MODULE – IV

- THREE DIMENSIONAL GEOMETRY
- VECTORS
- DETERMINANTS AND MATRICES
- PROBABILITY AND STATISTICS

CHAPTER 15

THREE DIMENSIONAL GEOMETRY

1. Distance Formula

The distance between the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

2. Division Formula

If R (x, y, z) divides join of

$P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$

internally in the ratio K : 1, then

$$x = \frac{Kx_2 - x_1}{K - 1}, y = \frac{Ky_2 + y_1}{K + 1}, z = \frac{Kz_2 + z_1}{k + 1}$$

If R (x, y, z) divides join of $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ externally in the ratio K : 1, then

$$x = \frac{Kx_2 - x_1}{K - 1}, \frac{Ky_2 - y_1}{K - 1}, z = \frac{Kz_2 - z_1}{K - 1}$$

The mid point of the line segment PQ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

3. Direction cosines and Direction ratios

If a line L makes angles α, β γ with + ve direction of X–, Y–, Z–axes respectively, then α, β γ are called he direction angles of L.

cos α, cos β, cos γ are the direction cosine of L.

$l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$

The numbers which are proportional to the d.c. of a line are called its direction ratios. *i.e.*

if $\frac{a}{l} = \frac{b}{m} = \frac{c}{n}$, then *a*, *b*, *c* are called the d.r.s, of L.

Note.

(i) $l^2 + m^2 + n^2 = 1$

(ii) D.C.s. of a line are unique.

(iii) D.r.s. of a line are infinitely many.

(iv) If < a, b, c > are direction ratios of a line then its direction cosines are

$$< \frac{a}{\pm\sqrt{a^2+b^2+c^2}}, \frac{b}{\pm\sqrt{a^2+b^2+c^2}}, \frac{c}{\pm\sqrt{a^2+b^2+c^2}} >$$

(v) If P (x, y, z) is any point on the line OP whose dcs are $<l, m, n>$ and $|OP| = r$ then $x = lr$, $y = mr$, $z = nr$.

(vi) The d. r. s of the line joining P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) are $<x_2-x_1, y_2-y_1, z_2-z_1>$.

Angle between two lines

Angle between two lines whose dcs are $<l_1, m_1, n_1>$ and $<l_2, m_2, n_2>$ is given by

$$\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$$

If $<a_1, b_1, c_1>$ and $<a_2, b_2, c_2>$ are their d.r.s, then

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Note. (i) Two lines are|| el $\Leftrightarrow \frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(iii) Two lines are $\perp$ r $\Leftrightarrow l_1l_2 + m_1m_2 + n_1n_2 = 0$
or $a_1a_2 + b_1b_2 + c_1c_2 = 0$

THE PLANE

Definition

A plane is a surface where the line joining any two points on it lies totally on the surface.

Equation :

1. One point form

The equation of a plane through (x_0, y_0, z_0) whose normal has *dcs*, $<l, m, n>$ is $l(x-x_0) + m(y-y_0) + n(z-z_0) = 0$. If $<a, b, c>$ be drs of normal then the equation is $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

2. Three point form

The plane through (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) has equation

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0.$$

3. Normal form

Theequation of plane in normal form is $lx + my + nz = P$.

Where <*l*, *m*, *n*> be *dcs* of normal and P is the perpendicular distance of the plane from origin.

4. Intercept form

The equation of plane in intercept form is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. Where $a = x$– intercept, $b = y$ – intercept, $c = z$ – intercept.

5. General form

The general form of equation of a plane is $Ax + By + Cz + D = 0$.

Where < A, B, C > are *drs* of normal to the plane.

Properties

6. Angle between two planes.

$A_1x + B_1y + C_1z + D_1 = 0$

$A_2x + B_2y + C_2z + D_2 = 0$ is θ which can be found out from

$$\cos\theta = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}}$$

Note: These two planes are parallel if $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$, ⊥r if $A_1A_2 + B_1B_2 + C_1C_2 = 0$ and identical if $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} = \frac{D_1}{D_2}$

7. The normal form of $Ax + By + Cz + D = 0$ is

$$\frac{A}{\pm\sqrt{A^2 + B^2 + C^2}}x + \frac{B}{\pm\sqrt{A^2 + B^2 + C^2}}y + \frac{C}{\pm\sqrt{A^2 + B^2 + C^2}}z = \frac{-D}{\pm\sqrt{A^2 + B^2 + C^2}}$$

Where one of signs + or – will be taken to make R.H.S. (+ve).

8. Distance of a plane $Ax + By + Cz + D = 0$ from any point (x_0, y_0, z_0) is

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\pm\sqrt{A^2 + B^2 + C^2}}$$

One will choose sign to make d (+ve).

9. Equation of planes bisecting the angle between two planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ are given by

$$\frac{A_1x+B_1y+C_1z+D_1}{\sqrt{A_1^2+B_1^2+C_1^2}}=\pm\frac{A_2x+B_2y+C_2z+D_2}{\sqrt{A_2^2+B_2^2+C_2^2}}$$

The straight line

A straight line in space is the set of all points of intersection of two planes.

Equation

1. **General equation**

   The equation of line obtained by intersection of two planes is the joint equation of these planes i.e. $a_1x + b_1y + c_1z + d_1 = 0$

   $a_2x + b_2y + c_zz + d_2 = 0$

2. **Symmetrical form (one point form)**

   The equation of line through (x_0, y_0, z_0) whose *drs* are $<a, b, c>$ is

$$\frac{x-x_0}{a}=\frac{y-y_0}{b}=\frac{z-z_0}{c}$$

3. **Two point form**

   The equation of line through (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\dfrac{x-x_1}{x_2-x_1}=\dfrac{y-y_1}{y_2-y_1}=\dfrac{z-z_1}{z_2-z_1}$

Properties

1. The line $\dfrac{x-x_0}{a}=\dfrac{y-y_0}{b}=\dfrac{z-z_0}{c}$ will lie on the plane $Ax + By + Cz + D = 0$ if

   (i) $Aa + Bb + Cc = 0$

   (ii) $Ax_0 + By_0 + Cz_0 + D = 0$

2. Two lines $\dfrac{x-x_1}{a_1}=\dfrac{y-y_1}{b_1}=\dfrac{z-z_1}{c_1}$ and

   $\dfrac{x-x_2}{a_2}=\dfrac{y-y_2}{b_2}=\dfrac{z-z_2}{c_2}$ are coplaner if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$

CHAPTER 16

VECTORS

Vector is a physical quantity which has both magnitude and direction.

