – (key element / pivot element)

CBi	c_j	6 x_1	8 x_2	0 s_1	0 s_2	sol^n	Ratio
0	s_1	5	$\boxed{10}$	1	0	60	$60/10 = 6$ (Outgoing vector)
0	s_2	4	4	0	1	40	$40/4 = 10$ (key row)
	z_j	0	0	0	0		
	$c_j - z_j$	6	8 ↑	0	0		

from the above table we conclude that $c_j - z_j$ are not ≤ 0. So optimality is not reached.

So, Max^m of $c_j - z_j = 8$ (incoming vector / key column)

2nd iteration –

CBi	c_j Basis	6 x_1	8 x_2	0 s_1	0 s_2	sol^n	ratio
8	x_2	5	$\boxed{10}$	1	0	60	$60/10 = 6$
0	s_2	4	4	0	1	40	$40/4 = 10$

Divide key element by 1/10

CBi	c_j	6 x_1	8 x_2	0 s_1	0 s_2	sol^n	ratio
8	x_2	$1/2$	1	$1/10$	0	6	$6/\frac{1}{2} = 12$
0	s_2	$\boxed{2}$	0	$-2/5$	1	16	$16/2 = 8$ →
	z_j	4	8	$4/5$	0		
	$c_j - z_j$	2	0	$-4/5$	0		

$4 - 1 \times 4 = 0$

$4 - 4 \times 1/2 = 2$

$0 - 4 \times 1/10 = -2/5$

$1 - 0 \times 4 = 1$

$40 - 4 \times 6 = 16$

from the above table we conclude that all $c_j - z_j$ are not ≤ 0. So optimality is not reached.

So Max^m of $c_j - z_j = 2$ (incoming vector)

2 is the pivot element (or) key element.

3rd iteration –

CBi	c_j Basis	6 x_1	8 x_2	0 s_1	0 s_2	sol^n	ratio
8	x_2	0	1	$1/5$	$-1/4$	2	
6	x_1	1	0	$-1/5$	$1/2$	8	

The standard form LPP can be represented by:— $\text{Max} z' = -x_1 + 3x_2 - 3x_3 + 0s_1 + 0s_2 + 0s_3$

Subject to $3x_1 - x_2 + 2x_3 + s_1 + 0s_2 + 0s_3 = 4$.

$$-2x_1 - 4x_2 + 0x_3 + 0s_1 + 1s_2 + 0s_3 = 12$$

$$-4x_1 + 3x_2 + 8x_3 + 0s_1 + 0s_2 + 1s_3 = 10$$

$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0 .$$

Solⁿ ①

The standard form of LPP problem can be represented by

$$\text{Max} z = 10x_1 + 15x_2 + 20x_3 .$$

Subject to $2x_1 + 4x_2 + 6x_3 + s_1 = 24$

$$3x_1 + 9x_2 + 6x_3 + s_2 = 30 .$$

$$x_1, x_2, x_3, s_1, s_2 \geq 0 .$$

s_1, s_2 are slack variable for balancing the constraints

1st iteration –

CB_i	c_j basis	10 x_1	15 x_2	20 x_3	0 s_1	0 s_2	solⁿ	ratio
0	s_1	2	4	6	1	0	24	$24/6 = 4$
0	s_2	3	9	6	0	1	30	$30/6 = 5$
	z_j	0	0	0	0	0		
	$c_j - z_j$	10	15	20 ↑	0	0		

from the above table we conclude that all $c_j - z_j$ are not ≤ 0. so optimality is not reached.

So max^m of $c_j - z_j = 20$ (incoming vector)

2nd iteration –

CB_i	c_j basis	10 x_1	15 x_2	20 x_3	0 s_1	0 s_2	solⁿ	ratio
20	x_3	$1/3$	$4/6$ $2/3$	1	$1/6$	0	4	$4/\frac{1}{3} = 12$
0	s_2	K.E $\boxed{1}$ $(3 - \frac{1}{3} \times 6)$	5 $(9 - \frac{4}{6} \times 6)$	0 $(6 - 1 \times 6)$	-1 $(0 - \frac{1 \times 6}{6})$	1 $(1 - \frac{0}{6})$	26 $(30 - \frac{4}{6})$	$6/1 = 6$ → O.V
	z_j	$20/3$	$\cancel{80/6}$ $40/3$	20	$20/6$	0	$\cancel{80}$	
	$c_j - z_j$	$10/3$ ↑	$5/3$	0.	$-\cancel{20/6}$ $-10/3$	0		

from the above table, we conclude that all $c_j - z_j$ are not ≤ 0. so optimality is not reached.

So max^m of $c_j - z_j = 10/3$ (incoming vector).

3rd iteration –

CB_i	c_j basis	10 x_1	15 x_2	20 x_3	0 s_1	0 s_2	soln	ratio
	x_3	0	-1	1	$1/2$	$-1/3$	2	
10	x_1	1	5	0	-1	1	6	
	z_j	10	30	20	0	$10/3$		
	$c_j - z_j$	0	-15	0	0	$-10/3$		

$1/3 - 1/3$

$2/3 - 5/3$

$0 - 1/3 \times 1$

$1/6 + 1/3$

$4 - 6 \times 1/3$

from the above table, we conclude that all $c_j - z_j$ is ≤ 0. so optimality is reached.

$$\boxed{\begin{array}{l} x_1 = \cancel{10}6 \\ x_2 = 0. \\ x_3 = \cancel{0}2 \end{array}}$$

~~Max z = 20(10) + 15(0) + 20(5)~~

~~Big M~~ ~~= 100 + 0 + 400~~ 60 +

= 500

Max z = $10(6) + 15(0) + 20(2)$

= $60 + 0 + 40$

= 100.

Big M (Artifical Variable Method):-

If some of the constraints are $\geq$ type then they will not contain any basic variable. Just to have basic variable in each of them a new variable called artifical variable will be introduced in each of some contraints a with a +ve. unit coefficient.

If the objective function is of maximization type then the co-efficient of artifical variable in the objective function should be $-M$. Otherwise it should be $+M$ when M is so large.

If the constraint is $\geq$ type then one can use surplus variable. (subtract something on LHS, inequation becomes eqn)

Q. Solve by simplex method.

$$\text{Min } z = 2x_1 + 3x_2$$
$$x_1 + x_2 \geq 6$$
$$7x_1 + x_2 \geq 14$$
$$x_1, x_2 \geq 0$$

Standard form:-

$$\text{Min } z = 2x_1 + 3x_2$$
$$x_1 + x_2 - s_1 + 0s_2 = 6$$
$$7x_1 + x_2 + 0s_1 + 1s_2 = 14$$
$$x_1, x_2, s_1, s_2 \geq 0$$

The above exp. can be converted into canonical form-

$$\text{Min } z = 2x_1 + 3x_2 + 0 \cdot s_1 + 0 \cdot s_2 + MR_1 + MR_2.$$
$$x_1 + x_2 - s_1 + 0s_2 + R_1 + 0R_2 = 6.$$
$$7x_1 + x_2 + 0s_1 + 1s_2 + 0R_1 + 1R_2 = 14$$
$$x_1, x_2, s_1, s_2, R_1 \text{ \& } R_2 \geq 0.$$

K.E

C_{Bi}	C_j	2	3	0	0	M	M		
	basis	x_1	x_2	s_1	s_2	R_1	R_2	soln	ratio
M	R_1	1	1	-1	0	1	0	6	$6/1 = 6$
M	R_2	$\boxed{7}$	1	0	-1	0	1	14	$14/7 = $
	z_j	8M	2M	-M	-M	M	M		
	$c_j - z_j$	2-8M	3-2M	M	M	0	0		

from the above table we conclude that for minimization problem all $c_j - z_j$ are not ≥ 0, so optimality is not reached. We choose most (-ve) as a incoming vector.

C_{Bi}	c_j basis	2 x_1	3 x_2	0 s_1	0 s_2	M R_1	M R_2	sol^n	ratio
M	R_1	0	k.e $\boxed{6/7}$	-1	1/7	1	-1/7	4	$\frac{4}{6/7} = 28/6$
2	x_1	1	1/7	0	-1/7	0	1/7	2	$\frac{2}{1/7} = 14$

z_j $\quad 2,\ \frac{6M}{7}+\frac{2}{7},\ -M,\ \frac{M}{7}-\frac{2}{7},\ M,\ -\frac{M}{7}+\frac{2}{7}$

$c_j - z_j$ $\quad 0,\ -\frac{6M}{7}+\frac{19}{7},\ M,\ -\frac{M}{7}+\frac{2}{7},\ 0,\ \frac{8M}{7}-\frac{2}{7}.$

So, the optimally is not reached. We choose most (-ve) as the incoming vector.

C_{Bi}	c_j basis	2 x_1	5 x_2	0 s_1	0 s_2	M R_1	M R_2	sol^n	ratio
3	x_2	0	1	-7/6	k.e $\boxed{1/6}$	7/6	-1/6	14/3	28
2	x_1	1 ~~0~~	~~1/3~~ 0	~~0~~ 1/6	-1/6	~~0~~ 1/6	1/6	4/3	-8

$0+\frac{7}{6}\frac{1}{3}\ \frac{1}{7}-$ $\qquad 2-\frac{14}{3}\frac{1}{7}$

z_j $\quad 2 \quad 3 \quad -19/6 \quad -1/6 \quad 19/6 \quad -1/6$

$c_j - z_j$ $\quad 0 \quad 0 \quad -19/6 \quad -1/6 \quad M-19/6 \quad M+1/16$

from the above table, we conclude that all $c_j - z_j$ are not ≥ 0. So optimality is not reached. So we chose most (-ve) as incoming vector.

C_{Bi}	c_j basis	2 x_1	3 x_2	0 s_1	0 s_2	M R_1	M R_2	sol^n	ratio
0	s_1	0	6	-7	1	7	-1	28	
2	x_1	1	1	-1	0	1	0	6	

z_j $\quad 2 \quad 2 \quad -2 \quad 0 \quad 2 \quad 0$

$c_j - z_j$ $\quad 0 \quad 1 \quad 2 \quad 0 \quad M-2 \quad M.$

from the above table, we conclude that all $c_j - z_j \geq 0$. So optimality is reached.

$x_1 = 6$, $s_2 = 28$, x_2.

$\boxed{\text{Min } z = 12}$

$$\text{Min } z = 10x_1 + 15x_2 + 20x_3.$$

$$2x_1 + 4x_2 + 6x_3 \geq 24$$

$$3x_1 + 9x_2 + 6x_3 \geq 30$$

$$x_1, x_2 \ \& \ x_3 \geq 0.$$

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DUALITY THEORY :-

The importance of duality-theory occurs due to two main reasons -

(1) If primal contain large no. of constraints and a small no. of variable, which can reduced by converting it into dual problem.

(2) The interpretation of dual variables from the cost or economic point of view proves extremely in making future decision in the activities being performed.

Formulation of dual Problem :-

~~Q. Max z =~~

Maximize $= c_1x_1 + c_2x_2 + c_3x_3 + \dots c_nx_n$

subject to $a_{11}x_1 + a_{12}x_2 + \dots a_{1n}x_n \le b_1$

$a_{21}x_1 + a_{22}x_2 + \dots a_{2n}x_n \le b_2$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + \dots a_{mn}x_n \le b_m$

s.t $x_1, x_2 \dots x_n \ge 0$. — (1)

To convert the following problem, we have to adopt the rules.

(i) The maximization problem is the primal becomes minimization problem in the dual and vice-versa.

(ii) $\le$ type in the maximization problem becomes $\ge$ type in the minimization problem and vice-versa.

(iii) The co-efficient $c_1, c_2 \dots c_n$ in the objective function of the primal becomes $b_1, b_2 \dots b_m$ in the objective function of the dual.

(iv) The co-efficient $b_1, b_2 \dots b_m$ ~~of the~~ in the constraint of the primal becomes $c_1, c_2 \dots c_n$ in the constraint of dual.

max $z = d_1x + d_2y$

$a_1x + b_1y \le c_1$

$p_1x + q_1y \le c_2$

$x, y \ge 0$.

min $z = c_1z_1 + c_2z_2$

$a_1z_1 + p_1z_2 \ge d_1$

$b_1z_1 + q_1z_2 \ge d_2$

$z_1, z_2 \ge 0$.

(v) If the primal has n variables & m constraints, dual will have m variables & n constraints i.e the transpose of the body matrix gives the body matrix of dual.

(vi) The variables of both primal are duals are non-negatives

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Let y_i be the corresponding dual of the primal (from eqⁿ ①).

Minimize $z = b_1y_1 + b_2y_2 + \dots b_ny_n$.

Subject to $a_{11}y_1 + a_{21}y_2 + \dots a_{m1}y_n \geq c_1$

$a_{12}y_1 + a_{22}y_n + \dots a_{m2}y_n \geq c_2$

$\vdots$

$a_{1n}y_1 + a_{2n}y_2 + \dots a_{mn}y_n \geq c_n$

s.t $y_1, y_2 \dots y_n \geq 0$.

Type	objective	Constraint	Nature
P	max	$\leq$	Restricted sign.
D	min	$\geq$	"
P	min	$\geq$	"
D	max	$\leq$	"
P	max	$=$	"
D	min	$\geq$	unrestricted
P	min	$=$	restricted
D	max	$\leq$	Unrestricted
P	max	$\leq$	unrestricted
D	min	$=$	restriction
P	max	$\geq$	unrestricted
D	min	$=$	restricted

Q. Max $z = 4x_1 + 10x_2 + 25x_3$

$2x_1 + 4x_2 + 8x_3 \leq 25$

$4x_1 + 9x_2 + 8x_3 \leq 30$

$6x_1 + 8x_2 + 2x_3 \leq 40$.

$x_1, x_2 \,\&\, x_3 \geq 0$.

Let y_i be the corresponding dual of the given primal.

Min $z = 25y_1 + 30y_2 + 40y_3$.

$2y_1 + 4y_2 + 6y_3 \geq 4$

$4y_1 + 9y_2 + 8y_3 \geq 10$

$8y_1 + 8y_2 + 2y_3 \geq 25$

$y_1, y_2, y_3 \geq 0$.

Q. Min $z = 20x_1 + 40x_2$

$2x_1 + 20x_2 \geq 40$

$20x_1 + 3x_2 \geq 20$

$4x_1 + 15x_2 \geq 30$

$x_1 \,\&\, x_2 \geq 0$

Let y_i be the corresponding dual of the given primal.

Max $z = 40z_1 + 20z_2 + 30z_3$.

$2y_1 + 20y_2 + 4y_3 \leq 20$

$20y_1 + 3y_2 + 15y_3 \leq 40$

$y_1, y_2, y_3 \geq 0$;

Q. Max $z = 4x_1 + 10x_2 + 25x_3$

$$2x_1 + 4x_2 + 8x_3 = 25$$
$$4x_1 + 9x_2 + 8x_3 = 30$$
$$6x_1 + 8x_2 + 2x_3 = 40$$
$$x_1, x_2, x_3 \geq 0.$$

Min $z = 25x_1 + 30x_2 + 40x_3$.

$$2y_1 + 4y_2 + 6y_3 \geq 0$$
$$4y_1 + 9y_2 + 8y_3 \geq 0$$
$$8y_1 + 8y_2 + 2y_3 \geq 0.$$
$$y_1, y_2, y_3 \geq 0.$$

Q. Max $z = 20x_1 + 40x_2$.

$$2x_1 + 20x_2 = 40.$$
$$20x_1 + 3x_2 = 20$$
$$4x_1 + 15x_2 = 30$$
$$x_1, x_2 \geq 0.$$

$$40x_1 + 20x_2 + 30x_3.$$

$$2y_1 + 20y_2 + 4y_3 \leq 20$$
$$20y_1 + 3y_2 + 15y_3 \leq 40$$

Min $z = 5x_1 + 8x_2$.

$$4x_1 + 9x_2 \geq 100$$
$$2x_1 + x_2 \leq 20 \quad = -2x_1 - x_2 \geq -20.$$
$$2x_1 + 5x_2 \geq 40.$$

Max $z = 100x_1 + 20x_2 + 20x_3$.

$$4y_1 \pm 2y_2 + 2y_3 \leq 05$$
$$9y_1 - 2y_2 + 5y_3 \leq -20.$$

Min $z = 6x_1 + 6x_2$.

$$9x_1 + 3x_2 \geq 20$$
$$2x_1 + 7x_2 = 40. \quad \left\{\begin{array}{l} 2x_1 + 7x_2 \geq 40 \\ 2x_1 + 7x_2 \leq 40 \end{array}\right\}.$$
$$x_1, x_2 \geq 0.$$

Max $z = 20x_1 + 40x_2$.

$$9y_1 + 2y_2 \leq 6.$$
$$3y_1 + 7y_2 \leq 6.$$

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DUAL SIMPLEX METHOD.

Dual simplex method, it is specialised form of simplex method in which optimality is maintained in all iterations.

Initially the solution may not be feasible, successive iteration will remove the infeasibility.

B.

→ If the problem is feasible in an iteration the procedure will be stopped. Because the solution is obtained is feasible & optimal at that stage.

This method is essential for integral programming method in which it is repeatedly used to remove the infeasibility due to additional constraints known as Gomory's cut.

Solve by Dual Simplex method.

Min $Z = 2x_1 + 4x_2$.

subject to $2x_1 + x_2 \geq 4$.

