MODULE – II

- RELATION AND FUNCTIONS
- TRIGONOMETRY
- LINEAR EQUATIONS IN TWO VARIABLE
- THE EQUATION OF A LOCUS
- ELEMENTARY ALGEBRA
- NEWTON’S BINOMIAL

CHAPTER 7

RELATION AND FUNCTIONS

7.1 : *Differential Calculus (Functions)*

Quantity : Quantity is that to which operations of Mathematics are applicable. For example : Numbers, time, volume, force, velocity etc., are all quantities.

Types of Quantities :

Let D be any set of real numbers and suppose 'x' is a symbol that, during any mathematical discussion, may be made to stand for any number of D, then x is called a variable and 'D' is called the domain of that variable. On the otherhand if a is '*a*' symbol that can stand precisely for one element of D, then '*a*' is called a constant. It is usual to represent variables by the letters, *x*, *y*, *z* and t etc. and the constants by *a*, *b*, *c*, *l*, m etc.

(a) Constant : Constant is a symbol which remains the same value through out a set of mathematical operations.

Constants are of two types :

(i) Absolute constants

(ii) Arbitrary constants

Absolute Constants : Constants do not change whatever operation we may perform and are known as absolute constants. Example 5, 15, 132, etc.

Arbitrary constants : In the equation $y = ax + b$, of a straight line are arbitrary constants as they remain fixed for particular straight line but vary from straight line to straight line, where 'a' and 'b' are arbitrary constants.

(b) Variable : Variable is a symbol which can take various numerical values.

If a variable can take all the numerical values or numerical values between any two given numbers, then it is said to be a continuous variable,

Variables are of two types : (i) Dependent (ii) Independent variables.

Consider the equation $y = e^x$ in which x can take any value. But for each value of x, there exists a value of y, Here x is called independent variable and y, Whose value depends upon that of x is called dependent variable.

Functions :

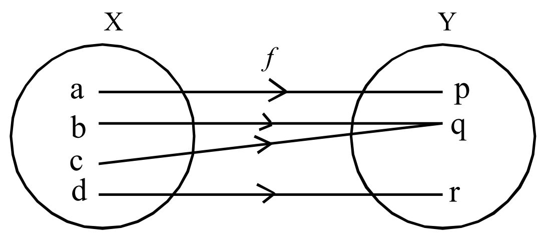
A function is a special case of relation. Let X, Y be two non-empty sets and 'R' be a relation from X to Y then R may not relate an element of X to an element of Y, Or it may relate an element of X to more than one element of Y. But a function relates each element of X to a unique element of Y.

Let X and Y be two non-empty sets. Then a function or mapping 'f' assigned from set X to the set Y is a sort of correspondence which associates to each element $x \in$ X, a unique element y $\in$ Y and is written as

$f : X \to Y$ (read as "f maps X into Y")

The elements y is called the image of x under f and is denoted by $f(x)$ i.e. $y = f(x)$ and x is called pre-image of y.

A function can be represented pictorially as shown in the ***(fig 7.1)*** given below.



(fig 7.1)

It must be noted here (i) that there may be some elements of set Y which are not associated to any elements of set X (ii) that each element of set X must be associated to one and only one element of set Y.

Domain : The set 'X' is called the domain of the function f.

Co-domain : The set 'Y' is called the co-domain of the function f.

Range : The set of all images of the elements of 'X' under the mapping f is called the range of f and is denoted by f(X).

In general, $f(X) \subseteq Y$.

Here $y = f(x)$, where x is called independent variable and 'y' is called dependent variable.

Real valued function of a real variable : A function whose domain and range are subsets of the set R of real numbers is called a real valued function. We will confine ourselves to real valued functions only.

Main features of a function :

(i) To each element $x \in X$, there exists a unique element $y \in Y$ such that $y = f(x)$.

(ii) Distinct elements of X may be associated with the same elements of Y.

(iii) These may be elements of Y which are not associated with any element of X.

Notation used :

If $f : X \to Y$, then domain of f is denoted by D_f. Thus $D_f = \{x : x \in R \text{ s.t } f(x) \in R\}$ and range is denoted by R_f.

Thus $R_f = \{f(x) : x \in D_f\}$

Intervals : Let 'a' and 'b' be two distinct real numbers, $a \neq b$. Let $a < b$. Then

(i) $[a, b] = \{x \in R : a \leq x \leq b\}$ is called closed interval from 'a' to 'b'. i.e. including the end points.

(ii) $(a, b) = \{x \in R : a < x < b\}$ is called open interval from 'a' to 'b'. i.e. excluding the end points.

(iii) $[a, b) = [x \in R : a \le x < b]$ is called semi closed and semi open intervals from 'a' to 'b'.

(iv) $(a, b] = \{x \in R : a < x \le b\}$ is called semi open and semi closed intervals from 'a' to 'b'.