Scalar is a physical quantity which has only magnitude.

Types of vectors

Unit Vector : A vector whose magnitude is unity is called unit vector.

Zero Vector : A vector whose magnitude is zero and direction is arbitrary is called a zero vector or a null vector. It is denoted by $\vec{0}$

Proper vector : A vector whose magnitude is not zero is called a proper vector.

Parallel vectors : Two vectors $\vec{a}$ and $\vec{b}$ are said to be paralle if there exists a scalar k such that $\vec{a} = k\vec{b}$.

Like vectors : Two vectors are said to be like vectors if they are parallel and have the same direction.

Unlike vectors : Two vectors are said to be unlike vectors if they are parallel and have the opposite directions.

Collinear vectors : Two vectors are said to be collinear if they lie on one line.

coinitial vectors : Vectors having the same initial point are called co-initial vectors.

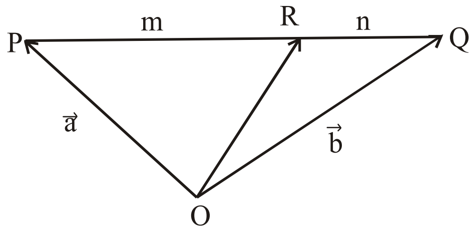
Coplaner vectors : Vectors are said to be coplanar if they lie on one plane.

Negative of a vector : A vector which has the same magnitude as the vector $\vec{a}$ but opposite direrction is called the negative of $\vec{a}$. It is denoted by $-\vec{a}$.

Position vector of a point : If 0 be a fixed point and P be any other point, then the vector $\vec{op}$ is defined as the position vector of P with respect to 0.

Note : If P and Q are two points whose position vectors are $\vec{a}$ and $\vec{b}$ respectively then $\vec{PQ} = \vec{OQ} - \vec{OP} = \vec{b} - \vec{a}$·

Note



If P and Q are two points whose position vectors are $\vec{a}$ and $\vec{b}$ respectively, then the position vector of a point R which divides the line segment joining P and Q internally in the ratio m : n is given by $\vec{OR} = \dfrac{m\vec{b}+n\vec{a}}{m+n}$

Note : If R is the middle point of PQ then the position vector of the point R is given by $\vec{OR} = \dfrac{\vec{a}+\vec{b}}{2}$

Note :

1. Position vector of a point p (x,y) w.r. to origin in coordinate plane is given by

   $\vec{OP} = x\hat{i} + y\hat{j}$

   Where $\hat{i}$ + $\hat{j}$ are unit vectors along x-axis, y-axis, respectively.

2. Position vector of a point P (x, y, z) w.r. to origin in space in given by

   $\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$

   Where $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit vectors along x-axis, y-axis, z-axis respectively.

3. If P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) are two points, then

   $\vec{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

Magnitude and Direction of a Vector : Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ be a vector. Then magnitude of $\vec{a}$ is defined by $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

Direction cosines of $\vec{a}$ are given by

$$\cos\alpha = \frac{a_1}{\sqrt{a_1^2 + a_2^2 + a_3^2}}, \cos\beta = \frac{a_2}{\sqrt{a_1^2 + a_2^2 + a_3^2}}, \cos\gamma = \frac{a_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$$

Note : Unit vector along a vector $\vec{a}$ is denoted by $\hat{a}$ and is given by $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

Algebra of vectors :

1. Addition

$[a_1, a_2, a_3] + [b_1, b_2, b_3] = [a_1 + b_1, a_2 + b_2, a_3 + b_3]$

2. Subtraction

$[a_1, a_2, a_3] - [b_1, b_2, b_3] = [a_1 - b_1, a_2 - b_2, a_3 - b_3]$

3. Multiplication

(I) Scalar Multiplication

$K [a_1, a_2, a_3] = [ka_1, ka_2, ka_3]$

(II) Scalar Product or Dot product

Definition : The scalar product of the vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a} \cdot \vec{b}$ and is defined by

$$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$$

Where θ is the angle between $\vec{a}$ and $\vec{b}$.

Note : $\vec{a}$ is ⊥ r to $\vec{b} \Leftrightarrow \vec{a}.\vec{b} = 0$

Note

$\hat{i}.\hat{i} = 1$	$\hat{i}.\hat{j} = 0$
$\hat{j}.\hat{j} = 1$	$\hat{j}.\hat{k} = 0$
$\hat{k}.\hat{k} = 1$	$\hat{k}.\hat{i} = 0$

Properties of dot product

1. Dot Product is Commutative.

   i.e. $\vec{a} . \vec{b} = \vec{b} . \vec{a}$

2. Dot product is distributive over vector addition.

   i.e. $\vec{a} .(\vec{b} + \vec{c}) = \vec{a} . \vec{b} + \vec{a} . \vec{c}$

Note : If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then $\vec{a} . \vec{b}$

$= a_1b_1 + a_2b_2 + a_3b_3.$

Note: 1. Scalar projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a} . \vec{b}}{|\vec{b}|}$

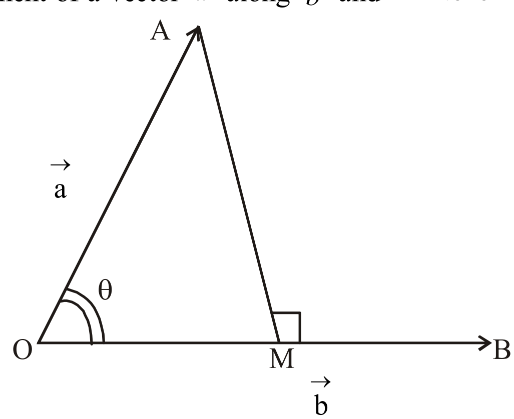
2. Vector projection of $\vec{a}$ on $\vec{b} = \left(\dfrac{\vec{a} . \vec{b}}{|\vec{b}|}\right)\hat{b}$

3. Scalar projection of $\vec{b}$ on $\vec{a}$ = $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

4. Vector projection of $\vec{b}$ on $\vec{a}$ = $\left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\hat{a}$

Note : Component of a vector along and perpendicular to a vector.