$x_1 + 2x_2 \geq 3$.

$2x_1 + 2x_2 \leq 12$. ≥ 12.

$x_1, x_2 \geq 0$.

Standard form of given LPP -

Min $Z = 2x_1 + 4x_2$

$-2x_1 - x_2 \leq -4$.

$-x_1 - 2x_2 \leq -3$.

$2x_1 + 2x_2 \leq 12$

$x_1, x_2 \geq 0$.

Min $Z = 2x_1 + 4x_2 + 0s_1 + 0s_2 + 0s_3$

$-2x_1 - x_2 + s_1 = -4$.

$-x_1 - 2x_2 + s_2 = -3$.

$2x_1 + 2x_2 + s_3 = 12$.

x_1, x_2, s_1, s_2 & $s_3 = 0$

key element

C_{Bi}	c_j / basis	2 / x_1	4 / x_2	0 / s_1	0 / s_2	0 / s_3	soln	ratio
0	s_1	$\boxed{-2}$	-1	1	0	0	-4	(Outgoing) leaving variable.
0	s_2	-1	-2	0	1	0	-3	
0	s_3	2	2.	0	0	1	12.	
	z_j	0	0	0	0	0		
	$c_j - z_j$	2	4	0	0	0		

incoming.

In above table we conclude that all $c_j - z_j \geq 0$ then optimality is reached but some of the values under the soln columns are (-ve). So these (-ve) value refer infeasibility.

The leaving variable which has more -ve.

Determination Of Entering Variable :-

⟹

Variable	x_1	x_2	s_1	s_2	s_3
$-(c_j - z_j)$	-2	-4	0	0	0
s_1	-2	-1	1	0	0
Ratio	1 ↑	4	0	–	–

entering variable.

for minimization problem, the entering variable is 1 which has the smallest ratio (1).

C_{Bi}	c_j basis	2	4	0	0	0	sol^n	ratio
2	x_1	1	1/2	-1/2	0	0	2	
0	s_2	0	-3/2	-1/2	1	0	-1. →	l_i
0	s_3	0	1	1	0	1	8	
z_j		2	1	-1	0	0		
$c_j - z_j$		0	3	1 ↑	0	0		

from the above table we conclude that all $c_j - z_j \geq 0$. so optimality is reached but in solution column some ~~colum~~ values are (-ve).

Variables	x_1	x_2	s_1	s_2	s_3
$-(c_j - z_j)$	0	-3	-1	0	0
s_2	0	-3/2	-1/2	1	0
ratio	–	2	2 ↑		

take any one of them

for minimization Problem, the entering vairble is 2 (s_1).

C_{Bi}	c_j basis	2 x_1	4 x_2	0 s_1	0 s_2	0 s_3	sol^n	ratio.
2	x_1	1	2.	0	-1	0	3.	
0	s_1	0	-3	1	-2	0	2	
0	s_3	0	-2	0	2	1	6.	
	z_j	2	4	0	-2	0		
	$c_j - z_j$	0	0	0	2	0		

$-\frac{1}{2} + 1 \cdot \frac{1}{2} = 0.$

$\frac{1}{2} + 3 \cdot \frac{1}{2} = 2.$

$2 + 1 \cdot \frac{1}{2} = 3$

$1 - 1 = 0$

from the above table that all $c_j - z_j \geq 0$ so optimality is reached. all values in sol^n column are (+ve).

$x_1 = 3$ $\quad s_1 = 2$

$x_2 = 0$ $\quad s_2 = 0$

$s_3 = 6.$

$Min\ z = 2(3) + 4(0)$

$\boxed{Min\ z = 6}$

Q. $Min\ z = 24x_1 + 30x_2$

subject to $2x_1 + 3x_2 \geq 10$

$4x_1 + 9x_2 \geq 15$

$6x_1 + 6x_2 \leq 20$

$x_1, x_2 \geq 0$.

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VOGEL'S APPROXIMATION METHOD :-

Transportation Method :-

defn:- special kind of LP problem in which goods are transported from a set of source to a set of destination subject to supply & demand of source & destination such that the cost of transportation is minimised.

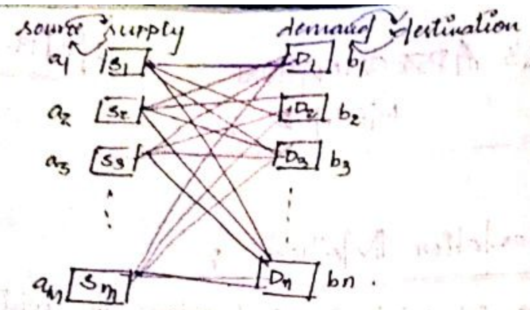
Source	Destination	Commodity	obj.
1. plants	market	finished goods.	minimising the total cost of shipping
2. supplier	plants	raw material	"

→ Let 'm' be the no. of sources and 'n' be the no. of destination.

→ Let 'ai' be the supplier at the source i, 'bj' be the demand at the destination j

c_{ij} be the unit of the transportation from a unit source i to a unit destination j.

x_{ij} be the no. of unit to be transported from the given source i to the given destination j



A mathematical problem for transportation problem is presented below.

$$\text{minimize } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} X_{ij}$$

Destination

$$\begin{array}{r|cccc|l} & & & & & \text{supply} \\ & C_{11} & C_{12} & \cdots & C_{1n} & a_1 \\ & C_{21} & C_{22} & \cdots & C_{2n} & a_2 \\ \text{source} & \vdots & & & & \vdots \\ & C_{m1} & C_{m2} & \cdots & C_{mn} & a_m \\ \hline \text{Demand} & b_1 & b_2 & \cdots & b_n & \end{array}$$

subject to constraints

$$\sum_{j=1}^{n} x_{ij} \leqslant a_i$$

$$\sum_{i=1}^{m} x_{ij} \geqslant b_j$$

$$x_{ij} \geqslant 0 .$$

The objective function minimize the total cost of transportation betn various source & destination

The 1st constraint 'i' in the set of constraint source i, the total unit to be transported $\leqslant$ or = to i/s supply.

The 2nd set of constraints ensures the total unit transported to the destination j is $\geqslant$ or = to j/s demand.

These are 2 types of transportation problems.

1. Balanced T P
2. Unbalanced "

Balanced Transportation Problem:

If sum of the supply of all sources = sum of the demands of all destination, then the problem is called balanced transportation problem.

$$\boxed{\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j}$$

Unbalanced Transportation problem:-

If sum of the supply of all sources ≠ sum of the demands of all destination.

$$\boxed{\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j}$$

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Example

Destination

Source				S
	5	10	15	20
	20	25	30	25
	17	18	22	30
D	15	10	20	

$$\sum_{i=1}^{m} a_i = 20+25+30 = 75$$

$$\sum_{j=1}^{n} b_j = 15+10+20 = 45.$$

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j .$$

Destination

Source					S
	5	10	15	0	20
	20	25	30	0	25
	17	18	22	0	30.
D	15	10	20	$\boxed{30}$	

Destination

Source				S
	5	10	15	20
	20	25	30	25
	17	18	22	30
D	15	40	30	

$\Sigma a_i = 75$ $\quad \Sigma b_j = 85$

$$\Sigma a_i \neq \Sigma b_j$$

Destination

Source				S
	5	10	15	20
	20	25	30	25
	17	18	22	30
	0	0	0	10
D	15	40	30	

There are 3 basic method to solve the transportation problem -

(a) North-west corner Method
(b) Least cost method / matrix minima method
(c) ~~Vogy~~ Vogel's approximation Method (VAM)

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smark ALGORITHM FOR NORTH WEST CORNER METHOD -

(1) find the minm of supply & demand value w.r.t the current N.W corner cell of the cost of the matrix.

(2) Allocate the minm value to the current N.W corner cell & subtract the minm from

N·W corner side.

③ check whether exactly one of the row or column w.r.t the N-W corner cell has zero supply or demand respectively. If yes go to step 4 otherwise go to step 5

④ Delete that row or column w.r.t the current N-W corner cell which has the zero supply or demand & go to step 6.

⑤ Delete both the row & column w.r.t the current N·W corner cell.

⑥ check whether exactly one row or column is left out. If yes go to step 7 otherwise go to step ②

⑦ Match supply & demand of that row or column with remaining demands & supply undeleted column or row.

Solve the TP by using North-west corner cell method

Q) destination

source	1	2	3	4	supply
1	3	1	7	4	300
2	2	6	5	9	400
3	8	3	3	2	500
demand	250	350	400	200	

solⁿ

$\sum a_i = 300+400+500 = 1200$

$\sum b_j = 250+350+400+200 = 1200$

$\sum a_i = \sum b_j$

So, the given TP is balanced type

[250] 3	1	7	4	~~300~~ 50
2	6	5	9	400
8	3	3	2	500
~~250~~ 0	350	400	200	

$\sum a_i = \sum b_j$

min (250,300) = 250, we can allocate the min 250 in cell(11) ~~Destination~~ then subtract the min value of supply & demand then delete.

destination

Source			supply
[50] 1	7	4	~~50~~ 0
6	5	9	400
3	3	2	500
demand ~~350~~ 300	400	200	

$\sum a_i = \sum b_j$

min (50,350) = 50 so 50 is ~~allocated~~ allocated.

[300] 6	5	9	~~400~~ 100
3	3	2	500
~~300~~ 0	400	200	

$\sum a_i = \sum b_j$

min (300,400) = 300

100 5	7	~~100~~ 0
3	2	500
~~400~~ 300	200	

min (400,100) = 100

300 3	200 2	500
300	200	

min (300,500) = 300

min (200,500) = 200.

So, the final table —

destination

					supply
source	250 3	50 1	7	9	300
	2	300 6	100 5	9	400
	8	~~300~~ 3	300 3	200 2	500
demand	250	350	400	200	

The total cost is evaluated by adding the product of transport cost per unit for each and every cell & the corresponding no. of units.

$3\times250 + 1\times50 + 6\times300 + 5\times100 + 3\times300 + 200\times2$

$= 750 + 50 + 1800 + 500 + 900 + 400$

= ₹4400/-

2600
500
900
400
4400

LEAST COST METHOD/ MATRIX MINIMA METHOD :-

1. Find the minm of the (undeleted values) in the cost matrix (i.e find the matrix minimum).
2. Find the minm of supply & demand value i.e (X) w.r.t the corresponding to the cell of matrix
3. Allocate x unit to the cell to the matrix minimum & also subtract x unit from supply & demand value to the cell with matrix minm.
4. Check whether exactly one of the row or column corresponding to the row with matrix minm has zero supply or demand. If yes go to st-5 otherwise go to st-6.
5. Delete that row or column w.r.t the cell with matrix minm which has zero supply or demand &. go to st-7.
6. Delete both row & column w.r.t to the cell with matrix minimum.
7. Check whether exactly one row or column is left out. If yes go to step-8 otherwise goto

st-2

8. Match the supply & demand value of that row & column with the remaining supply & demand of the undeleted column or row

Q. Solve the problem by least cost method

destination

					S
source	3	1	7	4	300
	2	6	5	9	400
	8	3	3	2	500
D	250	350	400	200	

$\Sigma a_i = \Sigma b_j$

so the transportation problem is balanced.

from the above table matrix min^m will in cell (1,2)

				S
3	$\boxed{300}$ 1	7	4	~~300~~
2	6	5	9	400
8	3	3	2	500
D 250	~~350~~ 50	400	200	

min^m cost

$\boxed{250}$ 2	6	5	9	~~400~~ 150
8	3	3	2	500
~~250~~ 0	50	400	200	

$\Sigma a_i = \Sigma b_j$

6	5	9	150
3	3	$\boxed{200}$ 2	~~500~~ 300
50	400	~~200~~ 0	

min (200,500) = 200

6	5	150
$\boxed{50}$ 3	3	~~300~~ 250
~~50~~ 0	400	

2 consecutive '3' are there.

$\boxed{150}$ 5	150
$\boxed{250}$ 3	250
400	

So, the final table —

3	$\boxed{300}$ 1	7	4
$\boxed{250}$ 2	6	$\boxed{150}$ 5	9
8	$\boxed{50}$ 3	$\boxed{250}$ 3	$\boxed{200}$ 2

total cost –

$= 250\times2 + 1\times300 + 150\times5 + 50\times3 + 250\times3 + 200\times2$

$= 500 + 300 + 750 + 150 + 750 + 400$

$=$ ₹ 2850/-

Solve the TP (~~unbalanced~~).

5	12	6	10	300
7	8	10	3	400
9	4	9	2	300
200	300	400	200	

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VOGEL'S APPROXIMATION METHOD (Penalty):-

In this method

(i) find row penality i.e difference b/w first minm & 2nd minm in each row.

(ii) If the two minm value are equal, row penality = 0.

(iii) find column penality i.e difference b/w 1st minm & 2nd minm in each column.

(iv) If the two minm value are equal

(v) find the maxm among the row penality & column penality & identify whether it occurs in a row or in a column choose randomly. If the maxm penality in a row go to st-(4) or ~~If maxm~~ step-(7)

(iv) Identify the cell for allocation which has least cost in that row.

(v) find the minm of supply & demand values w.r.t the selected cell.

(vi) Allocate the minm value to the cell & subtract the minm of supply & demand values w.r.t the selected cell & go to step (10)

(vii) Identify the cell for allocation which has least cost in that column.

(viii) find the minm of supply & demand value w.r.t to the selected cell.

Allocate the min^m value to the selected shell & substract this min^m from supply & demand

Check whether exactly one of the row or column corresponding to

If yes go to step-11

otherwise go to step-12

Delete row or column which has zero supply or demand & revise the corresponding row or column penality then to step-13.

Delete both the row & column to the selected shell then revise the row & column &

Check whether exactly one row or column is left out. If yes go to step-14 otherwise go to st-5.

Match the supply & demand of the left out-row or column with the remaining demand & supply of the undeleted column & rows.

Q.11 Destination

					S
Source	3	1	7	4	300
	2	6	5	9	400
	8	3	3	2	500
D	250	350	400	200	

$\Sigma a_i = \Sigma b_j$

The problem is balanced.

Destination

					supply	penality
	3	1	7	4	300	2
Source	[250] 2	6	5	9	~~400~~ 150	3* highest penality (least element)
	8	3	3	2	500	1
Demand	~~250~~ 0	350	400	200		
	1	2	2	2		

					penality
	[300] 1	7	4	~~300~~ 0	3* highest
	6	5	9	150	1
	3	3	2	500	1
	~~350~~ 50	400	200		
penality	2	2	2		

				S	penality
	6	5	9	150	1
	3	3	[200] 2	~~500~~ 300	1
D	50	400	~~200~~ 0		
penality	3	2	7*		

			S	penality
	6	5	150	1
	[50] 3	3	~~300~~ 250	0
D	~~50~~	400		
penality	0	3*	2	

[150] 5	150
[250] 3	250
400	

So the final table is

3	[300] 1	7	4
[250] 2	6	[150] 5	9
8	[50] 3	[250] 3	[200] 2

total cost =

300×1 + 250×2 + 50×3 + 150×5 + 250×3 + 200×2

= 300 + 500 + 150 + 750 + 750 + 400

= ₹ 2850/-

300
500
150
750
750
400
2850

STEPPING STONE METHOD :-

for finding an optimal solⁿ from the IBFS (Initial basic feasible solution) consisting of (m+n−1) occupied cell. In the independent position : cost effectiveness of goods to be transported on found out & then tested for each unoccupied cell.

for this select an unoccupied cell where allocation can be made from a closed path.

Here the movement can be done with only horizontal & vertical cell. As the cell at the turning point are considered to be on the closed path we may decide upon occupied & unoccupied cell. The cells which are placed on the turning point are called stepping stone on the path.

Now one can assign +ve & −ve to each of the corner cell of the closed path & then evaluated the net change in cost along the closed path by summing together the unit path of the cell of +ve sign and then subtracting the same cell of −ve sign. Continue these process as long as possible net change in cost is evaluated for each unoccupied cell.

If all net changes computed are ≥ zero then optima is reached but if not we leave a further chance of improving the solution & optimum solⁿ has not been reached. We can improved upon by selecting unoccupied cell which most −ve.