(v) $[a, \infty) = \{x \in R : x \ge a\}$

(vi) $(a, \infty) = \{x \in R : x > a\}$

(vii) $(-\infty, a) = \{x \in R : x < a\}$

(viii) $(-\infty, a] = \{x \in R : x \le a\}$

(ix) $(-\infty, \infty) = \{x \in R\}$

Value of a function : If $y = f(x)$ be any function of x and x = a is admissible value of x, then the value of the function at x = a is obtained simply by replacing x by 'a' in $y = f(x)$ and is $f(a)$.

Equality of functions : Two functions f and g are said to be equal written as $f = g$ if (i) $D_f = D_g$ (ii) $f(x) = g(x)$ for all $x \in D_f$ (or D_g).

Example : (i) $f(x) = x$ and $g(x) = \frac{x^3 + x}{x^2 + 1}$ are equal whereas $f(x) = x$ and $g(x) = \frac{x^2}{x}$ are not equal as $D_g = R - \{0\} \ne D_f$.

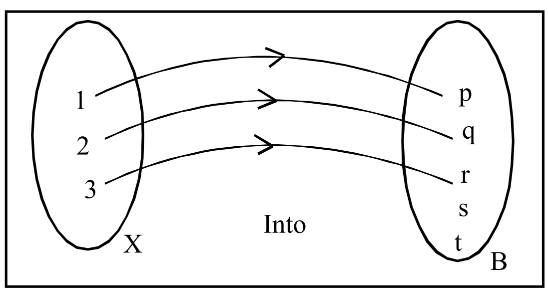
Classifications of Functions :

Into function : A function f defined from the set X to the set Y is said to be **into** function if range of f is a proper subset of in the co-domain Y.

(Or) The function $f : X \to Y$ is called into function if there exist at least one element of Y which does not correspond to any element of X. ***(fig 7.2)***

$X = \{1, 2, 3\}, Y = \{p, q, r, s, t\}$

$f = \{(1, p), (2, q), (3, r)\}$



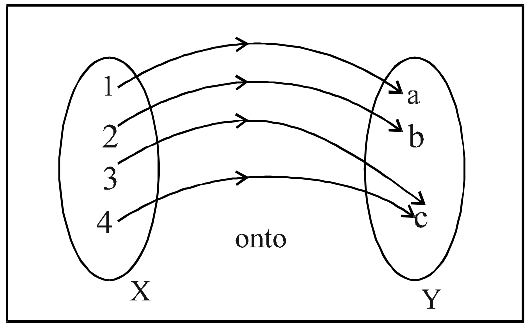
(fig . 7.2)

Onto Function or subjective mapping : A function f defined from the set X to the set Y is said to be an **onto** function if $f(X) = f(x) = Y$ i.e. if the range of f is equal to its co-domain. In other words if every member of Y appears as the image of at least one elements of X, then f is said to be function of X onto Y or f maps X onto Y, f is an onto function.

(Or) A function $f : X \to Y$ is said to be an onto if every element of Y is the image of some element in X, i.e. Y = range of f ***(fig. 7.3)***

$X = \{1, 2, 3, 4\}, \ Y = \{a, b, c\}$

$f = \{(1, a), (2, b), (3, c), (4, c)\}$



(fig. 7.3)

One-one Mapping : A function f defined from the set X to the set Y is said to be an one-one function if $f(x_1) = f(x_2)$, $\Rightarrow x_1 = x_2$, $\forall\ x_1, x_2 \in X$. (fig. 7.4)

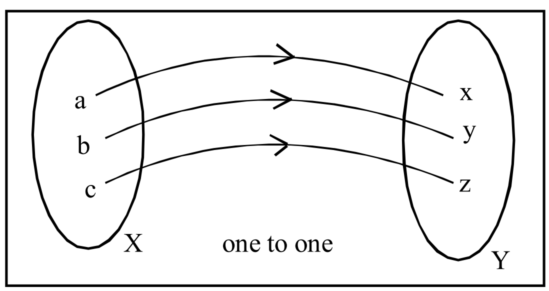
or equivalently,

$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$, $\forall\ x_1, x_2 \in X$.

In other words, if distinct elements of X have distinct image in Y, then the function is called **one-one** function.

X = {a, b, c}, Y = {x, y, z}

f = {(a, x), (b, y), (c, z)}



(fig. 7.4)

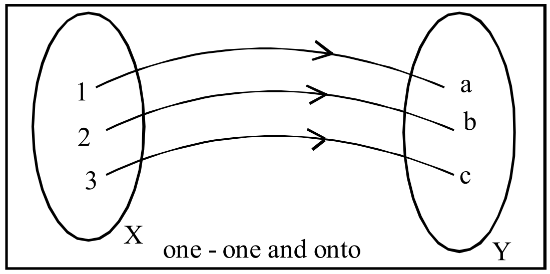
One-One and onto function or Bijection : A function which is such that (i) it is onto and (ii) it is one-one is called a bijection. In other words, a mapping $f : X \rightarrow Y$ is called **one-one and onto (Bijection)** if the following conditions are satisfied.