1. Component of a vector $\vec{a}$ along $\vec{b}$ and ⊥ r to $\vec{b}$



Let $\vec{a}$ and $\vec{b}$ are represented by $\vec{OA}$ and $\vec{OB}$ respectively.

Let θ be the angle between $\vec{a}$ and $\vec{b}$.

AM ⊥ r to OB is drawn.

$\vec{OM}$ is component of $\vec{a}$ along $\vec{b}$.

$\vec{MA}$ is component of $\vec{a}$ ⊥ r to $\vec{b}$.

$\vec{OM} = |\vec{OM}|\,\hat{b}$

$= OM\,\hat{b}$

$= OA\cos\theta\,\hat{b} \quad \left(\because \cos\theta = \dfrac{OM}{OA}\right)$

$= |\vec{a}|\cos\theta\,\hat{b}$

$= \dfrac{|\vec{a}||\vec{b}|\cos\theta}{|\vec{b}|}\,\hat{b}$

$$= \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \right) \hat{b}$$

So, component of $\vec{a}$ along $\vec{b}$ $= \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \right) \hat{b}$

$$\vec{MA} = \vec{OA} - \vec{OM}$$

$$= \vec{a} - \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \right) \hat{b}$$

So, component of $\vec{a} \perp r$ to $\vec{b}$

$$= \vec{a} - \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \right) \hat{b}$$

Note Component of $\vec{b}$ along $\vec{a}$ $= \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \right) \hat{a}$

Component of $\vec{b} \perp r$ to $\vec{a} = \vec{b} - \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \right) \hat{a}$

Note : $\vec{a}^{\,2} = \vec{a} \cdot \vec{a} = |\vec{a}|^2$

(III) Vector Product or Cross Product

Definition :

The Vector product of the vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a} \times \vec{b}$ and is defined by

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \, \hat{n}$$

Where θ is the angle between $\vec{a}$ and $\vec{b}$ $\hat{n}$ is the unit vector $\perp r$ to the plane containing $\vec{a}$ and $\vec{b}$.

Note : $\vec{a}$ is parallel to $\vec{b} \Leftrightarrow \vec{a} \times \vec{b} = \vec{0}$

Note :

$\hat{i}\times\hat{i}=\vec{0}$	$\hat{i}\times\hat{j}=\hat{k}$	$\hat{j}\times\hat{i}=-\hat{k}$
$\hat{j}\times\hat{j}=\vec{0}$	$\hat{j}\times\hat{k}=\hat{i}$	$\hat{k}\times\hat{j}=-\hat{i}$
$\hat{k}\times\hat{k}=\vec{0}$	$\hat{k}\times\hat{i}=\hat{j}$	$\hat{i}\times\hat{k}=-\hat{j}$

Note : If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ then

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

Properties of Cross Product

1. Cross Product is not Commutative.

   i.e. $\vec{a}\times\vec{b}=-\vec{b}\times\vec{a}$

2. Cross Product is distributive over vector addition. i.e.

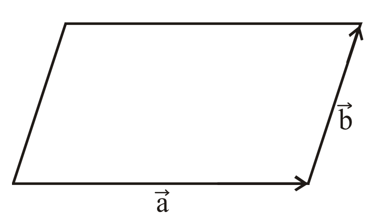
   $\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}$

   $(\vec{a}+\vec{b})\times\vec{c}=\vec{a}\times\vec{c}+\vec{b}\times\vec{c}$

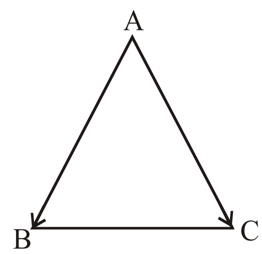
Note : Area of a parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$ $=|\vec{a}\times\vec{b}|$

Note : Area of a triangle ABC

$$=\frac{1}{2}|\vec{AB}\times\vec{AC}|$$



Note : The unit vector perpendicular to the vectors $\vec{a}$ and $\vec{b}$ $=\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$



(IV) Scalar Triple Product

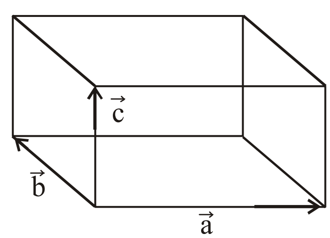
Definition : The scalar triple product of $[\vec{a}, \vec{b}, \vec{c}] = \vec{a}.(\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}).\vec{c}$

Note : $\vec{a}, \vec{b}, \vec{c}$ are coplaner $\Leftrightarrow [\vec{a}, \vec{b}, \vec{c}] = 0$

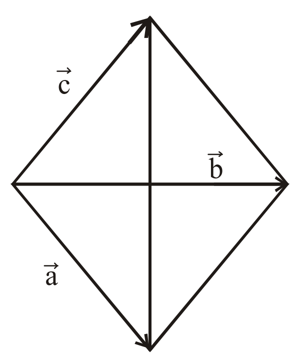
Note : If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then $[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Note : Volume of a parallelopiped whose edges are $\vec{a}, \vec{b}, \vec{c} = [\vec{a}, \vec{b}, \vec{c}]$



Note : Volume a tetrahedron whose sides are $\vec{a}, \vec{b}, \vec{c} = \frac{1}{6}[\vec{a}, \vec{b}, \vec{c}]$



Properties :

(i) If any two of the three vectors $\vec{a}, \vec{b}, \vec{c}$ are equal, then $[\vec{a}, \vec{b}, \vec{c}] = 0$

(ii) If any two of the three vectors $\vec{a}, \vec{b}, \vec{c}$ are parallel, then $[\vec{a}, \vec{b}, \vec{c}] = 0$

(iii) $[\vec{a}, \vec{b}, \vec{c} + \vec{d}] = [\vec{a}, \vec{b}, \vec{c}] + [\vec{a}, \vec{b}, \vec{d}]$

(iv) $[\vec{a}, k\vec{b}, \vec{c}] = k[\vec{a}, \vec{b}, \vec{c}]$

(v) **Vector Triple Product :**

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} . \vec{c})\vec{b} - (\vec{a} . \vec{b})\vec{c}$$

12. The vector equation of a straight line :

(i) The vector equation of a straight line passing through a point with position vector $\vec{a}$ and parallel to a vector $\vec{b}$ is $\vec{r} = \vec{a} + t\vec{b}$, where t is a parameter.