The maxᵐ no. of units that can be allocated to the cell with −ve sign on the closed path is to be found out.

find out the sum of the unit to all cell with +ve sign & find the difference of number from the cell on the closed path with +ve sign

Examine if the no. of occupied is (m+n−1) If Yes stop here otherwise continue the process till we get the optimum

Q11

	D_1	D_2	D_3	supply
O_1	4	2	4	400
O_2	2	2	3	900
O_3	3	3	2	600
Demand	200	900	800	

Sol^n: North west corner cell:-

[200] 4	2	4	~~400~~ 200
2	2	3	900
3	3	2	600
~~200~~ 0	900	800	

[200] 2	4	~~200~~ 0
2	3	900
3	2	600
~~900~~ 700	800	

[700] 2	3	~~900~~ 200
3	2	600
~~700~~ 0	800	

[200] 3	200
[600] 2	600
800	

So the final table:-

	D_1	D_2	D_3
O_1	[200] 4	[200] 2	(+1) 4
O_2	(-2) 2	[700] 2	[200] 3
O_3	(0) 3	(+2) 3	[600] 2

$m+n-1 = 3+3-1 = 5$

total cost-

$200\times4 + 200\times2 + 700\times2 + 200\times3 + 600\times2$

$= 800 + 400 + 1400 + 600 + 1200$

$= \boxed{₹4400/-}$

unoccupied cell-

O_1D_3, O_2D_1, O_3D_1, O_3D_2

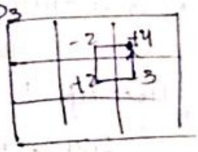
$O_1D_3 = 4 - 2 + 2 - 3 = +1.$

$O_2D_1 = +2 - 2 + 2 - 4 = -2$

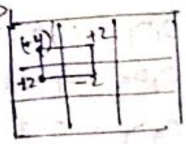
$O_3D_1 = 3 - 2 + 3 - 2 + 2 - 4 = 0.$

$O_3D_2 = +3 - 2 + 3 - 2 = +2$

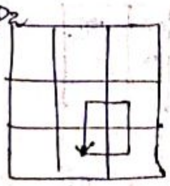
O_1D_3



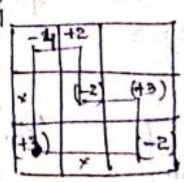
O_2D_1



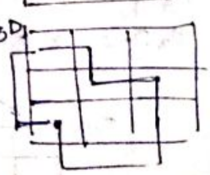
O_3D_2



O_3D_1



O_3D_1



So, this complete the evaluation of empty cell, the soln not optimal because O_2D_1 is negative, we can reduce the total transportation cost by moving some unit to empty cell. we choose the soln whose ckt has most (+ve) ckt.

4	400 / 2	4	400
200 / 2	500 / 2	200 / 3	900
3	3	600 / 2	600

We choose minm of (200, 400). It is done by subtracting smallest no. of unit i.e 200 from -ve cell & add thus no -ve from units of +ve cell. But the values of remaining cell does not change.

unoccupied cell -

O_1D_1, O_1D_3, O_3D_1, O_3D_2.

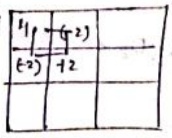
$O_1D_1 = 4 - 2 + 2 - 2 = 2$.

$O_1D_3 = 4 - 2 + 2 + 3 = 1$

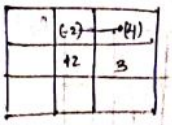
$O_3D_1 = 3 - 2 + 3 - 2 = 2$

$O_3D_2 = 3 - 2 + 3 - 2 = 2$.

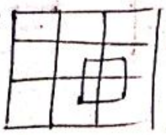
O_1D_1



O_1D_3



O_3D_2



O_3D_1

(+2) 4	400 / 2	4
200 / 2	500 / 2	200 / 3
(+2) 3	(+2) 3	600 / 2

Because all ckt cost is either (+ve) or zero so the optima is reached.

total cost = $400 \times 2 + 200 \times 2 + 500 \times 2 + 200 \times 3 + 600 \times 2$

$= 800 + 400 + 1000 + 600 + 1200$

$= \boxed{₹4000/-}$

18/8/18

MODI'S METHOD -

$d_{ij} \geq 0$

$d_{ij} = c_{ij} - (u_i + v_j)$

In stepping stone method, a closed path is traced for each unoccupied cell. Cell evaluation are found and cell with most -ve evaluation becomes the basic cell.

In MODI's method, cell evaluation of all unoccupied cells are calculated simultaneously & only one closed path for most -ve cell is traced. Thus, these provide considerable time saving over the stepping stone method.

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Solve the transportation problem & test the optimality.

	1	2	3	4	S
1	8	10	7	6	50
2	12	9	4	7	40
3	9	11	10	8	30
D	25	32	40	23	

$\Sigma a_i = \Sigma b_j$

balanced transportation problem.

$$\begin{array}{r} 50 \\ -23 \\ \hline 27 \end{array}$$

8	10	7	[23] 6	~~50~~ 27
12	9	[40] 4	7	~~40~~ 0
9	11	10	8	30
25	32	~~40~~ 40	~~23~~ 0	

8	10	7	27
12	9	[40] 4	~~40~~ 0
9	11	10	30
25	32	~~40~~ 0	

[25] 8	10	~~27~~ 2
9	11	30
~~25~~	32	

[2] 10	2
[30] 11	30
32	

So the final table

[25] 8	[2] 10	7	[23] 6
12	9	[40] 4	7
9	[30] 11	10	8

no. of allocation = 5

no. of allocation = $m+n-1$

$= 4+3-1$

$= 6$

$5 < 6$.

~~total cost~~.

total cost $= 25\times8 + 2\times10 + 23\times6 + 40\times4 + 30\times11$

$= 200 + 20 + 138 + 160 + 330$.

$=$ ₹ 848 /-.

So the problem is degeneracy

Optimality Test :-

→ first to calculate the value of the occupied cell i.e $c_{ij} = u_i + v_j$

→ first choosing maxm no. of allocation in made in which cell i.e Row 1.

maxm no. of allocation = 0

(8) 25	(10) 2	(7)	(6) 23	$u_1(0)$
(12)	(9)	(4) 40	(7) Δ	u_2
(9)	(11) 30	(10)	(8)	u_3

choose Δ for allocation

for occupied cell :-

$C_{11} = U_1 + V_1 = 8 \Rightarrow 0 + V_1 = 8$

$\Rightarrow \boxed{V_1 = 8}$

$C_{12} = U_1 + V_2 = 10 \Rightarrow 0 + V_2 = 10$

$\Rightarrow \boxed{V_2 = 10}$

$C_{14} = U_1 + V_4 = 6 \Rightarrow 0 + V_4 = 6$

$\Rightarrow \boxed{V_4 = 6}$

$C_{23} = U_2 + V_3 = 4 \Rightarrow 1 + V_3 = 4$

$\Rightarrow \boxed{V_3 = 3}$

$C_{24} = U_2 + V_4 = 7$

$\Rightarrow U_2 + 6 = 7$

$\Rightarrow \boxed{U_2 = 1}$

$C_{32} = U_3 + V_2 = 11$

$\Rightarrow U_3 + 10 = 11$

$\Rightarrow \boxed{U_3 = 1}$

for unoccupied cell :-

$C_{13} : U_1 + V_3 = 0 + 3 = 3$.

$C_{21} : U_2 + V_1 = 1 + 8 = 9$.

$C_{22} : U_2 + V_2 = 1 + 10 = 11$.

$C_{31} : U_3 + V_1 = 1 + 8 = 9$.

$C_{33} : U_3 + V_3 = 1 + 3 = 4$.

$C_{34} : U_3 + V_4 = 1 + 6 = 7$.

(8) 25	$\boxed{10}$ 2	(7)(3) $\boxed{4}$	$\boxed{6}$ 23
(12)(9) $\boxed{3}$	(9)(11) $\boxed{-2}$	(4) 40	(7) Δ
(9)(9) $\boxed{0}$	(11) 30	(10)(4) $\boxed{6}$	(8)(7) $\boxed{1}$

from the above table we conclude that all $d_{ij} \neq 0$ so optimality is not reached.

stepping stone.

	−Δ	$\boxed{4}$	Δ
$\boxed{3}$	$\boxed{-2}$ Δ		Δ −Δ
$\boxed{0}$		$\boxed{6}$	$\boxed{1}$

$\Downarrow$

		$\boxed{4}$	
$\boxed{3}$	Δ		
$\boxed{0}$		$\boxed{6}$	$\boxed{1}$

(8) 25	(10) 2	(7)	(6) 23	U_1
(12)	(9)	(4) 40	(7)	U_2
(9)	(11) 30	(10)	(8)	U_3

For occupied cell

$c_{11} = u_1 + v_1 = 8 \Rightarrow 0 + v_1 = 8$

$\boxed{v_1 = 8}$

$c_{12} = u_1 + v_2 = 10 \Rightarrow 0 + v_2 = 10$

$\Rightarrow \boxed{v_2 = 10}$

$c_{14} = u_1 + v_4 = 6 \Rightarrow 0 + v_4 = 6$

$\Rightarrow \boxed{v_4 = 6}$

$c_{22} = u_2 + v_2 = 9 \Rightarrow \boxed{u_2 = -1}$

$c_{23} = u_2 + v_3 = 4 \Rightarrow \boxed{v_3 = 5}$

$c_{32} = u_3 + v_2 = 11 \Rightarrow \boxed{u_3 = 1}$

For unoccupied cell.

$c_{13} = u_1 + v_3 = 0 + 5 = 5$

$c_{21} = u_2 + v_1 = -1 + 8 = 7.$

$c_{24} = u_2 + v_4 = -1 + 6 = 5$

$c_{31} = u_3 + v_1 = 1 + 8 = 9$

$c_{33} = u_3 + v_3 = 1 + 5 = 6$

$c_{34} = u_3 + v_4 = 1 + 6 = 7.$

(8) 25	(10) 2	(7)(5) $\boxed{2}$	(6) 23
(12)(7) $\boxed{5}$	(9) Δ	(4) 40	(7)(5) $\boxed{2}$
(9)(9) $\boxed{0}$	(11) 30	(10)(-1) $\boxed{9}$	(8)(7) $\boxed{1}$

From the above table, All $d_{ij} \geq 0$ so optimality is reached.

21/8/18

ASSIGNMENT PROBLEM:-

It is a special kind of transportation problem in which each source should have capacity to fulfill to the demand of any destination.

In otherwords, Operator should able to perform any job regardless of his skill although the cost will more if the job does not match the worker skill.

(operator)

$$\begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & \cdots & n \end{array} \\ (\text{job}) \begin{array}{c} 1 \\ 2 \\ \vdots \\ m \end{array} & \begin{bmatrix} t_{11} & t_{12} & t_{13} & \cdots & t_{1n} \\ t_{21} & t_{22} & t_{23} & \cdots & t_{2n} \\ \vdots & & & & \\ t_{m1} & t_{m2} & \cdots & & t_{mn} \end{bmatrix} \end{array}$$

m is a no. of jobs as well as the no. of operator.

t_{ij} be the processing time of the job i if it is assigned to the operator j.

Here the objective is to assign job to the operator such that the total processing is to be minimised.

Row entry	Column entry	Cell entry
job	operator	processing time.
operator	machines	processing time.
teachers	or subjects	student pass ratio.
Drivers of vehicle	route	travel time.

physician treatment no. of cases handled.

~~MIN Z~~

$$\text{Min } Z = \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} x_{ij}$$

subjected to $\sum_{j=1}^{m} x_{ij}$ $i = 1, 2, \ldots m$

$\sum_{i=1}^{m} c_{ij}$ $j = 1, 2 \ldots m$

$x_{ij} \geq 0$.

where x_{ij} = 0 or 1 $i = 1 \ldots m$

$j = 1, 2 \ldots m$

m = no. of rows (job) as well as columns (operator).

c_{ij} = time cost (processing time)

$$x_{ij} = \begin{cases} 1 & \text{If } i\text{th row assign the } j\text{th column.} \\ 0 & \text{otherwise.} \end{cases}$$

→ There are 2 type of assignment problem.

balanced assignment problem | unbalanced assignment problem

No. of job = no. of person

	j_1	j_2	j_3
P_1	2	3	5
P_2	3	5	7
P_3	2	4	1

No. of job ≠ No. of person.

	j_1	j_2	j_3
P_1	1	2	3
P_2	4	5	6

HUNGERIAN METHOD :-

ROW & COLUMN REDUCTION :- (ALGORITHM)

(i) step 0:- Consider the given net matrix

(ii) step 1:- obtain a next matrix by substracting minimum value of each value of from the entry of each row.

(iii) step 2: obtain the next matrix by substracting min

optimization of problem

(iv) step 3:- Draw minm no. of lines to cover all the zeroes of the matrix. The procedure for drawing minm no. of lines following methods should be adopted.

- step 3.1 :- Row scanning

starting from the 1st row as the following questions. Is there is exactly one zero? If Yes, ~~Draw~~ mark a square around in that zero entry and draw vertical lines around that zero otherwise skip that row.

3.3 After scanning the last row check whether all the zeroes covered with lines. If Yes, go to step ④. otherwise do column scanning

- step 3.2:- Column Scanning

starting from the 1st column as the following questions. Is there is exactly one zero? If Yes, mark as square around in that zero entry and draw horizontal lines around that zero otherwise skip that column.

(v) step 4 – check whether the no. of square marked is equal to the no. of rows of matrix if Yes g.

(vi) steps :- Identify the minm value of undeleted

(B)- step 5·1 :- copy the entries on the lines but not on the intersection pt. as such without any modification

- step 5·2 :- copy the entries of the intersection pt. of the present matrix after adding the minm undeleted cell value to the corresponding position of next matrix

subtract minm undeleted each cell value from the undeleted each value then copy them to the corresponding

proceeding in these manner till we get the optimal soln.

25/8/18

Q. Solve the problem by Hungarian method.

operator

Job	10	12	15	12	8
	7	16	14	14	11
	13	14	7	9	9
	12	10	11	13	10
	8	13	15	11	15

No. of rows = No. of columns.

Balanced assignment problem.

operator

						Row minm
Job	10	12	15	12	8	8
	7	16	14	14	11	7
	13	14	7	9	9	7
	12	10	11	13	10	10
	8	13	15	11	15	8

Row Reduction

2	4	7	4	0
0	9	7	7	4
6	7	0	2	2
2	0	1	3	0
0	5	7	3	7

2	4	7	4	0
0	9	7	7	4
6	7	0	2	2
2	0	1	3	0
0	5	7	3	7
column minimum: 0	0	0	2	0

Column Reduction

2	4	7	2	$\boxed{0}$
0	9	7	5	4
6	7	0	0	2
2	0	1	1	0
0	5	7	1	7

Row scanning :-

exactly 1 zero

2	4	7	2	$\boxed{0}$
$\boxed{0}$	9	7	5	4
6	7	$\boxed{0}$	0	2
2	$\boxed{0}$	1	1	0
0	5	7	1	7

So, we didn't get the optimal solⁿ.

Among the undeleted row & column 1 is the smallest element. then we substract from undeleted element & add with the intersection pt.

operator

job	I	II	III	IV	V
I	2	4	6	11	$\boxed{0}$
II	$\boxed{0}$	9	6	4	4
III	7	8	$\boxed{0}$	0	3
IV	2	$\boxed{0}$	0	0	0
V	0	5	6	$\boxed{0}$	7

every row & every column has assignment.

~~Optimal solution~~ :-

Optimal solution

Job	operator	Time
I	V	8
II	I	7
III	III	7
IV	II	10
V	IV	11

Total processing time = 43 hrs.

Q11 find the optimal solution of the following problem.

subjects.

		1	2	3	4	5
faculty	1	30	39	31	38	40
	2	43	37	32	35	38
	3	34	41	33	41	34
	4	39	36	43	32	36
	5	32	49	35	40	37
	6	36	42	35	44	42

No. of rows ≠ No. of column

So the problem is unbalanced.

subject

							Row min
faculty	30	39	31	38	40	0	0
	43	37	32	35	38	0	0
	34	41	33	41	34	0	0
	39	36	43	32	36	0	0
	32	49	35	40	37	0	0
	36	42	35	44	42	0	0

Row Reduction

	30	39	31	38	40	0
	43	37	32	35	38	0
	34	41	33	41	34	0
	39	36	43	32	36	0
	32	49	35	40	37	0
	36	42	35	44	42	0
column min^m	30	36	31	32	34	0

column Reduction

0	3	0	6	6	0
13	1	1	3	4	0
4	5	2	9	0	0
9	0	12	0	2	0
2	13	4	8	3	0
6	6	4	12	8	0

Row & Column Scanning:-

$\boxed{0}$	3	0	6	6	0
13	1	1	3	4	$\boxed{0}$
4	5	2	9	$\boxed{0}$	0
9	$\boxed{0}$	12	$\boxed{0}$	2	0
2	13	4	8	3	0
6	6	4	12	8	0

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$\boxed{0}$	3	0	6	07	01
12	0	$\boxed{0}$	2	4	0
3	4	1	8	$\boxed{0}$	0
~~1~~	~~12~~	~~3~~	~~7~~	3	
9	0	12	$\boxed{0}$	3	1
1	12	3	7	3	$\boxed{0}$
5	5	3	11	8	0

every column & every row does not have assignment.

Here the smallest element is 'y' add at the intersection of & subtract 'y' from each uncovered element

	I	II	III	IV	V	VI
I	0	3	$\boxed{0}$	6	7	2
II	12	$\boxed{0}$	0	2	4	1
III	3	4	1	8	$\boxed{0}$	1
IV	9	0	12	$\boxed{0}$	3	2
V	$\boxed{0}$	11	2	6	2	0
VI	4	4	2	10	7	$\boxed{0}$

every row & every column has assignment.

0	3	0	6	7	2
12	0	0	2	4	1
3	4	1	8	0	1
9	0	12	0	3	2
0	11	2	6	2	0
4	4	2	10	7	0

Optimal solution

faculty	subject
I	
II	
III	
IV	
V	
VI	

Maximization largest element to be find out subtracting from all the element convert to minimization one.

application

(I) travelling salesman problem

x_{ij} be the variable which is defined as.

$x_{ij} = \begin{cases} 1 & \text{if the salesman travel from } i\text{th city to } j\text{th city} \\ 0, & \text{otherwise} \end{cases}$

$\text{Min } z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$, $i \neq j$

c_{ij} → distance / time / cost of travelling from ith city to jth city

(II) worker & job.