(ii) The equation of a straight line through two points with position vectors $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + t(\vec{b} - \vec{a})$.

(iii) Equation of a straight line through a point with position vector $\vec{a}$ and perpendicular to two non-parallel vectors $\vec{b}$ and $\vec{c}$ is $\vec{r} = \vec{a} + t(\vec{b} \times \vec{a})\cdot$

13. The vector equation of plane :

(i) The vector equation of plane through a point $\vec{a}$ and perpendicular to n is $(\vec{r} - \vec{a}).\hat{n} = 0\cdot$

(ii) The equation of plane through a point $\vec{a}$ and parallel to non-parallel vectors $\vec{b}$ and $\vec{c}$ is $\vec{r} = \vec{a} + t\vec{b} + s\vec{c}$, where t and s are parameters.

(iii) Equation of the plane passing through the points $\vec{a}, \vec{b}$ and parallel to $\vec{c}$ is $\vec{r} = (1-t)\vec{a} + t\vec{b} + s\vec{c}$

(iv) Equation of the plane through three non collinear points $\vec{a}, \vec{b}, \vec{c}$ is $\vec{r} = (1-s-t)\vec{a} + t\vec{b} + s\vec{c}$

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CHAPTER 17

DETERMINANTS AND MATRICES

Determinants

$$|a| = a$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_2(b_2c_3 - b_3c_2) - b_1 (a_2c_3 - a_3c_2) + c_1 (a_2b_3 - a_3b_2)$$

Properties

1. The values of determinant remains unchanged by changing rows to columns and columns to rows.
2. The inter change of two adjacent rows or columns of a determinant changes the sign of determinant without changing numerical value.
3. If two rows or columns of a determinant are identical then the value of the determinant is zero.
4. If every element of any row or column is multiplied by a factor, then the determinant is multiplied by that factor.
5. A determinant remains unchanged by adding K times the elements of any row (or column) to corresponding elements of any other row (or column), where K is any number.
6. If every element of any row (or column) of a determinant be expressed as sum of two numbers then the determinant can be expressed as sum of two determinants.

Minors and Cofactors

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Minor of $a_{ij} = m_{ij}$ = Determinant obtained by omitting *i*th row and *j*th column.

Cofactor of $a_{ij} = c_{ij} = (-1)^{i+j} m_{ij}$

Crammer's Rule

To solve the following system of linear equations :

$a_1x + b_1y + c_1z = K_1$

$a_2x + b_2y + c_2z = K_2$

$a_3x + b_3y + c_3z = K_3$

$$\text{Let}\quad D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix},\ D_x = \begin{vmatrix} K_1 & b_1 & c_1 \\ K_2 & b_2 & c_2 \\ K_3 & b_3 & c_3 \end{vmatrix}$$

$$D_y = \begin{vmatrix} a_1 & K_1 & c_1 \\ a_2 & K_2 & c_2 \\ a_3 & K_3 & c_3 \end{vmatrix}\ D_z = \begin{vmatrix} a_1 & b_1 & K_1 \\ a_2 & b_2 & K_2 \\ a_3 & b_3 & K_3 \end{vmatrix}$$

$$\text{Then } x = \frac{D_x}{D}, y = \frac{D_y}{D}, z = \frac{D_z}{D} \text{ where } D \neq 0$$

Notes.

1. If $D \neq 0$, then the system has one solution.
2. If D = 0 and atleast one of D_1, D_2, D_3 is not zero then the system has no solution.
3. If $D = D_x = D_y = D_z = 0$, then the system has infinite number of solutions.

Consistent system of equations. A system of equations is said to be consistent if it has solution.

In consistent system of equation. A system of equations is said to be inconsistent if it has no solution.

Matrix. A matrix is an arrangement of elements in certain number of rows and in certain number of columns.

Order of matrix. No. of rows X No. of columns is called the order of a matrix

Row matrix. A matrix with a single row is called a row matrix.

Column matrix. A matrix with a single column is called a column matrix.

Square matrix. A matrix in which number of rows is equal to number of columns is called a square matrix.

Diagonal matrix. A square matrix in which the nondiagonal elements are all zero, is called a diagonal matrix.

Scalar matrix. A diagonal matrix in which the diagonal elements are all equal is called a scalar matrix.

Unit Matrix (Identity Matrix). A diagonal matrix in which the diagonal elements are all unity is called a unit matrix.

Zero matrix. A matrix in which the elements are all zero is called a zero matrix

Singular matrix. A square matrix whose determinant value is zero is called a Singular Matrix.

Non-Singular matrix. A square matrix whose determinant value is not zero is called a non-singular matrix.

Transpose of matrix. The transpose of a matrix A is the matrix obtained from A by changing its rows into columns and columns into rows. It is denoted by A' or A^T.

Algebra of Matrices

1. **Addition** If $A = (a_{ij})_{m\times n}$ and $B = (b_{ij})_{m\times n}$ then $A+B = (a_{ij}+b_{ij})_{m\times n}$
2. **Subtraction.** If $A = (a_{ij})_{m\times n}$ and $B = (b_{ij})_{m\times n}$ then $A-B = (a_{ij}-b_{ij})_{m\times n}$
3. **Multiplication**
   (i) Multiplication of matrix by a scalar
   If $A = (a_{ij})_{m\times n}$ and K is a scalar, then $KA = (ka_{ij})_{m\times n}$
   (ii) Multiplication of a matrix by a matrix
   If $A = (a_{ij})_{m\times n}$ and $B = (b_{ij})_{n\times p}$ then
   $AB = (c_{ij})_{m\times p}$ where $C_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$

Equality of two matrices

Two matrices A and B are said to be equal $\Leftrightarrow$

1. Order of A and B are the same.
2. Each element of A is equal to the corresponding element of B.

Adjoint of matrix

The transpose of matrix of cofactors of a matrix A is called the adjioint of A. It is denoted by adj.A.

Inverse of matrix

If A is a non singular matrix, then its inverse matrix

A^{-1} is defined as $A^{-1} = \frac{1}{|A|}$ adj.A.

Where |A| is the determinant of matrix A.

Solution of a system of linear equations

Let the system of linear equations is

$a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

$$\Rightarrow \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$\Rightarrow$ $AX = b$

$\Rightarrow$ $X = A^{-1}b$

$\Rightarrow$ $X = \frac{1}{|A|}$ adj. $A\,b$

Symmetric Matrix:

A square matrix A is said to be a symmetric matrix if $A = A'$

Example : $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

Skew Symmetric Matrix :

A square matrix A is said to be a Skew Symmetric matrix if $A = -A'$.

Example : $\begin{bmatrix} 0 & h & g \\ -h & 0 & f \\ -g & -f & o \end{bmatrix}$

Note: Every square matrix can be uniquely expressed as a sum of symmetric and a skew symmetric matrix.

i.e., $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

Elementary Row Operations:

1. Interchanging two rows.
2. Multiplying a row by a non zero scalar.
3. Adding to a row a scalar times another row.

Note: We can find inverse of a non-singular matrix by using elementary row operations.

CHAPTER 18

PROBABILITY AND STATISTICS

Random experiment : An experiment is called random experiment if its outcomes are uncertain.

Out come : The results of an experiment are called out comes.

Sample space : The set of all possible outcomes of a random experiment is called the sample space. It is denoted by S.

Sample points : An element of a sample space is called a sample point.

Event : Any subset of a sample space is called an event.

Simple event : An event containing only one sample point is called a simple event.

Compound event : An event containing more than one sample point is called a compound event.

Impossible event : An event having no sample point is called an impossible event. It is denoted by ϕ.

Sure event : The whole sample space S is called a sure event.

Mutually exclusive events : Two events are said to be mutually exclusive if they have no common element.

Exhaustive events : The events are said to be exhaustive events if their union is the sample space.

Equally likely outcomes : Two out comes are said to be equally likely if they have equal chance of occurrence.

Probability : The Probability of an event A is denoted by P(A) and is defined by $P(A) = \frac{|A|}{|S|}$, where S is the sample space.

Properties

1. $O \leq P(A) \leq 1$ for any event A.
2. $P(S) = 1, P(\phi) = 0$
3. If $A \subseteq B$, then $P(A) \leq P(B)$
4. $P(A^1) = 1 - P(A)$
5. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
6. $P(A-B) = P(A) - P(A \cap B)$
7. $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$
   $- P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

Equiprobable space : If each sample point in a finite probability space has the same probability, then the probability space is called an equiprobable space.

Non-uniform space : A sample space which is not equiprobable is called a non-uniform space.

Independent events : Two events A and B are independent if $P(A \cap B) = P(A)\,P(B)$.

Conditional Probability : Let B be an event. The conditional probability of B relative to event A is denoted by P (B/A) and is defined by

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

Note : If A and B are independent, then $P(A \cap B) = P(A)\,P(B)$.

If A and B are not independent, then $P(A \cap B) = P(A).\,P(B/A)$

Extension : $P(C \cap B \cap A) = P(A).\,P(B/A).\,P(C / A \cap B)$

The law of total probability : Let S be the sample space and let $E_1, E_2, \ldots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A be any event which occurs with E_1 or E_2 or .... or E_n, then $P(A) = P(E_1)\,P(A / E_1) + P(E_2).\,P(A / E_2) + \ldots + P(E_n).\,P(A / E_n)$

Baye's theorem : Let S be the sample space and $E_1, E_2, \ldots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A be any event which occurs with E_1, or E_2 or .... or E_n then

$$P(E_i / A) = \frac{P(E_i).\,P(A / E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A / E_i)}$$

Where i = 1, 2,.....n.

Random Variable and Binomial Distribution

Definition : A random variable is a numerically valued function defined on the sample space S. It is a rule that assigns a numerical value to each possible outcome of an experiment.

Probability mass function :

Let X be a discrete random variable, which takes the possible values $x_1, x_2, \ldots, x_n$. With each x_i we associate a real number $p_i = P(X=x_i)$, i = 1,2....n satisfying the following conditions :

(i) $P_i \geq 0$ for each i

(ii) $\sum_{i=1}^{n} p_i = 1$

The function $p_i = P(X=x_i)$ is called the probability mass funcion.

The set of all ordered pairs (x,p(x)) is called the probability distribution of X.

Mean and Variance :

Let X be a random variable having the following probability distribution :

x :	$x_1\ x_2\ x_3$ ....x_n
p(x) :	$p_1 p_2 p_3$ ....p_n

Mean of random variable $X = \overline{x} = \sum_{x=1}^{n} x_i p(x_i)$

Variance of random variable $X = \sigma^2 = \sum_{x=1}^{n} (x_i - \overline{x})^2 p(x_i)$

$= = \sum_{i=1}^{n} x_i^2 p(x_i) - (\overline{x})^2$

The positive square root of the variance of X is called the standard deviation of X and is denoted by σ.

Trial : Each time an experiment if performed is a trial.

Bernouli trial : Trials of a random experiment are called Bernoulli trial if the following conditions are satisfied:

(1) The number of trials is finite

(2) Trials are independent

(3) The outcomes are dichotomous (success or failure)

(4) The probability of success (or failure) in each trial is constant.

Binomial Distribution :

If the number of trials is n, the probability of a success in each trial = P and the probability of a failure in each trial = q, then P (r successes in n trials = $^nC_r p^r q^{n-r}$

Mean = $\sum_{r=0}^{n} rp(r) = np$

Variance = Σr^2 p (r) – $(\Sigma rp(r))^2$ = npq

STATISTICS :

Statistics deals with the collection, classification, tabulation, analysis and interpretation of numerical data.

A few terms commonly used in statistics are :

Data : It is a collection of Observations expressed in numerical figures by measuring or counting.

In otherwords, information in numerical terms collected though census or survey or any other method is called **data** or **raw data.**

Population : It is a collection of all possible individuals, objects or measurements.

For example a population might consists of all workers in a plant, all items produced by a machine on a particular day etc.

Sample : It is a part or portion of the population. It is examined with a view to assessing the characteristics of the population.