27/8/18

REVISED SIMPLEX METHOD

ALGORITHM :-

Step 1:- Write the standard form of given LPP & convert it into maximization time. If it is minimization type i.e

$Max\ x = Cx$

subject to $Ax = b$

$x \geq 0$.

$C^T = [c_1, c_2, c_3 \ldots c_n]$ profit co-efficient

column of A as $A_1, A_2, A_3 \ldots A_n$.

$\pi = (\pi_1, \pi_2 \ldots)$ simplex multiplier

x_B = Basic vector

c_B^T = profit co-efficient in the matrix

B = Basic matrix

B^{-1} = Basic inverse

$\bar{c_j}$ = Net evolution.

j = index of the non-basic variable

$\bar{b}$ = current BFS.

Step 2:- $B = I$

$B^{-1} = I$

else for other iteration find $B = [x_{Bi}] = [A_{xBi}]$ and hence find B^{-1}.

Step 3:- Calculate $\pi = c_B^T B^{-1}$ and $\bar{b} = B^{-1} \cdot b$ (current solution)

$\bar{c_j} = \pi A_j - c_j$

Decision = If all $\bar{c_j} \geq 0$ then current BFS is optimal. else select the most -ve of $\bar{c_j}$ (say) $\bar{c_k}$ then x_k will be the entering variable & $\bar{A_k}$ = key column = $B^{-1}A_k$

Step 4: PROCEDURE FOR FINDING RSM :-

x_B	B^{-1}	$\bar{b}$	entering variable	key column	ratio

encircle the key element obtain from the minimum ratio ($\bar{b}$/key column) element corresponding to the key element, will be depart from x_B i.e leaving variable.

Step 5:- Go to step 2 repeat the procedure until optimal BFS is obtained.

Q// Use Revise simplex Method to solve LPP.

$Max\ z = 5x_1 + 2x_2 + 3x_3$

subject to $x_1 + 2x_2 + 2x_3 \leq 8$

$3x_1 + 4x_2 + x_3 \leq 7$.

$x_1, x_2, x_3 \geq 0$

soln The standard form of LPP is given by

$Max\ z = 5x_1 + 2x_2 + 3x_3 + 0s_1 + 0s_2$.

$x_1 + 2x_2 + 2x_3 + s_1 + 0s_2 = 8$

$3x_1 + 4x_2 + x_3 + 0s_1 + 1s_2 = 7$

$x_1, x_2, x_3, s_1, s_2 \geq 0$

Body matrix -

$$\begin{pmatrix} 1 & 2 & 2 & 1 & 0 \\ 3 & 4 & 1 & 0 & 1 \end{pmatrix}$$

$A_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $A_2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $A_3 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $A_4 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $A_5 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$b = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$

Let us consider the index of the variable x_1 be 1, x_2 be 2, x_3 be 3 ...

$x_B = (S_1 \; S_2)$, $B = [A_4 \; A_5] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$C_B^{T} = (0, 0)$

$\bar{b} = B^{-1} \cdot b$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 7 \end{bmatrix}$

$= \begin{bmatrix} 8 \\ 7 \end{bmatrix}$

$J = 1, 2, 3$

$\pi = C_B^{T} \cdot B^{-1}$

$= (0, 0) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= (0, 0)$

$= (\pi_1, \pi_2)$

Net evolution

$\bar{c}_1 = \pi A_1 - c_1$

$= (0\,0)\begin{pmatrix} 1 \\ 3 \end{pmatrix} - 5 \longleftarrow$

$= -5$

$\bar{c}_2 = \pi A_2 - c_2$

$= (0\,0)\begin{pmatrix} 2 \\ 1 \end{pmatrix} - 2$

$= -2$

$\bar{c}_3 = \pi A_3 - c_3$

$= (0\,0)\begin{pmatrix} 8 \\ 1 \end{pmatrix} - 3$

$= -3$

Here all $\bar{c}_j$ are $\neq 0$ so optimal is not reached. We choose the most -ve as the entering variable (-5)

Key element column $= B^{-1} A_k$

$= B^{-1} A_1$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

x_B	B^{-1}	$\bar{b}$	entering variable	key column	Ratio
S_1	1 0	8	x_1	1	$8/1$
S_2	0 1	7		3	$7/3$

Departing variable : S_2.

Iteration - 2 :-

$x_B = (S_1 \; x_1)$ $\quad B = [A_4 \; A_1] = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$

$B^{-1} = \dfrac{\begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}}{3}$

$= \begin{bmatrix} 1 & -1/3 \\ 0 & 1/3 \end{bmatrix}$

$\bar{b} = B^{-1} \cdot b$

$= \begin{pmatrix} 1 & -1/3 \\ 0 & 1/3 \end{pmatrix}\begin{pmatrix} 8 \\ 7 \end{pmatrix} = \begin{pmatrix} 8 - 7/3 \\ 7/3 \end{pmatrix} = \begin{pmatrix} \frac{24-7}{3} \\ 7/3 \end{pmatrix} = \begin{pmatrix} 17/3 \\ 7/3 \end{pmatrix}$

$J = 2, 3, 5$

$\pi = C_B^T \cdot B^{-1}$

$= \begin{pmatrix} 0 & 5 \end{pmatrix} \begin{pmatrix} 1 & -1/3 \\ 0 & 1/3 \end{pmatrix}$

$= \begin{pmatrix} 0 & 5/3 \end{pmatrix}$

$\bar{c}_2 = \pi A_2 - c_2$

$= \begin{pmatrix} 0 & 5/3 \end{pmatrix} \begin{pmatrix} 8 \\ 4 \end{pmatrix} - 2$

$= 20/3 - 2$

$= \frac{20-6}{3}$

$= 14/3$.

$\bar{c}_3 = \pi A_3 - c_3$.

$= \begin{pmatrix} 0 & 5/3 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} - 3$

$= 5/3 - 3$

$= -4/3 \leftarrow$

$\bar{c}_5 = \pi A_5 - c_5$

$= \begin{pmatrix} 0 & 5/3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} - 0$.

$= 5/3$.

Here all $\bar{c}_j \not> 0$

$-4/3$ is the entering variable.

Key column $= B^{-1}A_3 = \begin{pmatrix} 1 & -1/3 \\ 0 & 1/3 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 5/3 \\ 1/3 \end{pmatrix}$

x_B	B^{-1}		$\bar{b}$	entering variable		key column	Ratio
s_1	1	$-1/3$	$14/3$	~~$14/5$~~	x_3	$5/3$	
x_1	0	$1/3$	$7/3$	~~$7/3$~~		$1/3$	

x_B	B^{-1}		$\bar{b}$	entering variable	key column	Ratio
s_1	1	$-1/3$	$14/3$	x_3	$5/3$	
x_1	0	$1/3$	$7/3$		$1/3$	

Iteration 3 :-

$x_B = (x_3 \ x_1)$ $\quad B = [A_3, A_1] = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$

$B^{-1} = \dfrac{\begin{matrix} 3 & -1 \\ -1 & 2 \end{matrix}}{5} = \begin{matrix} 3/5 & -1/5 \\ -1/5 & 2/5 \end{matrix}$

$J = 2, 4, 5$

$\bar{b} = B^{-1} \cdot b = \begin{pmatrix} 3/5 & -1/5 \\ -1/5 & 2/5 \end{pmatrix} \begin{pmatrix} 8 \\ 7 \end{pmatrix} = \begin{pmatrix} 17/5 \\ 6/5 \end{pmatrix}$

$\pi = C_B^T \cdot B^{-1}$

$\bar{c}_2 = 26/5$

$\bar{c}_4 = 4/5$

$\bar{c}_5 = 7/5$

Here all $c_j \geq 0$ so optimality is reached

$x_1^* = 6/5$

$x_2^* = 0$

$x_3^* = 17/5$

$\text{Max } z = 5x_1 + 2x_2 + 3x_3$

$= 5(6/5) + 2(0) + 3(17/5)$

$= 6 + \frac{51}{5}$

$= \frac{30+51}{5}$

$= 81/5$

22/2/18

$\text{Max } z = 12x_1 + 20x_2$

$6x_1 + 8x_2 \leq 100$

$7x_1 + 12x_2 \leq 120$

$x_1, x_2 \geq 0$

$\text{Min } z = 12x_1 + 20x_2$

$6x_1 + 8x_2 \geq 100$

$7x_1 + 12x_2 \geq 120$

$x_1, x_2 \geq 0$

Solve by using Revised simplex method.

The standard form of LPP is given by

$\text{Max } z = 12x_1 + 20x_2 + 0s_1 + 0s_2$

$6x_1 + 8x_2 + s_1 + 0s_2 = 100$

$7x_1 + 12x_2 + 0s_1 + s_2 = 120$

$x_1, x_2, s_1, s_2 \geq 0$

Body matrix

$$\begin{pmatrix} 6 & 8 & 1 & 0 \\ 7 & 12 & 0 & 1 \end{pmatrix}$$

$A_1 = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$ $A_2 = \begin{pmatrix} 8 \\ 12 \end{pmatrix}$ $A_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $A_4 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$b = \begin{pmatrix} 100 \\ 120 \end{pmatrix}$

Let us consider the index of the variable x_1 be 1, x_2 be 2, s_1 be 3 & s_2 be 4.

$x_B = (s_1 \quad s_2)$ $B = (A_3 \quad A_4) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$C_B^T = (0, 0)$

$\bar{b} = B^{-1} b$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 100 \\ 120 \end{pmatrix}$

$= \begin{pmatrix} 100 \\ 120 \end{pmatrix}$

$J = 1, 2$

$\pi = C_B^T \cdot B^{-1}$

$= (0 \quad 0)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= (0 \quad 0)$

$= (\pi_1, \pi_2)$

Net evaluation

$\bar{c}_1 = \pi A_1 - c_1$

$= (0, 0)\begin{pmatrix}6\\7\end{pmatrix} - 12$

$= -12$

$\bar{c}_2 = \pi A_2 - c_2$

$= (0\ 0)\begin{pmatrix}8\\12\end{pmatrix} - 20 \leftarrow$

$= -20$.

Here all $\bar{c}_j \ngeq 0$ so optimality is not reached so we choose the most -ve the entering variable

Key column $= B^{-1}A_K$

$= \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}8\\12\end{pmatrix}$

$= \begin{pmatrix}8\\12\end{pmatrix}$

x_B	B^{-1}	$\bar{b}$	entering Variable	key column element	ratio
S_1 S_2	1 0 0 1	100 120	x_2	8 12	$100/8 = 25/2 = 12.5$ $120/12 = 10$

Departing variable - S_2.

Iteration - 2 :-

$x_B = (S_1\ x_2)$ $\quad B = [A_3\ A_2] = \begin{pmatrix}1 & 8\\0 & 12\end{pmatrix}$

$B^{-1} = \dfrac{\begin{bmatrix}12 & -8\\0 & 1\end{bmatrix}}{12} = \begin{pmatrix}1 & -2/3\\0 & 1/12\end{pmatrix}$

$\bar{b} = B^{-1}\cdot b$

$= \begin{pmatrix}1 & -2/3\\0 & 1/12\end{pmatrix}\begin{pmatrix}100\\120\end{pmatrix}$

$= \begin{bmatrix}100 - \frac{240}{3}\\10\end{bmatrix}$

$= \begin{pmatrix}\frac{300-240}{3}\\10\end{pmatrix}$

$= \begin{bmatrix}60/3 = 20\\10\end{bmatrix}$

$= \begin{bmatrix}20\\10\end{bmatrix}$.

$J = \{1, 4\}$

$\pi = C_B^T \cdot B^{-1}$

$= (0.\ 20)_{1\times2}\begin{pmatrix}1 & -2/3\\0 & 1/12\end{pmatrix}_{2\times2}$

$= (0\ \ 20/12)$

$= (0\ \ 5/3)$

$\bar{c}_1 = \pi A_1 - c_1$

$= (0\ \ 5/3)\begin{pmatrix}6\\7\end{pmatrix} - 12$

$= 35/3 - 12$

$= -1/3 \leftarrow$

$\bar{c}_4 = \pi A_4 - c_4$.

$= (0\ \ 5/3)\begin{pmatrix}0\\1\end{pmatrix} - 0$

$= 5/3$.

Here all $\bar{c}_j \neq 0$ so $-1/3$ is the entering variable

key column $= B^{-1}A_1$

$$= \begin{pmatrix} 1 & -2/3 \\ 0 & 1/12 \end{pmatrix}\begin{pmatrix} 6 \\ 7 \end{pmatrix}$$

$$= \begin{pmatrix} 6 - 14/3 \\ 7/12 \end{pmatrix}$$

$$= \begin{pmatrix} 4/3 \\ 7/12 \end{pmatrix}$$

x_B	B^{-1}	$\bar{b}$	entering variable	key column	ratio
s_1	$1 \quad -2/3$	20	x_1	$4/3$	26.6 $60/4$ 15
x_2	$0 \quad 1/12$	10		$7/12$	5.8 $70/12$ 17.14

Departing variable = s_1

$x_B = (x_1 \quad x_2)$ $\quad B = [A_1 \quad A_2]$

$$= \begin{matrix} 6 & 8 \\ 7 & 12 \end{matrix}$$

$$B^{-1} = \left[\frac{\begin{matrix} 12 & -8 \\ -7 & 6 \end{matrix}}{16}\right]$$

$$= \begin{bmatrix} 4/3 & -1/2 \\ -7/16 & 3/8 \end{bmatrix}$$

$\bar{b} = B^{-1}b$.

$$= \begin{bmatrix} 4/3 & -1/2 \\ -7/16 & 3/8 \end{bmatrix}\begin{bmatrix} 100 \\ 120 \end{bmatrix}.$$

$$= \begin{bmatrix} \frac{400}{3} - \frac{120}{2}. \\ -\frac{700}{16} + \frac{360}{8} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{220}{3} \\ 5/4 \end{bmatrix}.$$

$J_1 = 3, 4.$

$\pi = C_B^T \cdot B^{-1}$

$$= (0 \quad 0)_{1\times 2}\begin{bmatrix} 4/3 & -1/2 \\ -7/16 & 3/8 \end{bmatrix}_{2\times 2}$$

$$= \underset{x_1}{(0} \quad \underset{x_2}{0)}.$$

$\pi = (12, 20)\begin{pmatrix} 4/3 & -1/2 \\ -7/16 & 3/8 \end{pmatrix}.$

$\pi_1 = \pi A_1 - c_1.$

$= (0 \quad 0)\begin{pmatrix} 6 \\ 7 \end{pmatrix} - 12.$

$= -12.$

$\pi_2 = \pi A_2 - c_2.$

$= (0, 0)\begin{pmatrix} \quad \end{pmatrix} - 20$

$= -20.$

~~The standard form of LPP is~~

$Max(z) = Min(-z)$

$= -12x_1 - 20x_2.$

$6x_1 + 8x_2 - 0s_1 + 0s_2 + R_1 + 0R_2 = 100$

$7x_1 + 12x_2 + 0s_1 - s_2 + 0R_1 + 0R_2 = 120.$

$x_1, x_2, s_1, s_2 \geq 0.$

$R_1, R_2 \geq 0.$

The standard form of LPP is written in the form

$Max(z) = Min(-z)$

$= -12x_1 - 20x_2 + 0s_1 + 0s_2 - MR_1 - MR_2.$

$A_1 = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$ $A_2 = \begin{pmatrix} 8 \\ 12 \end{pmatrix}$ $A_3 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ $A_4 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$ $A_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $A_6 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$b = \begin{pmatrix} 100 \\ 120 \end{pmatrix}$

$x_B = (R_1 \ R_2) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = B$

$\bar{b} = B^{-1}.b.$

$= \begin{pmatrix} 100 \\ 120 \end{pmatrix}$

$C_B^T = (-M, -M)$

$J = 1, 2, 3, 4.$

$\pi = C_B^T . B^{-1}$

$= (-M, -M)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$

$= (-M \ -M)$

$= (\pi_1, \pi_2)$

Net evolution

$\bar{C_1} = \pi A_1 - C_1$

$= (-M \ -M)\begin{pmatrix} 6 \\ 7 \end{pmatrix} - (-12)$

$= (-6M - 7M) + 12.$

$= -13M + 12.$

$\bar{C_2} = \pi A_2 - C_2.$

$= (-M \ -M)\begin{pmatrix} 8 \\ 12 \end{pmatrix} - (-20)$

$= (-M8 - 12M) + 20$

$= \boxed{-20M + 20.}$ (most -ve) entering variable.

$\bar{C_3} = \pi A_3 - C_3.$

$= (-M \ -M)\begin{pmatrix} -1 \\ 0 \end{pmatrix} - (-0).$

$= M$

$\bar{C_4} = \pi A_4 - C_4.$

$= (-M - M)\begin{pmatrix} 0 \\ -1 \end{pmatrix} - (0)$

$= M$

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key column $= B^{-1}A_2$.

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 8 \\ 12 \end{pmatrix}$

$= \begin{pmatrix} 8 \\ 12 \end{pmatrix}$

x_B	B^{-1}	$\bar{b}$	Entering variable	key column	ratio
R_1	1 0	100	x_2	8	12.5
R_2	0 1	120		12	10

$\boxed{\text{Departing variable } - R_2}$.

Iteration 2 :-

$x_B = (R_1 \quad x_2)$

$B = (A_5, A_2)$ $\qquad B^{-1} = \dfrac{\begin{pmatrix} 12 & -8 \\ 0 & 1 \end{pmatrix}}{12} = \begin{pmatrix} 1 & -2/3 \\ 0 & 1/12 \end{pmatrix}$

$= \begin{pmatrix} 1 & 8 \\ 0 & 12 \end{pmatrix}$

$\bar{b} = B^{-1}.b$

$= \begin{pmatrix} 1 & -2/3 \\ 0 & 1/12 \end{pmatrix}\begin{pmatrix} 200 \\ 120 \end{pmatrix}$

$= \begin{pmatrix} 20 \\ 10 \end{pmatrix}$

$J = 1, 3, 4, 6$

$\pi = C_B^T . B^{-1}$

$= (-M \; -20)\begin{pmatrix} 1 & -2/3 \\ 0 & 1/12 \end{pmatrix}$

$= \left(-M \quad 2M/3 - 20/12\right)$.

$= \left(-M \quad \dfrac{2M-5}{3}\right)$

$\bar{c_1} = \pi A_1 - c_1$

$= \left(-M \quad \dfrac{2M-5}{3}\right)\begin{pmatrix} 6 \\ 7 \end{pmatrix} - (-12)$.

$= \left(-6M + \dfrac{14M-35}{3}\right) + 12$.

$= \left(\dfrac{-18M + 14M - 35}{3}\right) + 12$.

$= \dfrac{-4M-35}{3} + 12$.

$= \dfrac{-4M+1}{3} \longleftarrow$ entering.

$\bar{c_3} = \pi A_3 - c_3$.

$= \left(-M \quad \dfrac{2M-5}{3}\right)\begin{pmatrix} -1 \\ 0 \end{pmatrix} - (0)$

$= M$

$\bar{c_4} = \pi A_4 - c_4$.

$= \left(-M \quad \dfrac{2M-5}{3}\right)\begin{pmatrix} 0 \\ -1 \end{pmatrix} - 0$

$= \dfrac{-2M+5}{3}$

$\bar{c_6} = \pi A_6 - c_6$

$= \left(-M \quad \dfrac{2M-5}{3}\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix} - 0 \; . = \dfrac{2M-5}{3}$

Scanned by CamScanner

Key column = $B^{-1}A_1$

$$= \begin{pmatrix} 1 & -8/3 \\ 0 & 1/12 \end{pmatrix} \begin{pmatrix} 6 \\ 7 \end{pmatrix}$$

$$= \begin{pmatrix} 6 - 14/3 \\ 7/12 \end{pmatrix}$$

$$= \begin{pmatrix} 4/3 \\ 7/12 \end{pmatrix}$$

x_B	B^{-1}	$\bar{b}$	entering variable	key column	ratio
R_1	$1 \quad -2/3$	20	x_1	$4/3$	15
x_2	$0 \quad 1/12$	10		$7/12$	17.14

Departing variable − R_1

$x_B = (x_1 \quad x_2)$

$B = (A_1 \quad A_2)$

$$= \begin{pmatrix} 6 & 8 \\ 7 & 12 \end{pmatrix}$$

$$B^{-1} = \frac{\begin{pmatrix} 12 & -7 \\ -8 & 6 \end{pmatrix}}{72-56} \qquad \frac{\begin{pmatrix} 12 & -8 \\ -7 & 6 \end{pmatrix}}{16} = \begin{pmatrix} 3/4 & -1/2 \\ -7/16 & 3/8 \end{pmatrix}$$

$$\frac{\begin{pmatrix} 12 & -7 \\ -8 & 6 \end{pmatrix}}{16} = \begin{matrix} 3/4 & -7/16 \\ -8/16 & 6/16 \end{matrix}$$

$J = \{3, 4, 5, 6\}$

$\pi = C_B^T B^{-1}$

$$= (-12 \;\; -20) \begin{pmatrix} 3/4 & -1/2 \\ -7/16 & 3/8 \end{pmatrix}$$

$$= \begin{pmatrix} -3/4 & -3/2 \end{pmatrix}.$$

$$\bar{b} = B^{-1} \cdot b = \begin{bmatrix} 15 \\ 5/4 \end{bmatrix}$$

$\bar{C_3} = \pi A_3 - C_3 = \begin{pmatrix} -3/4 & -3/2 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix} - 0 = 3/4.$

$\bar{C_4} = \pi A_4 - C_4 = \begin{pmatrix} -3/4 & -3/2 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \end{pmatrix} - 0 = 3/2$

$\bar{C_5} = \pi A_5 - C_5 = \begin{pmatrix} -3/4 & -3/2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \overset{(-M)}{0} = M - 3/4$

$\bar{C_6} = \pi A_6 - C_6 = \begin{pmatrix} -3/4 & -3/2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} - (-M) = M - 3/2$

$$\boxed{\begin{matrix} x_1^* = 15 \\ x_2^* = 5/4 \end{matrix}}$$

NON-LINEAR PROGRAMMING PROBLEM :- (NLPP)

1/9/18

It is a mathematical technique like linear programming for determining optimal soln. related many business problem that can be solved by NLPP

In an NLPP either the objective function is non-linear or one or more constraint have non-linear relationship or both.

NLPP deals with problem of optimising objective function subject to inequality constraints or equality constraints.

optimise $z = x_1^2 + x_2^2 + x_3^3$

$x_1 - x_2 + x_3 \leq 1$

$x_1, x_2 \& x_3 \geq 0$

The principle of finding optimal soln of non-linear based on the principle that one or more lies corner pt of the feasible region.

It is very difficult to distinguish local optimum & global optimum due to non-linearity of objective function or constraints. Also it sometime difficult to test optimality NLPP unless the objective function is concave for maximisation & convex for minimisation.

It can be ensure that local minimum or local maximum is also global minm or global maxm

GENERAL FORMAT OF NLPP :-

Mathematical formulation of general LPP can written as -

Maximize or minimize $(x) = C(x_1, x_2, x_3 \ldots x_n)$

subject to constraints $a_1(x_1, x_2 \ldots x_n) \{\leq \text{ or } = \text{ or } \geq\} b_1$

$a_2(x_1, x_2 \ldots x_n) \{\leq \text{ or } = \text{ or } \geq\} b_2$

$\vdots$

$a_m(x_1, x_2 \ldots x_n) \{\leq \text{ or } = \text{ or } \geq\} b_m$

and the non negative restriction

$x_j \geq 0 \;\; \forall j = 1, 2, 3 \ldots n$

where $C(x_1, x_2, \ldots x_n)$

$a_i(x_1, x_2 \ldots x_n) \;\; \forall i = 1, 2, \ldots m$

both are non-linear.

Unconstrained Optimisation Problem :-

Mathematically, a function $f(x)$ has maximum at pt x_0 for all $|h|$ sufficiently small so we can write

$f(x_0 + h) - f(x_0) < 0$

$f(x_0 + h) - f(x_0) > 0$

local max^m or local min^m can be expressed as global max^m or global min^m.

A necessary condition for continuous function $f(x)$ is continuous 1st & 2nd partial derivatives to have an extremum pt. at x_0 is that the 1st order partial derivative of $f(x)$ vanishes or equal to zero.

$$\nabla f(x_0) = 0$$

$$\nabla = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial x_3} \right)$$

→ gradient.

A sufficient condition for x is the stationary point at x_0 to be an extremum pt is that the Hessian Matrix (H) evaluated at x_0.

Negative Definite -
when x_0 is the max^m pt.

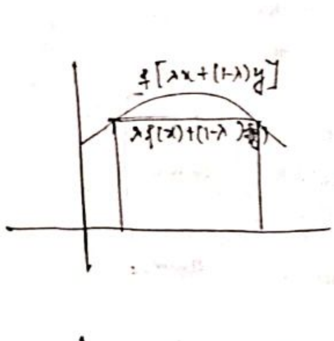
positive Definite -
when x_0 is the min^m pt.

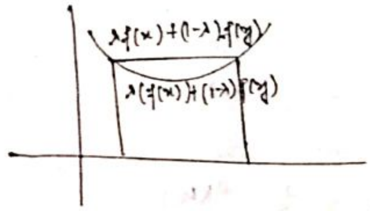
A function $f(x)$ is said to concave over a region S if for any 2 pts. (x, y) in S the following condition must be satisfied

$$f[\lambda x + (1-\lambda) y] \geq \lambda f(x) + (1-\lambda) f(y)$$
$$0 \leq \lambda \leq 1$$

A function $f(x)$ is said to convex over a region S if for any 2 pts (x, y) in S the following condition must be satisfied

$$f[\lambda x + (1-\lambda) y] \leq \lambda f(x) + (1-\lambda) f(y)$$
$$0 \leq \lambda \leq 1.$$





local or valid max^m or global max^m

$$\frac{d^2y}{dx^2} \leq 0 \ \forall x \text{ (concave)}$$

$$\frac{d^2y}{dx^2} \geq 0 \ \forall x \text{ (convex)}.$$

Hessian Matrix :-

A function $f(x_1, x_2 \ldots x_n)$ is a convex function iff the matrix of the 2nd derivative or Hessian matrix is positive semidefinite. & principal minor of the determinant of the matrix are all non-negative.

$$H(x) = \frac{\partial^2 f}{\partial x_i \partial x_j}$$

$i = 1, 2 \ldots m$

$j = 1, 2 \ldots n$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix}$$

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1k} \\ a_{21} & a_{22} & \cdots & a_{2k} \\ \vdots & & & \\ a_{k1} & a_{k2} & \cdots & a_{kk} \end{vmatrix}$$

The function $f(x_1, x_2 \ldots x_n)$ is concave if $H(x)$ is negative semi-definite.

$-a_{11} \quad a_{12}$

$a_{21} \quad a_{22}$

minor $a_{11} = a_{22}$

$a_{12} = a_{21}$

$a_{21} = a_{12}$

$a_{22} = a_{11}$

Q. Find maxm & minm of given function

$f(x_1) = x_1^2 + x_2^2 + x_3^2 - 3x_1 - 6x_2 - 10x_3 + 48$

N.C. – $\nabla f = 0$

$\frac{\partial f}{\partial x_1} = 2x_1 - 3$

$\frac{\partial f}{\partial x_2} = 2x_2 - 6$

$\frac{\partial f}{\partial x_3} = 2x_3 - 10$

$2x_1 - 3 = 0$, $2x_1 = 3$, $\boxed{x_1 = 3/2}$

$2x_2 - 6 = 0$, $2x_2 = 6$, $\boxed{x_2 = 6/2 = 3}$

$2x_3 - 10 = 0$, $2x_3 = 10$, $\boxed{x_3 = 5}$

$X = (3/2, 3, 5)$.

Sufficient Condition

By checking sufficient condition we must determine whether the pt. is maxm or min.

$$H(x) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} \\ \frac{\partial^2 f}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3^2} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= |2| \qquad \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} \qquad \begin{vmatrix} \cdot & \cdot & \cdot \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}$$

$$2 \qquad 4 \qquad 8$$

Hence, it is positive definite. (min^n)

Q. $f(x) = x_1^2 + x_2^2 + x_3^2 + x_1 x_2 - 2x_2 = 0$.

$$\frac{\partial f}{\partial x_1} = 2x_1 + x_2 \qquad 2x_1 + x_2 = 0.$$

$$\frac{\partial f}{\partial x_2} = 2x_2 - 2 + x_1 \qquad 2x_1 + 4x_1 = -2$$

$$\frac{\partial f}{\partial x_3} = 2x_3. \qquad \boxed{x_3 = 0}.$$

Constrained External Problem :-

The optimisation problem having continuous objective function & equality & inequality type constraint are called constrained external problem.

The sol^n of such problem having differentiable objective function & equality type constraints can be obtained by different type of method. One such method i.e. Langragian multiplier method

Problem with all equality constraints :-

Let us consider a simple 2 variable problem having single equality type constraint.

Suppose we want find an optimum of differentiable function i.e. maximise or minimise $z = f(x, y)$

$$x = (x_1, x_2, x_3, \dots x_n) \in R^n$$

$$y = (y_1, y_2, y_3 \dots y_n) \in R^n$$

subject to constraints $g(x,y) = 0$

such that $x, y \geq 0$.

$$\boxed{L(x,y,\lambda) = f(x,y) - \lambda g(x,y)}$$

↳ symbol

$$\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4 \dots \lambda_n.)$$

The necessary condition for an unconstraint optimum of 'L' are also necessary condition for a constraint optimum of $f(x,y)$ provided that matrix of partial derivative $\frac{\partial g_i}{\partial x_j}$ & $\frac{\partial g_i}{\partial y_j}$ have rank 'm'. The necessary condition for minimum or max^m for $f(x,y)$ are the system of (m+n) equation are given by

$$\frac{\partial L}{\partial x_j} = \frac{\partial f}{\partial x_j} - \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial x_j} = 0.$$

$$\frac{\partial L}{\partial y_j} = \frac{\partial f}{\partial y_j} - \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial y_j} = 0$$

$$\frac{\partial L}{\partial \lambda_i} = -g_i = 0.$$

These necessary condition also become sufficient condition for max^m (min^n) of the objective function (concave (convex).

Sufficient Condition (SC) for max^m or min^m of objective function with single equality constant.

The SC for max^m or min^m for computation of (n-1) principal - of the determinant of each of the stationary pt.

$$D_{n+1} = \begin{vmatrix} 0 & \frac{\partial g}{\partial x_1} & \frac{\partial g}{\partial x_2} & \cdots & \frac{\partial g}{\partial x_n} \\ \frac{\partial g}{\partial x_1} & \frac{\partial^2 f}{\partial x_1^2} - \lambda\frac{\partial^2 g}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} - \lambda\frac{\partial^2 g}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} - \lambda\frac{\partial^2 g}{\partial x_1 \partial x_n} \\ \frac{\partial g}{\partial x_2} & \frac{\partial^2 f}{\partial x_2 \partial x_1} - \lambda\frac{\partial^2 g}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} - \lambda\frac{\partial^2 g}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} - \lambda\frac{\partial^2 g}{\partial x_2 \partial x_n} \\ \vdots & & & & \\ \frac{\partial g}{\partial x_n} & \frac{\partial^2 f}{\partial x_n \partial x_1} - \lambda\frac{\partial^2 g}{\partial x_n \partial x_1} & & \cdots & \frac{\partial^2 f}{\partial x_n^2} - \lambda\frac{\partial^2 g}{\partial x_n^2} \end{vmatrix}$$

If the sign of the minor D_3, D_4, D_5 are alternatively +ve & -ve the stationary point is a local ~~min~~ max^m & min.

Assignment Q. $f(x) = x_1^2 - 2x_1^2 x_2 + x_2^3$.
find out the N.C & S.C

Q. Solve the NLPP

$\text{Max } z = 4x_1 - x_1^2 + 8x_2 - x_2^2$
subject to $x_1 + x_2 = 2$,
$x_1, x_2 \geq 0$.

Sol^n $f(x_1, x_2) = 4x_1 - x_1^2 + 8x_2 - x_2^2$.
$g(x_1, x_2) = x_1 + x_2 - 2$

m = equ^n / constraint
n = variable

$L(x_1, x_2, \lambda) = f(x_1, x_2) - \lambda(g(x_1, x_2))$
$= 4x_1 - x_1^2 + 8x_2 - x_2^2 - \lambda(x_1 + x_2 - 2)$

$\frac{\partial L}{\partial x_1} = 4 - 2x_1 - \lambda$ ——— (1)

$\frac{\partial L}{\partial x_2} = 8 - 2x_2 - \lambda$ ——— (2)

$\frac{\partial L}{\partial \lambda} = -(x_1 + x_2 - 2)$ ——— (3)

Solving (1) and (2)

$4 - 2x_1 - \lambda = 0$.
$8 - 2x_2 - \lambda = 0$
$-4 - 2x_1 + 2x_2 = 0$.

$\boxed{x_2 - x_1 = 2}$ ——— (4).

$x_1 + x_2 = 2$ ——— (5)

Solving (4) and (5).

$x_2 - x_1 = 2$.
$x_2 + x_1 = 2$
$-2x_1 = 0$.

$\boxed{x_1 = 0}$

$\boxed{x_2 = 2}$

$\boxed{\lambda = 4}$

~~$\lambda = 0, \lambda \neq 0$~~

The sufficient condition for determining whether the sol^n result max^m or min^m of the objective function involve in the

$n-1=1$ (determinant of order 2)

$\boxed{n=2}$

$$D_3 = \begin{vmatrix} 0 & \frac{\partial g}{\partial x_1} & \frac{\partial g}{\partial x_2} \\ \frac{\partial g}{\partial x_1} & \frac{\partial^2 f}{\partial x_1^2} - \lambda\frac{\partial^2 g}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} - \lambda\frac{\partial^2 g}{\partial x_1 \partial x_2} \\ \frac{\partial g}{\partial x_2} & \frac{\partial^2 f}{\partial x_2 \partial x_1} - \lambda\frac{\partial^2 g}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} - \lambda\frac{\partial^2 g}{\partial x_2^2} \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 1 & 1 \\ 1 & -2 & 0 \\ 1 & 0 & -2 \end{vmatrix}$$

$= 0(4-0) - 1(-2-0) + 1(0+2)$

$= 2+2$

$= 4 > 0$ (maxm)

So, D_3 is +ve. The soln maximises the objective function at $\boxed{x_1 = 0}$ & $\boxed{x_2 = 2}$

$\text{Max } z = 4x_1 - x_1^2 + 8x_2 - x_2^2$

$= 16 - 4$

$= 12$

Q// Determine the optimal soln for the following NLPP and check whether it maximise the objective function.