Variable : A measurable characteristic is called a variable or variate, Examples are age, height, income, weight etc.

Continuous and discrete Variables :

A continuous variable can assume any real value within limits, where as, a discrete variable can assume only isolated values.

For example, the height of a person is a continuous variable, because height can be measured within certain limits. Other examples of continuous variables are weight, temperature, time etc.

Examples of discrete variables are number of workers in a factory, meter reading on a taxi or tempo, prices of articles etc.

Frequency : The number of times a particular value occurs in a given series of observations is called the frequency of that value.

For example, in the raw data 2, 2, 3, 2, 4, 4; 2 occurs 3 times, 4 occurs 2 times, 3 occurs once. Hence, we can say that frequencies of 2, 4 and 3 are respectively 3, 2, 1.

Grouped Frequency Distribution :

It is a statistical table which shows that data has been organised into groups, with their corresponding frequencies.

The following table is a grouped frequency distribution.

Marks	Frequency
0 - 10	4
10 - 20	8
20 - 30	6
30 - 40	5

Mid value or class mark

The mid value of the class interval is the value exactly at the middle of the class interval.

Midvalue of class interval 10 - 20 is $\frac{10+20}{2}=15$.

10 is called lower limit and 20 is called upper limit of the class interval 10 – 20. The difference between lower and upper limit is called class size or class width. In the class interval 10 – 20, class width = 20 – 10 = 10.

In many frequency distribution, the tabulated values shows small frequencies at the beginning and at the end and very high frequencies at the middle of the distribution. This indicates that the typical values of the variable lie near the central part of the distribution and other values cluster around these central values. This phenomena of the data about the concentration of the values in the central part of the distribution is called **central tendency** of data.

The **cumulative frequency (*c.f*)** corresponding to any value is the number of observations, which are less than that value. The *c.f* corresponding to a class is the sum of all frequencies upto and including that class.

MEASURE OF LOCATION OR MEASURE OF CENTRAL TENDENCY

The average of a set of data is that value which gives us an idea about the entire distribution. The type of averages that are incommon use are :

(i) Arithmetic Mean (ii) Geometric Mean
(iii) Harmonic Mean (iv) Median (v) Mode.

(i) Arithmetic Mean or Simply Mean :

The mean $\overline{X}$ of n observations $x_1, x_2, \ldots, x_n$ is defined as

$$\overline{X} = \frac{x_1 + x_2 + x \ldots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Note : $\sum_{i=1}^{n} x_i$ can be writtan as Σx $\quad \therefore \ \overline{X} = \frac{\Sigma x}{n}$

Arithmetic mean in continuous series can be obtained by any of the following methods.

(i) Direct method (ii) Deviation method
(iii) Step deviation method

Direct method,

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \ldots + f_n x_n}{f_1 + f_2 + \ldots + f_n}$$

$$= \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} \text{ or } \frac{\sum fx}{\sum f},$$

where f_i is the corresponding frequency of x_i,
$i = 1, 2, \ldots, n$.

Deviation method :

Let $x_1, x_2, \ldots, x_n$ be the set of n observations and $d_1, d_2, \ldots, d_n$ be the deviations of the observations from any arbitrary value 'a' (called as **assumed mean** or **provisional mean** or **working mean**).

$$\therefore \ d_i = x_i - a \Rightarrow x_i = d_i + a$$

$$\therefore \ \bar{x} = \frac{\Sigma x_i}{n} = \frac{\Sigma(d_i + a)}{n} = \frac{\Sigma d_i + \Sigma a}{n}$$

$$= \frac{\Sigma d_i + na}{n} = a + \frac{\Sigma d_i}{n} \text{ or } a + \frac{\Sigma d}{n}$$

Again, $$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{\sum_{i=1}^{n} f_i (d_i + a)}{\sum_{i=1}^{n} f_i}$$

$$= a + \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i} \quad \text{or} \quad a + \frac{\sum fd}{\sum f}$$

$$\therefore \ \bar{x} = a + \frac{\sum fd}{\sum f}$$

WEIGHTED MEAN

If $x_1, x_2, \ldots, x_n$ be the values of a variable X and $w_1, w_2, \ldots, w_n$ denotes respectively their weights, then their weighted mean $\bar{X}_W$ is given by

$$\bar{X}_W = \frac{x_1 w_1 + x_2 w_2 + \ldots + x_n w_n}{w_1 + w_2 + \ldots + w_n}$$

$$= \frac{\sum_{i=1}^{n} x_i w_i}{\sum_{i=1}^{n} w_i} \ \textbf{or} \ \frac{\Sigma xw}{\Sigma w}$$

SOME IMPORTANT OBSERVATIONS

(1) The sum of the observations is equal to the product of the number of observations and their A.M.

i.e., $\bar{x} = \frac{\sum x}{n} \Rightarrow \sum x = n\,\bar{x}$

(2) The sum of the deviations of the observations from the arithmetic mean is always zero i.e., if $x_1, x_2, \ldots, x_n$ are the n observations and $\bar{x}$ is mean then,

$$\sum_{i=1}^{n}(x_i - \bar{x}) = (x_1 - \bar{x}) + (x_2 - \bar{x}) + \ldots + (x_n - \bar{x})$$

$$= (x_1 + x_2 + \ldots + x_n) - n\bar{x} = n\bar{x} - n\bar{x} = 0$$

(3) If each of the n given observations is doubled, then their mean is doubled.

(4) If $\bar{x}$ is the mean of $x_1, x_2, \ldots, x_n$, then the mean of $ax_1, ax_2, \ldots, ax_n$ where $a \neq 0$ is $a\bar{x}$.

(5) If $\bar{x}$ is the mean of $x_1, x_2, \ldots, x_n$, then mean of $x_1 \pm a, x_2 \pm a, \ldots, x_n \pm a$ is $\bar{x} \pm a$.

(6) If $\bar{x}$ is the mean of $x_1, x_2, \ldots, x_n$, then mean of $\frac{x_1}{a}, \frac{x_2}{a}, \ldots, \frac{x_n}{a}$ is $\frac{\bar{x}}{a}$.