Optimise $z = x_1^2 - 10x_1 + x_2^2 - 6x_2 + x_3^2 - 4x_3$

$x_1 + x_2 + x_3 = 7$

$x_1, x_2, x_3 \geq 0$.

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Extremal problem with more than 1 equality constraints:-

Let H^B be the Hessian ~~product~~ bordered matrix, computed at the stationary pt are obtained by solving principle minor of the Lagrangian function $L(x, \lambda)$.

$$H^B = \begin{bmatrix} O & P \\ P^T & Q \end{bmatrix}$$

O – $m \times m$ null matrix

$$P = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \cdots & \frac{\partial g_1}{\partial x_n} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} & \cdots & \frac{\partial g_2}{\partial x_n} \\ \vdots & & & \\ \frac{\partial g_m}{\partial x_1} & \frac{\partial g_m}{\partial x_2} & \cdots & \frac{\partial g_m}{\partial x_n} \end{bmatrix}_{m \times n} \begin{array}{l} \text{1st eq} \\ \text{2nd eq} \\ \\ \\ \end{array}$$

P^T = transpose of P

$$Q = \left[\frac{\partial^2 L}{\partial x_i \partial x_j}\right]$$

$i = 1, 2 \cdots m$

$j = 1, 2 \cdots n$

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$$Q = \begin{bmatrix} \frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 L}{\partial x_1 \partial x_n} \\ \frac{\partial^2 L}{\partial x_2 \partial x_1} & \frac{\partial^2 L}{\partial x_2^2} & \cdots & \frac{\partial^2 L}{\partial x_2 \partial x_n} \\ \vdots & & & \\ \frac{\partial^2 L}{\partial x_n \partial x_1} & & & \frac{\partial^2 L}{\partial x_n^2} \end{bmatrix}$$

If $X = (x^*, \lambda^*)$ be the stationary pt for the function $L(x, \lambda)$ & H^B is the corresponding boarded hessian matrix, the sufficient but not necessary condition for maxima & minima is determined by the sign of last $n-m$ principle minor of H^B.

Starting with principle minor of order $2m+1$

If x^* maximizes the function if signs starting with ~~minus~~ $(-1)^{m+n}$ & minimizes the function if all the sign are same & of the type $(-1)^m$

n = no. of variable

m = constraints/eqn.

Q1 Solve the following Q. the NLPP using Langragian multiplier techniques.

optimize $Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$.

subject to $x_1 + x_2 + x_3 = 15$

$2x_1 - x_2 + 2x_3 = 20$.

$x_1, x_2, x_3 \geq 0$.

Soln:- $L(x, \lambda) = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2 - \lambda_1(x_1 + x_2 + x_3 - 15) - \lambda_2(2x_1 - x_2 + 2x_3 - 20)$

The necessary conditions for the maxima & minima of the objective function -

$$\frac{\partial L}{\partial x_1} = 8x_1 - 4x_2 - \lambda_1 - 2\lambda_2$$

$$\frac{\partial L}{\partial x_2} = 4x_2 - 4x_1 - \lambda_1 + \lambda_2.$$

$$\frac{\partial L}{\partial x_3} = 2x_3 - \lambda_1 - 2\lambda_2.$$

$$\frac{\partial L}{\partial \lambda_1} = -x_1 - x_2 - x_3 + 15$$

$$\frac{\partial L}{\partial \lambda_2} = -2x_1 + x_2 - 2x_3 + 20$$

$8x_1 - 4x_2 - \lambda_1 - 2\lambda_2 = 0$ — (1)

$4x_2 - 4x_1 - \lambda_1 + \lambda_2 = 0$ — (2)

$2x_3 - \lambda_1 - 2\lambda_2 = 0$ — (3)

$(x_1 + x_2 + x_3 - 15 = 0) \times 2$ — (4)

$-2x_1 + x_2 - 2x_3 + 20 = 0$ — (5)

$8x_1 - 4x_2 - \lambda_1 - 2\lambda_2 = 0$.

$-4x_1 + 4x_2 - \lambda_1 + \lambda_2 = 0$

$(+) \quad (-) \quad (+) \quad (-)$

$12x_1 - 8x_2 - 3\lambda_2 = 0$.

$$\begin{aligned} x_1 + \cancel{x_2} + x_3 &= 10 \\ 2x_1 - \cancel{x_2} + 2x_3 &= 20 \\ \hline 3x_1 + 3x_3 &= 30 \end{aligned}$$

$$\begin{aligned} \cancel{2x_1} + 2x_2 + \cancel{2x_3} - 30 &= 0 \\ -\cancel{2x_1} + x_2 - \cancel{2x_3} + 20 &= 0 \\ \hline 3x_2 - 10 &= 0 \end{aligned}$$

$$3x_2 = 10$$

$$\boxed{x_2 = 10/3}$$

$$(4x_2 - 4x_1 - \lambda_1 + \lambda_2 = 0) \times 2.$$

$$-4x_2 + 8x_1 - \lambda_1 - 2\lambda_2 = 0$$

$$\begin{aligned} 8x_2 - \cancel{8x_1} - 2\lambda_1 + \cancel{2\lambda_2} &= 0 \\ -4x_2 + \cancel{8x_1} - \lambda_1 - \cancel{2\lambda_2} &= 0 \\ \hline 4x_2 - 3\lambda_1 &= 0. \end{aligned}$$

$$40/3 - 3\lambda_1 = 0.$$

$$\boxed{\lambda_1 = \frac{40}{9}}$$

$x_1 = 11/3$, $x_2 = 10/3$, $x_3 = 8$, $\lambda_1 = \frac{40}{9}$, $\lambda_2 = \frac{52}{9}$

Next to determine whether the following stationary pt is maxima or minima, the following Hessian bordered matrix can be computed / calculated are as follows:-

$$H^B = \left[\begin{array}{c|c} 0 & P \\ \hline P^T & Q \end{array}\right]$$

$$H^B = \left[\begin{array}{cc|ccc} 0 & 0^{2\times 2} & \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \frac{\partial g_1}{\partial x_3} \\ 0 & 0 & \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} & \frac{\partial g_2}{\partial x_3} \\ \hline \multicolumn{2}{c|}{P^T} & \multicolumn{3}{c}{\frac{\partial^2 L}{\partial x_i \partial x_j} =} \end{array}\right]$$

$$\begin{matrix} \frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \partial x_2} & \frac{\partial^2 L}{\partial x_1 \partial x_3} \\ \frac{\partial^2 L}{\partial x_2 \partial x_1} & \frac{\partial^2 L}{\partial x_2^2} & \frac{\partial^2 L}{\partial x_2 \partial x_3} \\ \frac{\partial^2 L}{\partial x_3 \partial x_1} & \frac{\partial^2 L}{\partial x_3 \partial x_2} & \frac{\partial^2 L}{\partial x_3^2} \end{matrix}$$

$$= \left[\begin{array}{cc|ccc} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & -1 & 2 \\ \hline 1 & 2 & 8 & -4 & 0 \\ 1 & -1 & -4 & 4 & 0 \\ 1 & 2 & 0 & 0 & 2 \end{array}\right]$$

$n = 3$ (variable) $m = 2$ (eqn)

$n - m = 1.$

$2m + 1 = 5.$

There means the only principle minor of H^B is of order 5. need to be calculated.

For maximization, the sign should be [illegible]

For minimization, the sign should be [illegible]

$$= -1\left[1(0) - 2(0)\right] + 2\left[1(0-8) - 2(0-4) - 4(2+1)\right]$$

[illegible] Calculation [illegible]

$H^B = [illegible] > 0$.

So it minimizes the objective function.

$Z_{min} = (\quad)$

Q/ Optimize $z = 4x_1 + 9x_2 - x_1^2 - x_2^2$

$4x_1 + 3x_2 = 15$

$3x_1 + 5x_2 = 14$.

$x_1, x_2 \geq 0$.

Sol^n

$$L(x, \lambda) = 4x_1 + 9x_2 - x_1^2 - x_2^2 - \lambda_1(4x_1 + 3x_2 - 15) - \lambda_2(3x_1 + 5x_2 - 14).$$

The necessary conditions for the maxima & minima of the objective function -

$$\frac{\partial L}{\partial x_1} = 4 - 2x_1 - 4\lambda_1 - 3\lambda_2 \quad \text{—— (1)}$$

$$\frac{\partial L}{\partial x_2} = 9 - 2x_2 - 3\lambda_1 - 5\lambda_2 \quad \text{—— (2)}$$

$$\frac{\partial L}{\partial \lambda_1} = -4x_1 - 3x_2 - 15 \quad \text{—— (3)}$$

$$\frac{\partial L}{\partial \lambda_2} = -3x_1 - 5x_2 - 14 \quad \text{—— (4)}$$

eqn (3) & (4).

$x_1 = 3 \qquad x_2 = 1$

From eqn (1) & eq (11) $4 - 6 - 4\lambda_1 - 3\lambda_2 = 0 \Rightarrow -2 - 4\lambda_1 - 3\lambda_2 = 0$

$9 - 2 - 3\lambda_1 - 5\lambda_2 = 0 \Rightarrow 7 - 3\lambda_1 - 5\lambda_2 = 0.$

$\lambda_1 = \frac{31}{11}, \quad \lambda_2 = -\frac{34}{11}$

$$H^B = \left[\begin{array}{c|c} 0 & P \\ \hline P^T & Q \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 0 & 0 & 4 & 3 \\ 0 & 0 & 3 & 5 \\ \hline 4 & 3 & -2 & 0 \\ 3 & 5 & 0 & -2 \end{array}\right]$$

$$= 4\begin{vmatrix} 0 & 0 & 5 \\ 4 & 3 & 0 \\ 3 & 5 & -2 \end{vmatrix} - 3\begin{vmatrix} 0 & 0 & 3 \\ 4 & 3 & -2 \\ 3 & 5 & -2 \end{vmatrix}$$

$= 4 \times 5(20-9) - 3 \times 3(20-9)$

$= 20(11) - 9(11)$

$= 220 - 99.$

$= 121 > 0.$

→ it minimises the objective function.

$x(\min) =$

22/9/18

K.T PROBLEM :— (Kuhn-Tucker condition)

Non-linear programming problem by K.T condition

Consider NLPP having a inequality constraints of the form $\boxed{\text{Max}(z) = f(x)}$

subject to $g(x) \le b_i$

$x \ge 0 \quad i = 1, 2, 3 \dots m.$

Introducing the slack variable s in the form of s^2 so as to ensure that it is always non-negative. The constraint can be modified as

$h(x) + s^2 = 0.$

$\{h(x) = g(x) - b_i \le 0.\}$

The problem can be expressed in the form of Max $z = f(x)$

subject to $h(x) + s^2 = 0.$

$x \ge 0$

Which is an $(N+1)$ variable single equality constraint optimisation problem & can be solved by using Langragian multiplier technique.

$L[x, s, \lambda] = f(x) - \lambda[h(x) + s^2].$

Necessary condition for stationary point -

$$\frac{\partial L}{\partial x_j} = \frac{\partial f}{\partial x_j} - \lambda \frac{\partial h}{\partial x_j} = 0 \quad \text{—— (I)}$$

$$\frac{\partial L}{\partial \lambda} = -[h(x) + s^2] = 0 \quad \text{—— (II)}$$

$$\frac{\partial L}{\partial s} = -2s\lambda = 0 \quad \text{—— (III)}$$

from (III)

$-2s\lambda = 0$.

$\boxed{s = 0 \text{ or } \lambda = 0}$

→ But both cannot act zero simultaneously

→ If $s = 0$, $\frac{\partial L}{\partial \lambda} = 0$ which gives $h(x) = 0$

→ Thus we conclude that either $\boxed{\lambda = 0}$ or $\boxed{h(x) = 0}$

⇒ $\lambda \cdot h(x) = 0$.

→ since $s^2 \geq 0$, $h(x) \leq 0$, $x = 0$.

→ If $\lambda > 0$, $h(x) = 0$.

→ Necessary conditions for the maximisation problem can be summarised.

$$\boxed{\begin{array}{c} \frac{\partial f}{\partial x_j} - \lambda \frac{\partial h}{\partial x_j} = 0 \\ \lambda h(x) = 0 \\ h(x) \leq 0 \\ \lambda \geq 0 \end{array}}$$ {K-T condition}

similar argument ~~hold~~ can hold for minimisation NLP problem.

$$\boxed{\begin{array}{c} \frac{\partial f}{\partial x_j} - \lambda \frac{\partial h}{\partial x_j} = 0 \\ \lambda h(x) = 0 \\ h(x) \geq 0 \\ \lambda \geq 0 \end{array}}$$

for max^m problem, ~~when~~ $f(x)$ is concave $h(x)$ is convex. and min problem both $f(x)$ & $h(x)$ are convex.

Q// Solve the NLPP by KT condition.

Max $z = 4x_1 - x_1^3 + 2x_2$

subject $x_1 + x_2 \leq 1$.

$x_1, x_2 \geq 0$.

Sol^n

$f(x) = 4x_1 - x_1^3 + 2x_2$.

$h(x) = x_1 + x_2 - 1$

The problem is of maximisation, the K-T condition -

$$\frac{\partial f}{\partial x_j} - \lambda \frac{\partial h}{\partial x_j} = 0$$

$\lambda h(x) = 0$.

$h(x) \leq 0$

$\lambda \geq 0$.

$L[x, s, \lambda] = 4x_1 - x_1^3 + 2x_2 - \lambda(x_1 + x_2 - 1)$.

$j = 0, 1$. {two variable}

$$\frac{\partial f}{\partial x_1} - \lambda \frac{\partial h}{\partial x_1} = 0.$$

$$\frac{\partial f}{\partial x_2} - \lambda \frac{\partial h}{\partial x_2} = 0.$$

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$\lambda h(x) = 0$.

$h(x) \le 0$

$\lambda \ge 0$.

$4 - 3x_1^2 - \lambda = 0$ —— (I)

$2 - \lambda = 0$. —— (II)

$\lambda(x_1 + x_2 - 1) = 0$. —— (III)

$x_1 + x_2 - 1 \le 0$. —— (IV)

$\lambda \ge 0$. —— (V)

from (II)

$\boxed{\lambda = 2}$

$(\lambda \ge 0)$

from (III)

$2(x_1 + x_2 - 1) = 0$.

$x_1 + x_2 - 1 = 0$.

This result satisfied condition (IV) & (V)

$4 - 3x_1^2 - 2 = 0 \Rightarrow 2 - 3x_1^2 = 0$.

$\Rightarrow 2 = 3x_1^2$

$2 + 3x_1^2 = 0$. $\Rightarrow \sqrt{2/3} = x_1$

$x_1 = \sqrt{2/3} = 0.81$ $\boxed{x_1 = 0.81}$

$x_2 = 1 - \sqrt{\frac{2}{3}} = 0.19$. $\boxed{x_2 = 0.19}$

It can be observed that $f(x)$ is concave since while $h(x)$ is a convex function (min)

$$\boxed{\begin{matrix} x_1 = 0.81 \\ x_2 = 0.19 \end{matrix}}$$

x_1, x_2 Maximize the objective function.

$Max(x) = 4(0.81) - (0.81)^3 + 2(0.19) = 3.088$

Q. Solve the NLPP by using K.T conditions.

$Max(x) = 2x_1^2 - 7x_2^2 + 12x_1x_2$.

subject to $2x_1 + 5x_2 \le 98$

$x_1, x_2 \ge 0$.

Soln

$f(x) = 2x_1^2 - 7x_2^2 + 12x_1x_2$.

$h(x) = 2x_1 + 5x_2 - 98$

The problem is of maximisation, the K.T condition is

$\frac{\partial f}{\partial x_j} - \frac{\lambda \partial h}{\partial x_j} = 0$.

$\lambda h(x) = 0$.

$h(x) \le 0$.

$\lambda \ge 0$.

$L[x, s, \lambda] = 2x_1^2 - 7x_2^2 + 12x_1x_2 - \lambda[2x_1 + 5x_2 - 98]$

$\frac{\partial f}{\partial x_1} - \frac{\lambda \partial h}{\partial x_1} = 0. \Rightarrow 4x_1 + 12x_2 - \lambda(2) = 0$.

$\frac{\partial f}{\partial x_2} - \frac{\lambda \partial h}{\partial x_2} = 0. \Rightarrow -14x_2 + 12x_1 - \lambda(5) = 0$.

$\lambda h(x) = 0$.

$h(x) \le 0$.

$\lambda \ge 0$.

(I) $4x_1 + 12x_2 - \lambda(2) = 0$ $\times 5$.

(II) $-14x_2 + 12x_1 - \lambda(5) = 0$ $\times 2$

(III) $\lambda(2x_1 + 5x_2 - 98) = 0$.

$2x_1 + 5x_2 - 98 \le 0$.

$\lambda \ge 0$.

$20x_1 + 60x_2 - 10\lambda = 0$

$-20x_1 + 24x_2 - 10\lambda = 0$

(a) (-) (+)

$0x_1 + 36x_2 = 0$

If $\lambda = 0$.

$-4x_1 + 12x_2 = 0$ $12x_1 + 12x_2$

$-14x_2 + 12x_1 = 0$.

$x_1 \neq x_2 = 0$ so it doesn't satisfy.

$2x_1 + 5x_2 - 78 = 0$.

$x_1 = 44$

$x_2 = 2$

$\lambda = 100$

Q11 Max $z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$

$2x_1 + x_2 \leq 5$.

$x_1, x_2 \geq 0$.

21/9/18

NLPP WITH MORE THAN ONE INEQUALITY CONSTRAINTS

Max $z = f(x)$

subject to $g^i(x) \leq b_i$, $i = 1, 2, \ldots, m$.

the constraint eqn can be written in the form

$$h^i(x) = g^i(x) - b^i \leq 0.$$

which can be further modified to equality constraints by introducing slack variables i.e.

$$h^i(x) + s_i^2 = 0 \quad i = 1, 2, \ldots, m.$$

→ formation of Langrangian's function

$$L(x, s, \lambda) = f(x) - \sum_{i=1}^{m} \lambda_i \left[h^i(x) + s_i^2 \right].$$

for maximisation problems, the necessary conditions are as follows.

$$\frac{\partial L}{\partial x_j} = \frac{\partial f(x)}{\partial x_j} - \sum_{i=1}^{m} \lambda_i \frac{\partial h^i(x)}{\partial x_j} = 0.$$

j = no. of variables.

$$\frac{\partial L}{\partial \lambda_i} = -\left[h^i(x) + s_i^2 \right] = 0 \quad — (2)$$

$$\frac{\partial L}{\partial s_i} = -2 s_i \lambda_i = 0 \quad — (3)$$

Solving (2) & (3) in case of use of single inequality constraint

$$\lambda^i h^i(x) = 0 \quad \text{------} \quad (iv)$$
$$h^i(x) \le 0 \quad \text{------} \quad (v)$$
$$\lambda^i \ge 0 \quad \text{------} \quad (vi)$$

$$\boxed{\begin{aligned} & f^i_j(x) - \sum_{i=1}^{m} \lambda^i h^i_j(x) = 0 \\ & \lambda^i h^i(x) = 0 \\ & h^i(x) \le 0 \\ & \lambda^i \ge 0 \end{aligned}}$$ maximization problem.

$i = 1, 2, \dots m$

$j = 1, 2 \dots \dots n$

Similar argument can be done for minimization problem.

For NLPP with more than one equality constraint.

$$\boxed{\begin{aligned} & f^i_j(x) - \sum_{i=1}^{M} \lambda^i h^i_j(x) = 0 \\ & \lambda^i h^i(x) = 0 \\ & h^i(x) \ge 0 \\ & \lambda^i \ge 0 \end{aligned}}$$ minimization problem

Q. Solve the NLPP

$$\text{Max: } 7x_1^2 + 6x_1 + 5x_2^2$$

s.t. $x_1 + 2x_2 \le 10$

$x_1 - 3x_2 \le 9$.

$x_1, x_2 \ge 0$

Sol^n $f(x) = 7x_1^2 + 6x_1 + 5x_2^2$

$h_1(x) =$

$h^1(x) = x_1 + 2x_2 - 10$

$h^2(x) = x_1 - 3x_2 - 9$.

K.T condition for maximization problem

$$f_j^i - \sum_{i=1}^{M} \lambda^i h^i_j(x) = 0$$

$$\lambda^i h^i(x) = 0$$

$$h^i(x) \le 0$$

$$\lambda^i \le 0 .$$

$$L(X, S, \lambda) = 7x_1^2 + 6x_1 + 5x_2^2 - \lambda_1(x_1 + 2x_2 - 10) - \lambda_2(x_1 - 3x_2 - 9).$$

The necessary condition for maximization problem –

$$\frac{\partial L}{\partial x_1} = 14x_1 + 6 - \lambda_1 - \lambda_2 = 0 \quad \text{------} \quad (i)$$

$$\frac{\partial L}{\partial x_2} = 10x_2 - 2\lambda_1 + 3\lambda_2 = 0 \quad \text{------} \quad (ii)$$

$$\frac{\partial L}{\partial \lambda_1} = -(x_1 + 2x_2 + 10) = 0 \quad \text{------} \quad (iii)$$

$$\frac{\partial L}{\partial \lambda_2} = -(x_1 - 3x_2 - 9) = 0 \quad \text{------} \quad (iv)$$

$$\lambda^i h^i(x) = 0 .$$

$$\lambda_1(x_1 + 2x_2 - 10) = 0 \quad \text{---} \quad (iii)$$

$$\lambda_2(x_1 - 3x_2 - 9) = 0 \quad \text{---} \quad (iv)$$

$$x_1 + 2x_2 - 10 \le 0 \quad \text{------} \quad (v)$$

$$x_1 - 3x_2 - 9 \le 0 . \quad \text{------} \quad (vi)$$

$\lambda_1, \lambda_2 \geq 0$.

$\lambda_1 = 0, \lambda_2 = 0$
$\lambda_1 = 0, \lambda_2 \neq 0$
$\lambda_1 \neq 0, \lambda_2 = 0$
$\lambda_1 \neq 0, \lambda_2 \neq 0$

Here we have 2 Lagrangian multiplier λ_1 & λ_2 which can take zero & non zero by using 4 different conditions.

Case-I

$\lambda_1 = 0, \lambda_2 = 0$

$14x_1 + 6 = 0$.

$x_1 = \frac{-6}{14}$

$10x_2 = 0$. 00, -ve

$x_2 = 0$

which is an infeasible soln.

Case II $\lambda_1 = 0, \lambda_2 \neq 0$.

eqn (ii) = 0

if $\lambda_2 \neq 0$ then.

$x_1 - 3x_2 - 9 = 0$

$14x_1 + 6 - \lambda_2 = 0$.

$10x_2 + 3\lambda_2 = 0$.

$x_1 - 3x_2 - 9 = 0$

$x_1 = 9/34$ $x_2 = \frac{-99}{34}$ $\lambda_2 = \frac{165}{17}$.

infeasible

Case III

$\lambda_1 \neq 0, \lambda_2 \neq 0$

$x_1 + 2x_2 - 10 = 0$.

$14x_1 + 6 - \lambda_1 = 0$

$10x_2 - 2\lambda_1 = 0$

$x_1 = 38/33$, $x_2 = \frac{146}{33}$ & $\lambda_3 = \frac{730}{33}$

feasible soln

(put it in objective function)

$Z_{max} =$?

Case IV –

$\lambda_1 \neq 0; \lambda_2 \neq 0$

$x_1 + 2x_2 - 10 = 0$.

$x_1 - 3x_2 - 9 = 0$

$14x_1 + 6 - \lambda_1 - \lambda_2 = 0$
$10x_2 - 2\lambda_1 - 3\lambda_2 = 0$ $\Big\rangle$ $\lambda_1 = ?$ $\lambda_2 = ?$

$x_1 + 2x_2 - 10 = 0$

$x_1 - 3x_2 - 9 = 0$

$x_1 = 48/5$ $x_2 = 1/5$

Max $z = 4(48/5)^2 + 6(48/5) + 5(1/5)^2$

$= \underline{\underline{402.92}}$

more +ve (soln).

Assign

Maximize $Z = 2x_1 + 3x_2 - (x_1^2 + x_2^2)$

$x_1 + x_2 \leq 1$

$2x_1 + 3x_2 \leq 6$

$x_1, x_2 \geq 0$.

Optimize $z = 2x_1 + 3x_2 - (x_1^2 + x_2^2 + x_3^2)$

s.t $x_1 + x_2 \le 1$

$2x_1 + 3x_2 \le 6$

$x_1, x_2 \ge 0$

$H^B = \begin{bmatrix} 0 & P \\ P^T & Q \end{bmatrix}$

If optimiz ques then given in HB find

$n = 3$, $m = 2$

$$\left[\begin{array}{cc|ccc} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 3 & 0 \\ \hline 1 & 2 & -2 & 0 & 0 \\ 1 & 3 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{array}\right]$$

$H^B = -10$

for minimisation

$(-1)^{m+n} = -1$.

Max $z = 2x_1 + 3x_2 - (x_1^2 + x_2^2 + x_3^2)$.

$x_1 + x_2 \le 1$.

$2x_1 + 3x_2 \le 6$.

$x_1, x_2 \ge 0$

Proceed by ~~takin~~ previous problem by using K.T condition.

Q11 Max $z = 4x_1^2 - 6x_1 + 5x_2^2$

$x_1 + 2x_2 \le 1$

$x_1 - 3x_2 \le 9$

$x_1, x_2 \ge 0$

If min $z = 4x_1^2 - 6x_1 + 5x_2^2$.

$x_1 + 2x_2 \ge 1$

$x_1 - 3x_2 \ge 9$

$x_1, x_2 \ge 0$.

25/9/18

Quadratic Programming

Quadratic programming deals with non-linear programming problem of minimising or maximising ~~problem~~ the quadratic objective function subject to the set of a linear inequality constraints.

The quadratic programming problem can be expressed as

$$\text{Maximise } f(x) = \sum_{j=1}^{n} c_j x_j + \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} x_j d_{jk} x_k \quad -(1)$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i$$

$$x_j \geq 0.$$

where $i = 1, 2, \ldots m$.
$j = 1, 2, \ldots n$.

WOLF'S METHOD :-

Wolf's method is an extension of simplex method has been modified to solve the NLPP problem.

I. Convert the inequality constraints to eqn by introducing slack variable i.e s_i^2 & r_i^2 into constraints as discussed earlier.

II. from the Langrangian function and derive K.T condn.

III. Introduce the artifical variable $j = 1, 2 \ldots n$ in the K.T condition & construct objective function of the type

$$\text{Min } x = \sum_{j=1}^{n} A_j$$

IV. Obtain an IBFS to the problem i.e

$$\text{Min } x = \sum_{j=1}^{n} A_j$$

subject to $\sum_{k=1}^{n} a_{ij} d_{jk} + \sum_{i=1}^{m} \lambda_i a_{ij} - u_j + A_j = c_j$

$$\sum_{i=1}^{m} a_{ij} x_j + s_i^2 = b_i \quad i = 1, 2, \ldots m$$

$$\lambda_i, x_j, u_j, s_i, A_j \geq 0.$$

$$\lambda_i s_i = 0.$$

$$\lambda_i x_j = 0$$

Apply 2 phase simplex method to obtain optimal solⁿ to the above problem.

These optimal solⁿ of the given

Solve the QPP by using Wolfe's method.

$$\text{Max } Z = 15x_1 + 30x_2 + 4x_1x_2 - 2x_1^2 - 4x_2^2$$

subject to $x_1 + 2x_2 \le 30$.

$x_1, x_2 \ge 0$.

Soln :- Consider the non-(ve) constraints as inequality constraints & add slack variable to all the inequality constraint. The problem can be converted.

$$\text{Max } Z = 15x_1 + 30x_2 + 4x_1x_2 - 2x_1^2 - 4x_2^2$$

$$x_1 + 2x_2 + s_1^2 = 30$$

$$-x_1 + r_1^2 = 0.$$

$$-x_2 + r_2^2 = 0.$$

$$x_1, x_2, r_1, r_2, s_1 \ge 0.$$

$$L(x_1, x_2, s_1, \lambda_1, \mu_1, \mu_2, r_1, r_2) = -15x_1 + 30x_2 + 4x_1x_2 - 2x_1^2 - 4x_2^2 - \lambda_1(x_1 + 2x_2 + s_1^2 - 30) - \mu_1(-x_1 + r_1^2) - \mu_2(-x_2 + r_2^2).$$

The necessary & sufficient condition for the minimization of L.

$$\frac{\partial L}{\partial x_1} = 15 + 4x_2 - 4x_1 - \lambda_1 + \mu_1 = 0.$$

$$\frac{\partial L}{\partial x_2} = 30 + 4x_1 - 8x_2 - 2\lambda_1 + \mu_2 = 0.$$

$$\frac{\partial L}{\partial \lambda} = x_1 + 2x_2 + s_1^2 = 30 = 0.$$

$$\frac{\partial L}{\partial s_1} = 2\lambda_1 s_1 = 0.$$

$$\frac{\partial L}{\partial \mu_1} = -x_1 + r_1^2 = 0.$$

$$\frac{\partial \lambda}{\partial \mu_2} = -x_2 + r_2^2 = 0.$$

$$\frac{\partial L}{\partial r_1} = 2\mu_1 r_1 = 0.$$

$$\frac{\partial L}{\partial r_2} = 2\mu_2 r_2 = 0.$$

After simplification

$$4x_1 - 4x_2 + \lambda_1 - \mu_1 = 15.$$

$$-4x_1 + 8x_2 + 2\lambda_1 - \mu_2 = 30.$$

$$x_1 + 2x_2 + s_1^2 = 30.$$

$$\lambda_1 s_1 = 0.$$

$$\mu_1 r_1 = \mu_2 r_2 = 0.$$

$$\mu_1 x_1 = \mu_2 x_2 = 0.$$

$$x_1, x_2, \lambda_1, \mu_1, \mu_2, s_1 \ge 0.$$

Now introduce artifical variable A_1 & A_2 in the first 2 constraints and replace s_1^2 by s_1 in third constraints. The modified QPP can be written as following manner.

$$\text{Min } Z = A_1 + A_2.$$

$$4x_1 - 4x_2 + \lambda_1 - \mu_1 + A_1 = 15$$

$$-4x_1 + 8x_2 + 2\lambda_1 - \mu_2 + A_2 = 30.$$

$$x_1 + 2x_2 + s_1 = 30.$$

$$x_1, x_2, \lambda_1, \mu_1, \mu_2, A_1, A_2, s_1 \ge 0.$$

The artifical variable A_1 & A_2 & slack variable s_1 can be taken the initial basic feasible soln

~~$x_1 = x_2 = \lambda_1 = \mu_1 = \mu_2 = 0$.~~

$$x_1 = x_2 = \lambda_1 = \mu_1 = \mu_2 = 0$$

Scanned by CamScanner

$A_1 = 15$, $A_2 = 30$, $s_1 = 30$.

$\text{Min } z = 0x_1 + 0x_2 + 0\lambda_1 + 0\mu_1 + 0\mu_2 + A_1 + A_2$

C_{Bi}	C_j / basis	0 x_1	0 x_2	0 λ_1	0 μ_1	0 μ_2	0 s_1	1 A_1	1 A_2	sol^n	ratio
1	A_1	4	-4	1	-1	0	0	1	0	15	[illegible]
1	A_2	-4	$\boxed{2}$	2	0	-1	0	0	1	30	15
0	s_1	1	2	0	0	0	1	0	0	30	
	z_j	0	4	3	-1	-1	0	1	1		
	$c_j - z_j$	0	-4 ↑	-3	1	1	0	0	0		

entering variable

C_{Bi}	C_j / basis	0 x_1	0 x_2	0 λ_1	0 μ_1	0 μ_2	0 s_1	1 A_1	sol^n	Ratio
1	A_1	2	0	2	-1	-1/2	0	1	30	15
0	x_2	-1/2	1	1/4	0	-1/8	0	0	30/2	-
0	s_1	$\boxed{2}$	0	-1/2	0	1/4	1	0	45/2	45/4 → outgoing
	z_j	2	0	2	-1	-1/2	0	1		
	$c_j - z_j$	-2 ↑	0	-2	1	1/2	0	0		

entering variable.

C_{Bi}	C_j / basis	0 x_1	0 x_2	0 λ_1	0 μ_1	0 μ_2	0 s_1	1 A_1	sol^n	ratio
1	A_1	0	0	$\boxed{5/2}$	-1	-3/4	-1	1	15/2	3 →
0	x_2	0	1	1/8	0	-3/16	-1/4	0	27/2	
0	x_1	1	0	-1/4	0	1/8	1/2	0	45/4	
	z_j	0	0	5/2	-1	-3/4	-1	1		
	$c_j - z_j$	0	0	-5/2 ↑	1	3/4	1	0		

C_{Bi}	C_j / basis	0 x_1	0 x_2	0 λ_1	0 μ_1	0 μ_2	0 s_1	sol^n
0	λ_1	0	0	1	-2/5	-3/10	-2/5	3
0	x_2	0	1	0	1/20	-3/20	-1/5	9
0	x_1	1	0	0	-1/10	-1/20	-1/20	12
	z_j	0	0	0	0	0	0	
	$c_j - z_j$	0	0	0	0	0	0.	

Hence all $c_j - z_j \geq 0$. so the optimality is reached.

$\lambda_1 = 3$, $x_1 = 12$ & $x_2 = 9$.

$\text{Max } z = 15(12) + 30(9) + 4(12)(9) - 2(12)^2 - 4(9)^2$
$= 270.$

Assignment problem

$\text{Max } z = 2x_1 + x_2 - x_1^2$.

subject to $2x_1 + 3x_2 \leq 6$.
$2x_1 + x_2 \leq 4$.
$x_1, x_2 \geq 0$.

7/10/18 (Tuesday)

QUEUING THEORY

Queuing models are those where facilities perform a service. Queuing problem arises when current service rate of facilities short of the current flow rate of customer.

eg- we are very much familiar of long queue for a bus, a movie ticket, bank counters and for various other situations

→ long queue - railway booking office, post office, bank counter particularly in a large cities,

Meaning of Queue Of Waiting line -

Ordinarily the line that forms in front of service facilities is called a queue or waiting line. A queue thus involves arriving costumer or item who wait to be serviced at the facility / which provide the service they want to have.