(7) If the mean of n observations $ax_1, ax_2, \ldots, ax_n$ is aX then $(ax_1 - a\text{X}) + (ax_2 - a\text{X}) + \ldots + (ax_n - a\text{X}) = 0$

GEOMETRIC MEAN

The geometric mean (G.M) of n observations $x_1, x_2, ..., x_n$ is defined as the positive nth root of their product i.e.,

G.M = $(x_1 . x_2 . x_3 ... x_n)^{1/n}$, where none of the observations is zero.

In case of grouped data, G.M. of n observation $x_1, x_2......, x_n$ is

G.M. = $\left(x_1^{f_1} \cdot x_2^{f_2} ...... x_n^{f_n}\right)^{1/N}$, where $N = \sum_{i=1}^{n} f_i$.

Properties of Geometric Mean :

1. If the given values of a variable are all equal, then the geometric mean will be equal to their common value.

Let the n values of a variable X are $x_1, x_2, ...x_n$ & each equal to c.

$\therefore$ G.M. = X_G

$$= (c.c.....n\ times)^{\frac{1}{n}} = (c^n)^{\frac{1}{n}} = c$$

For example : G.M. of 7, 7, 7, 7, 7 is $(7^5)^{\frac{1}{5}} = 7$.

2. If G_1 and G_2 are geometric means of two variables X and Y, having m & n observations and G is the geometric mean of the combined series, then

$$G = (x_1 . x_2 .....x_m . y_1 . y_2 ... y_n)^{\frac{1}{m+n}}$$

$$= \left((x_1 . x_2 .....x_m)^{\frac{1}{m}}\right)^{\frac{m}{m+n}} \cdot \left((y_1 . y_2 .....y_n)^{\frac{1}{n}}\right)^{\frac{n}{m+n}} = G_1^{w_1} \cdot G_2^{w_2},$$

where $w_1 = \dfrac{m}{m+n}$ and $w_2 = \dfrac{n}{m+n}$.

3. The geometric mean of the ratio of the two variables X and Y is the ratio of their geometric means.

4. If $y = ax$, then $y_G = ax_G$.

HARMONIC MEAN

The Harmonic mean (H.M) of n observations $x_1, x_2, ..., x_n$ is defined by the total number of observations divided by the sum of their reciprocals i.e.,

$$\text{H.M} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_n}} = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}.$$

If $x_1, x_2,....., x_n$ are n observations which occur with frequencies $f_1, f_2,.....f_n$ respectively,

then H.M. $= \dfrac{\sum_{i=1}^{n} f_i}{\sum_{i=1}^{n} \left(f_i / x_i\right)}$.

Realtion between A.M, G.M and H.M

For a given set of observations,

A.M $\geq$ G.M. $\geq$ H.M.

Here equality sign holds only when all observations are equal.

Proof :

Taking two positive values x_1 and x_2,

we have, $A.M = \dfrac{x_1 + x_2}{2}$, $G.M = \sqrt{x_1 \, . \, x_2}$, $H.M = \dfrac{2}{\dfrac{1}{x_1} + \dfrac{1}{x_2}} = \dfrac{2x_1x_2}{x_1 + x_2}$

Now, $\left(\sqrt{x_1} - \sqrt{x_2}\right)^2 \geq 0$

$\Rightarrow x_1 + x_2 - 2\sqrt{x_1x_2} \geq 0 \Rightarrow \dfrac{x_1 + x_2}{2} \geq \sqrt{x_1 \, . \, x_2}$

i.e. A.M $\geq$ G.M ......(1)

Again, $x_1x_2 - \left(\dfrac{2x_1x_2}{x_1 + x_2}\right)^2$

$$= \frac{x_1x_2(x_1 + x_2)^2 - 4x_1^2 \, x_2^2}{(x_1 + x_2)^2} = \frac{x_1^3x_2 + x_1x_2^3 - 2x_1^2x_2^2}{(x_1 + x_2)^2}$$

$$= \frac{x_1x_2(x_1 - x_2)^2}{(x_1 + x_2)^2} \geq 0$$

$\therefore \; x_1 \, . \, x_2 \geq \left(\dfrac{2x_1x_2}{x_1 + x_2}\right)^2 \Rightarrow \sqrt{x_1 \, . \, x_2} \geq \dfrac{2x_1x_2}{x_1 + x_2}$ i.e., G.M $\geq$ H.M ......(2)

Combining (1) and (2),

we get A.M $\geq$ G.M $\geq$ H.M.

Note : A.M $\times$ H.M = $(G.M)^2$

MEDIAN

If a set of values of a variable are arranged in ascending order or descending order of their magnitudes, then the middlemost observation or the central value is called **median**.

Calculation of median :

(A) **Median of Individual series :**

In case of raw data, if total number of observations is *n,* then

(α) For *n* odd, median = value of $\left(\dfrac{n+1}{2}\right)$th observation

(β) For *n* even, median

$$= \frac{\dfrac{n}{2}\text{th observation} + \left(\dfrac{n}{2} + 1\right)\text{th observation}}{2}$$

Note :

$$\sum_{i=1}^{n}(x_i - A)^2 \text{ is least if } A = \bar{x}$$

and $\sum_{i=1}^{n}|x_i - A|$ is least when A = median.

MODE

The mode of a set of observations is that value, which occurs with maximum frequency.

If each observations occurs equal number of times, the data is said to have no mode. We say that mode is zero or the series have zero mode.

For example, the observation : 3, 5, 3, 5, 3, 5, 3, 5, has no mode.

Calculation of Mode

When all the classes are of equal width, mode is Calculated by the formula

$$\text{Mode} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2}(l_2 - l_1),$$

where l_1 = lower limit of the modal class

(i.e., the class having maximum frequency)

l_2 = upper limit of the modal class

f_1 = frequency corresponding to the modal class

f_0 = frequency of the class preceeding the modal class

f_2 = frequency of the class following the modal class.

MEASURES OF DISPERSION

In order to get a proper idea about the overall nature of a given set of values, it is necessary to know, besides average, the extent to which the given values differe among themselves or equivalently, how they are scattered about the average. This feature of frequency distribution which represents the variability of the given values or reflects how scattered the values are, is called is **dispersion.**

Various measures of dispersion are

(1) Range

(2) Mean deviation

(3) Standand deviation etc.

Range :

It is the difference between the greatest and least of its given set of values.

MEAN DEVIATION

Mean deviation of a set of observations is the arithmetic mean of all the absolute deviations from a fixed value. The fixed value may be mean, median or mode. It is an absolute measure of dispersion.

Mean deviation of n observations $x_1, x_2, ..., x_n$ from their mean is defined by

(i) **Mean deviation (M.D)**

$$= \frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{n} = \frac{\sum_{i=1}^{n} |d_i|}{n} \text{ or } \frac{\sum |d|}{n}$$

where $\bar{x}$ is arithmetic mean, $d_i = x_i - \bar{x}$ is called deviation of x_i from $\bar{x}$.

(ii) **Mean deviation from median (M)** is defined by $\text{M.D.} = \frac{\sum_{i=1}^{n} |x_i - \text{M}|}{n}$

(iii) **For a simple frequency distribution, mean deviation from mean**

$$(\bar{x}) = \frac{\sum f\ |x - \bar{x}|}{\sum f} \text{ or } \frac{\sum f\ |d|}{\sum f},$$

where x = value or mid value according as data is ungrouped or grouped.

(iv) **Mean deviation from median** (M)

$$= \frac{\sum f\ |x - \text{M}|}{\sum f}$$

If 'a' be a fixed real number, we can define **Mean deviation of $x_1, x_2, ..., x_n$ from 'a'** by

$$\text{M.D.} = \frac{\sum_{i=1}^{n} |x_i - a|}{n}$$

If f_i be the corresponding frequencies of x_i $(i = 1, 2, ..., n)$, then **Mean deviation from 'a'** is defined as

$$\text{M.D.} = \frac{\sum_{i=1}^{n} f_i\ |x_i - a|}{\sum_{i=1}^{n} f_i}$$

Note :

(1) If y = a + bx, then M.D of y about its mean = |b| . (M.D. of x about its mean.)

(2) The mean deviation about median is least.

STANDAND DEVIATION

It is defined as

$$\sigma = \left(\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right)^{\frac{1}{2}}, \text{ for raw data}$$

For frequency distribution of x_1, x_2,....., x_n with frequencies $f_1, f_2, ...f_n$ respectively, the standand deviation is given by

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{n} f_i (x_i - X)^2}, \text{ where, } N = \sum_{i=1}^{n} f_i$$

or $$\sigma = \sqrt{\frac{\sum f_i x_i^2}{\Sigma f_i} - \left(\frac{\sum f_i x_i}{\Sigma f_i}\right)^2}$$

In case of grouped frequency distribution,

$$\sigma = C\sqrt{\frac{1}{N}\sum f_i u_i^2 - \left(\frac{1}{N}\sum f_i u_i\right)^2},$$

where $u_i = \frac{1}{C}(x_i - A)$,

$N = \sum_{i=1}^{n} f_i$, A is the working mean & C is the width of the class interval.

Note :

1. If each observation 1, 2, 3, ...,n have frequency 1, then σ can be obtained by using the formula
$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}.$$
2. Standard deviation of first n natural numbers is $\sqrt{\frac{n^2-1}{12}}$.
3. Standard deviation of a, a + d, a + 2d, ..., a + 2nd, d > 0 is $\sqrt{\frac{n(n+1)}{3}}.d$
4. Mean deviation of a, a + d, a + 2d, ..., a + 2nd, d > 0 is $\frac{n(n+1)}{2n+1}.d$
5. Standand deviation a, a +d, a + 2d,... a + (n – 1) d, d > 0 is $\frac{(n^2-1)d^2}{12}$
6. If the values 0, 1, 2, 3, ..., n have frequencies $^nC_0$, $^nC_1$, $^nC_2$, ..., ^nC_n respectively, then standand deviation is $\frac{1}{2}\sqrt{n}$.
7. If the values 0, 1, 2, ..., n have frequencies p^n, $^nC_1 .p^{n-1} q$, $^nC_2 .p^{n-2} q^2$,....., q^n respectively, where p + q = 1, the standand deviation is $\sqrt{npq}$
8. If a variable assumes the values a and b & other (n – 2) values are all equal to $\frac{a+b}{2}$, the S.D of these n observations is $\frac{|a-b|}{\sqrt{2n}}$
9. The standand deviation of first n odd natural numbers is equal to the standand deviation of first n even numbers.

Properties of standand deviation :

1. If all the values of a variable are equal, then standand deviation is **zero**.
2. If y = a + bx is the relation betwen x and y, then $S_y = |b| . S_x$.

Hence the standand deviation is independent of change of origin (a) but depends on change of scale (b).

3. If S.D of $x_1, x_2, ..., x_n$ is σ, then S.D of $x_1 \pm a, x_2 \pm a, ..., x_n \pm a$ is σ.

4. If $y = x - c$ where c is a constant, then S.D of y = S.D of x.

5. S.D of $x_1, x_2, ..., x_n$ is equal to k times the S.D of $\frac{x_1}{k}, \frac{x_2}{k}, ..., \frac{x_n}{k}$.

6. If σ be the S.D of $x_1, x_2, x_3, ..., x_n$, then the S.D of $kx_1, kx_2, ..., kx_n$ is $k\sigma$, $k > 0$.

7. If a group of n_1 observation has mean $\bar{x}_1$ and S.D σ_1 and another group of n_2 observations has mean $\bar{x}_2$ and S.D σ_2, then S.D (σ) of combined group $n_1 + n_2$ (= n) observations is obtained by the formula :

$$\mathbf{n\,\sigma^2 = \left(n_1\sigma_1^2 + n_2\sigma_2^2\right) + \left(n_1 d_1^2 + n_2 d_2^2\right)}$$

where $d_1 = \bar{x}_1 - \bar{x}, d_2 = \bar{x}_2 - \bar{x}$

and $N\,\bar{x} = n_1\bar{x}_1 + n_2\bar{x}_2$.

OR, $\mathbf{n\,\sigma^2 = n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)}$

where $\bar{x}$ is the combined mean,

$$d_1 = \bar{x}_1 - \bar{x},\ d_2 = \bar{x}_2 - \bar{x}.$$

Note : If $n_1 = n_2$ & $\bar{x}_1 = \bar{x}_2$, then $\sigma^2 = \frac{1}{2}\left(\sigma_1^2 + \sigma_2^2\right)$

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