Customers arrivals → Probable waiting line or queue → getting service → departure of customers

Basic terminology in Queuing Theory :-

① Customer :- the person or unit arriving at a station ... customer is either person, m/c & other item.

② Service Station :-

The point where service is to be provided

③ Waiting time :-

Time a customer spends in the queue before being serviced.

→ Time spent by a customer in the system

$\boxed{\text{Waiting time + service time}}$

→ No. of customer in the system

no. of customer in the queue + no. of customer being service

④ Queue length :-

no. of customer waiting in the queue

⑤ Jockeying :-

Joining the other queue & leaving the 1st one.

⑥ Reneging :-

Joining the queue & leaving it afterwards.

⑦ Balking :-

customers decides not to join the queue.

Q. Queuing System:

System consisting arrival of customer waiting in the queue picked up for service acc. to certain discipline being serviced & the departure of customers.

OBJECT OF QUEUING THEORY :-

- If there are queue then customer have to wait for some time before service. the time lost in waiting is expensive in terms of money, equipment & etc.
- At such conditions, there are cost associated with waiting in line commonly known as waiting time cost.
- In other word, if there are no queues member of service station stay idle & these prove burdensome.
- Cost associated with service or facility are known as service cost.
- The object of queuing theory is to achieve a good economic balance b/w these two type of cost & optimum solⁿ is arrived at a pt. where the sum of the waiting time & service cost is minⁿ.
- In other word, the object of any queuing problem is to minimize the total waiting & service cost.

CHARACTERISTICS OF QUEUING THEORY :-

- → Avg. arrival rate
- → Avg. service rate
- → Avg. length of queue
- → Avg. waiting time
- → Avg. time spent in the system

ELEMENTS OF QUEUING SYSTEM :-

1. I/p process / arrivals.
2. Service mechanism
3. Queue discipline
4. O/p of the queue.

I/p process —

The customer arrives at a service station for service. They do not come at regular interval but arrivals into the system occurs acc. to some chance mechanisms.

The arrival occur at random & independent of what has previously occured.

The arrival occur at a constant rate or may be with some probability distribution such as poisson's distribution, normal distribution etc. in connection with the i/p process the following terms:-

1. Arrival distribution
2. Interarrival "
3. Mean arrival rate / avg. no of customer arriving in 1 unit of time (λ)

④ Mean time b/w arrival $\left(\frac{1}{\lambda}\right)$.

② Service Mechanism :- 1/11/18 (Thursday)

Service Mechanism concerns itself with the service time & the service ~~time~~ facility.

→ Ⓐ Single channel facility :-

no. of queues - 1.

1 service station facility, this means that there is only one queue in which the customer waits till the service pt. is ready to take them for servicing.

→ 1 queue i.e several station facility -

In this case costumer waits in single queue until one of the service station is ready to take them servicing.

→ Several queues (1 service station) -

In such situation, there are several queues & the costumer can join any one of this but the service station is only one.

→ Ⓑ Multiple channel facility :-

Many service station are required for getting service.

→ Multistage channel facility :-

In this case, costumer requires several types of services & different service station are there. Each situation providing a specialised service and the costumer passes through each of the service station before leaving station.

~~service~~

The following information are obtained for the point of view of queuing theory.

① Distribution of no. of costumer service.

② Distribution of time taken to service customers

③ Avg. no. of customers being serviced in 1 unit of time at a particular pt. of a service station so this can be denoted by the greek letter 'μ'

④ Avg. time taken to service a customer It is represented by $\frac{1}{\mu}$.

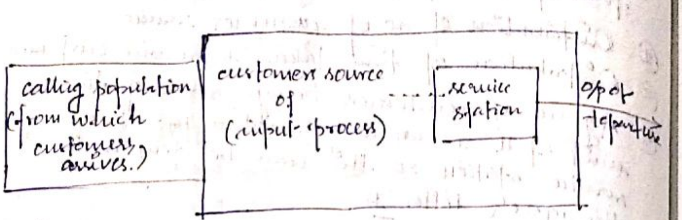
- Queue discipline - may refer to many things one of such things is the order in which the service station selects the next customer from the waiting line to be served.

In this context, queue ~~disciplin~~ discipline may be like LIFO & $\boxed{\text{FIFO}}$.

Output of the Queue :-

In a single channel facilities of, the o/p of the queue does not pose any problem after receiving of the service but the o/p of the queue become important when the system is at multichannel facility. becoz the possibility of the service break down on the queue.

the time before the breakdown to lengthen & the time following the breakdown become diminished. The idea of the queuing system can be represented by the following diagram.



SIMPLE QUEUING MODEL —

Arrival & services occurring following a given time schedule i.e. fixed arrival & fixed service.

Random arrival when there is one service pt or when there are ~~serv~~ several service pt.

Fixed arrival & fixed service —

The simplest situation occurs when arrival occurs at regular specified interval & service is the same length of the time of each interval. When arrival & service time are fixed are same then no backlog occurs.

Random arrivals —

Random arrivals of customers may be from an infinite population or a finite population when there is one service station or multiple service station in such situation ...

possible queuing model defined as follows.

— Assumption of simplest queuing model in case of random arrival.

1. 1 queue and it sufficiently large, ~~1 service~~
2. 1 service stations.
3. queue discipline (FIFO) → (First come first served)
4. Calling population is infinite.
5. Arrival & service of customer take place continuously.
6. Arrival & service occurs in accordance with the poisson process (The arrival are independent of each other service are also independent; & the mean arrivals (λ) & services (μ) do not change over time.
7. The time interval betn arrival. i.e the waiting time betn successive events or what is called interarrival time follow exponential distribution & such mean interval time is represented as $1/\lambda$ (mean inter-arrival time).

Similarly the time interval betn services follow exponential distribution & as such the mean time taken to service a unit is represented as ~~$1/\lambda$~~ $1/\mu$ (mean service rate).

Scanned by CamScanner

Traffic intensity = λ/μ

If it is greater than 1, the queue will grow without end but if the ratio = 1, no change in the queue length & if $\lambda/\mu < 1$, then the length of queue will go on diminishing gradually.

① Expected no. of units (customers) in the waiting line or being serviced i.e. expected no. of customers in

$$\boxed{E(n) = \frac{\lambda}{\mu - \lambda}}$$

② expected no. of units (customers) in the queue.

$$\boxed{E(nq) = \frac{\lambda^2}{\mu(\mu - \lambda)}}$$

③ Avg. waiting time in the queue of an arrival

$$\boxed{E(w) = \frac{E(nq)}{\lambda}}$$

$$\boxed{E(w) = \frac{\lambda}{\mu(\mu - \lambda)}}$$

④ Avg. time an arrivals spends in the system

$$\boxed{E(v) = \frac{1}{\mu - \lambda}}$$

⑤ Probability that the no. of queue of being serviced $n > K$

$$\boxed{P(n > k) = \left(\frac{\lambda}{\mu}\right)^{k+1}}$$

⑥ The expected proportional of time, a facility will be idle

$$\boxed{E(\text{idle}) = 1 - \frac{\lambda}{\mu}}$$

⑦ The probability that at a given time the exact no. of customer say 'n' in the system

$$\boxed{P_r(n) = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)}.$$

~~exact~~ probability of expect exactly (1-λn) customer waiting in the queue.

The probability that waiting time of customer exceed 't'

λ = avg no. of customers arriving in unit of time

μ = avg. no. of customer being serviced in 1 unit of time

n = no. of customer in the system i.e waiting or being serviced.

nq = no. of customer in the queue.

w = time of arrival must wait in the queue.

v = time of arrival must spend in the system

K = 0, 1, 2, 3 . . .

Q. Given the following information arrival of services follow poisson's process. Customer arrive at a rate of 8 per hour. service rate of customer is 10 per hr. Answer the following question.

1. What is the avg. no. of customer waiting for service?
2. What is the avg. time a customer must wait in the queue.
3. What is the avg. time for a customer to be in the system.

Soln

Mean arrival rate of customer λ = 8 per hr.
Mean service rate μ = 10 per hr.

(i) $F(nq) = \frac{\lambda^2}{\mu(\mu-\lambda)}$

$= \frac{(8)^2}{10(10-8)} = \frac{64^{32}}{10(2)} = \underline{3.2}$ hr.

(ii) $E(w) = \frac{E(nq)}{\lambda} = \frac{3.2}{8} = \underline{0.4}$ hr

(iii) $E(v) = \frac{1}{\mu-\lambda} = \frac{1}{10-8} = \frac{1}{2} = 0.5$ hrs.

$= \underline{30 \text{ mins}}$.

Q.

Q. Arrival in an ATM counter are consider poisson with avg. time of 10min betn 2 consecutive arrivals. The length of an ATM cash withdrawals is consider exponential with mean 4 min. Find the avg no. of person waiting in the system.

Soln

Mean inter-arrival time $\frac{1}{\lambda} = 10$ min

$\lambda = \frac{1}{10} = 0.1$ mins.

Mean service rate $= \frac{1}{\mu} = 4$ mins.

$\mu = \frac{1}{4}$ mins $= 0.25$ mins.

$E(n) = \frac{\lambda}{\mu-\lambda} = \frac{0.1}{0.25 - 0.1} = 0.66$ mins.

→ $P(w>0) = 1 - P(w=0)$

$= 1 - P(\text{no. customer})$.

$= 1 - P_0$

$= 1 - \left(1 - \frac{\lambda}{\mu}\right) = \frac{\lambda}{\mu} = \frac{0.1}{0.25} = 0.4$.

→ % time of time an arrival need not wait

$= \frac{1}{4} = 0.25 \times 100 = 25\%$.

→ What is the probability that it will take more than 8 mins altogether to wait for the use of ATM facility & complete its cash withdrawal?

$P(w>8) = e^{-(\mu-\lambda)8}$

$= e^{-(0.25-0.1)8}$

$= 0.3011$

→ Compute the transaction of the day when the ATM counter is found to be busy.

Probability of ATM idle $P(n=0) = P_0$

$= 1 - \frac{\lambda}{\mu}$ arrival time / service time

$= 1 - \frac{0.1}{0.25}$

$= 0.6$.

(7.) 0.6 (0.6×60) probability(ATM busy) $= 1 - P(\text{idle})$.

$= 1 - 0.69$

$= 0.4$.

0.6×60 = 3.6

The time of the day to be busy $= 0.4 \times 24$
$= 9.6$ hrs
$= 9$ hr 36 min

Assignment

(vii) find out the % of customer who have to wait prior to getting into cinema hall?

Q. Customers arrive at a single window cinema hall ticket counter according to poisson process with mean arrival time 20 mins. Customer spend an avg. of 15 min in the ticket customer.

(i) What is the expected no. of customer in cinema ticket counter?

(ii) What is the expected no. of customers in the queue.

(iii) find out the % of time arrival can watch straight into the cinema hall chairs without having to wait.

(iv) What is the time duration that the customer of cinema expect to spent in the cinema hall.

(v) What is the avg. time customers of the cinema halls spend in the queue.

(vi) what is the probability that the waiting time in the system will be more than 20 ...

Q. A certain type of machine breakdown at a avg. rate of 5 hrs. The breakdown are in accordance with poisson process. Cost of idle machine hrs to ₹ 15 per hrs. 2 repairmen A & B have been interviewed, A charge ₹ 8 per hr. & this services breakdown machine at the rate of 7 per hrs. where as B charges ₹ 10 per hrs and this services the ~~same~~ 1st machine at an avg. rate of 9 per hour. Which repairmen service should be used and why?

Soln

Repairmen-A

Mean arrival rate (λ) $\boxed{₹5 \text{ per hrs.}}$

cost of idle m/c = ₹ 15 "

Mean service rate (μ) = ₹ 7 "

Hour to be charged = ₹ 8 "

expected no. of breakdown of m/c $E(n)$

$$\frac{\lambda}{\mu - \lambda} = \frac{5}{7-5} = \frac{5}{2} = 2.5$$

Total no. of machine require $- 2.5 \times 8 = 20$ m/c hr.

Higher charges of repairmen $= 8 \times 8 = ₹64$.

Total cost of idle Machine $= 20 \times 15 = 300$

Total cost of 1st repairmen $= 300 + 64$
$= ₹364$

Repairmen-B

Mean service rate = ₹ 9.

Hour to be charged = ₹ 10

$$E(n) = \frac{\lambda}{\mu - \lambda} = \frac{5}{9-5} = 5/4 = 1.25$$

there case loss of 1.25 m/c in an hrs in chart

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Total no of machine required per hour = 1.25 × 8 = 10.

total cost of second repairment = 8 × 10 + 10 × 15
= 80 + 150
= ₹230

= (Higher charge of repairmen + total cost of idle m/c)

Q// A telephone exchange receive 1 call for every 5 mins and connect 1 call for every 2 mins. If the rate of arrival is poisson distribution & service time follow exponential

(a) find out expected waiting time for a call

(b) find out the expected time in the system.

(c) what is the expected no. of customer in the system.

(d) what is the expected no. of customer in the queue.

Soln

$\lambda = 1/5$ per min

$\mu = 1/2$ per min.

(a) $E(nq) = \frac{\lambda}{\mu(\mu-\lambda)}$

(b) $= 1 - \frac{\mu}{\lambda}$

(c) $= \frac{\lambda}{\mu-\lambda}$

(d) $= \frac{\lambda^2}{\mu(\mu-\lambda)}$

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(i) $\frac{\lambda}{\mu-\lambda}$

(ii) $\frac{\lambda^2}{\mu(\mu-\lambda)}$

(iii) $P(0) = 1 - \lambda/\mu$

(iv) $\frac{1}{\mu-\lambda}$

(v) $E(nq) = \frac{\lambda^2}{\mu(\mu-\lambda)}$

(vi) $e^{-(\mu-\lambda)t}$

(vii) $P(w>0) = 1 - P(w=0) = 1 - (1 - \lambda/\mu) = \lambda/\mu$.

Q// A certain petrol pumps customer arrives in a poisson process in an avg. time of 5 min bet^n arrival. The time interval betn services at the petrol pump follow exponential distribution and as such the mean time taken to service a unit is 2 min. On the basis of this information you are required to answer the following question.

(i) what would be the expected avg. queue length?

(ii) what would be the avg. no. of customers in the queue system?

(iii) How long on avg. a customer does wait in the queue?

(iv) How much time on an avg. a customer does spent in this system.

(v) By how much should the flow of customer to increase the justify the opening of second service point if the management is willing to open the same provided the customer has to wait for 5 min for the service.

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Sol^n Mean arrival rate/min $= \frac{1}{5} = 0.2$

/hr $= 0.2 \times 60 = 12$

service rate/min $\mu = 1/2 = 0.5$

/hr $= 0.5 \times 60 = 30$

(i) Expected avg. queue length $= \frac{\lambda^2}{\mu(\mu-\lambda)}$

(ii) $\frac{\lambda}{\mu-\lambda} = \frac{12}{30-12} = \frac{12}{18} = 2/3 = 0.66$.

(iii) Avg. waiting time $= \frac{\lambda}{\mu(\mu-\lambda)}$

(iv) $E(V) = \frac{1}{\mu-\lambda}$

(v) $E(W) = \frac{\lambda'}{\mu(\mu-\lambda')}$

$E(W) = 1/12$

• $\frac{1}{12} = \frac{\lambda'}{30(30-\lambda')} \Rightarrow 30(30-\lambda') = 12\lambda'$

$\Rightarrow 900 - 30\lambda' = 12\lambda'$

$\Rightarrow 900 = 42\lambda'$

$\Rightarrow \lambda' = \frac{900}{42} =$

Q. Given the following information.

Mean arrival rate = 4 patient/hr.

" service rate = 5 patient/hr.

Concerning patient plant operates 20 per/hrs a day. Cost per reach for doct because of waiting is 20. find out the avg. cost per day for waiting.

Impact of increasing the mean service rate to 6 per hr on the avg. cost per day from waiting.

The saving in daily cost as a result of μ' becoming 6 per hr.

Probability that no. of customer in the queue and being serviced greater than '3' & greater than '4'.

Probability that the no. of customer in the system at any point of time is 4.

The probability that the no. of customer in the system at any point is 4.

Probability that the no. of customers in the queue is exactly.

METHOD :-

method consist the service as consisting of k-phases. each phase taking an avg. time $\frac{1}{K}$ where as the avg. service time is $\frac{1}{\mu}$. so it is assume that actual time taken in each phase of the service is accordance of exponential function distribution. But the mean time remains same i.e $\frac{1}{\mu}$ ... — method various will as follows.

(i) expected no. of units in the system

$$\boxed{E(n) = \frac{k+1}{2k}\cdot\frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu}}$$

(ii) expected no. of units in the queue

$$\boxed{E(nq) = \frac{k+1}{2k}\,\frac{\lambda^2}{\mu(\mu-\lambda)}}$$

(iii) the avg. waiting time in the queue of an arrival

$$\boxed{E(w) = \frac{k+1}{2k}\,\frac{\lambda}{\mu(\mu-\lambda)}}$$

(iv) the avg. time an arrival spends in the system

$$\boxed{E(v) = \frac{k+1}{2k}\,\frac{\lambda}{\mu(\mu-\lambda)} + \frac{1}{\mu}}$$