(i) $f(x_1) = f(x_2) \Rightarrow x_1 = x_2, x_1, x_2 \in X$.

(ii) Given any element $y \in Y$, there exists an element $x \in X$, such that $y = f(x)$ i.e. every element of Y has a pre-image.

(Or) The function $f : X \rightarrow Y$ is called bijective function if f is one -one and also onto. ***(fig. 7.5)***

X = {1, 2, 3}, Y = {a, b, c} f = {(1, a), (2, b), (3, c)}



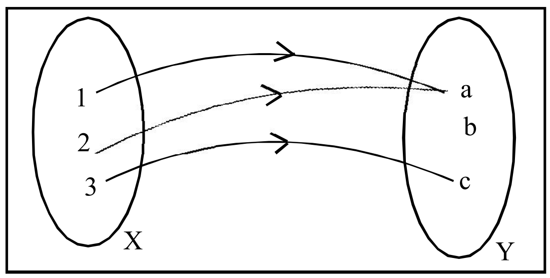
(fig. 7.5)

Note 1 : Every bijective function has it's inverse function.

Note 2 : Every function is a relation but converse is not true.

Many- one Function : A function f from the set X to the set Y is said to be **many-one** if there exist atleast one element in Y which has more than one pre-image in X. ***(fig 7.6)***

X = {1, 2, 3}, Y = {a, b, c} f = {(1, a), (2, a), (3, c)}



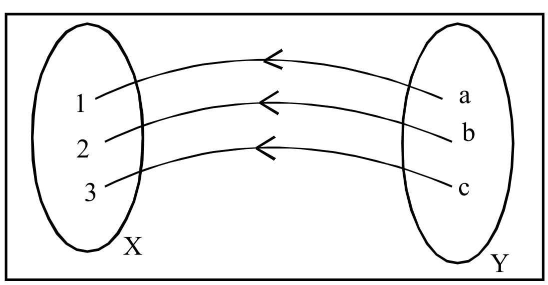
(fig 7.6) **Many one function**

Inverse Image of an element : Let f be a function defined from the set X to the set Y, then the inverse image of an element b $\in$Y under f is denoted by f^{-1}(b) to be read as f inverse b. $\therefore$ f^{-1}(b) = {x : x $\in$ X and f(x) = b} i.e. f^{-1} (b) is the set of all those elements in X which has b as their pre-image.

Inverse function : If f is a bijective function defined from X to Y then it's inverse function is defined from Y to X which is denoted by f^{-1}. ***(fig. 7.7)***

If for $x \in$ X and y $\in$ Y such that y = $f(x)$ then it's inverse function is given by x = f^{-1}(y)

X = {1, 2, 3 }, Y = {a, b, c}



(fig. 7.7)

Here f^{-1}(a) = 1, f^{-1}(b) = 2, f^{-1}(c) = 3

A function which possesses an inverse is called invertible.

Some Results based on it :

If f is one-one and onto function then f^{-1} is also one-one onto (bijective)

Some useful results :

(i) $|x| \leq l \Leftrightarrow -l \leq x \leq l$

(ii) $|x| \geq l \Leftrightarrow x \leq -l \leq$ or $x \geq l$

If a and b $\in$ R and a < b, then

(iii) $(x-a)(x-b) \leq 0 \Leftrightarrow a \leq x \leq b$

(iv) $(x-a)(x-b) \geq 0 \Leftrightarrow x \leq a$ or $x \geq b$.

Useful points to be remembered for finding Domain of a function :

(i) Take the terms under the square root as non-negative if it is in the numerator and positive if it is in the denominator.

(ii) Take the denominator as non-zero.

(iii) Take the logarithm of the number as positive.

Domains and Ranges of real functions :

Generally real functions in calculus are described by some formula are their domains are not explicitly stated. The domain is the set of all real numbers x for which $f(x)$ is a real number.

Example : **Find the domain of the function**

$$f(x) = \frac{x}{x^2 - 3x + 2}$$

Solution : Clearly, $f(x)$ is not defined, when

$x^2 - 3x + 2 = 0$

Now $x^2 - 3x + 2 = 0$

$\Rightarrow (x-1)(x-2) = 0$

$\Rightarrow x = 1, 2$

So, $f(x)$ is not defined for x = 1, 2

Hence, it's domain is R – {1, 2}

Working rule of finding the range of a real function.

(i) Put $f(x) = y$

(ii) End the value of x to obtain $x = \phi(y)$

(iii) Find the values of y for which the values of x obtained from $x = \phi(y)$ are in the domain of f.

(iv) The set of values of y obtained in step (iii) is the range of f.

7.2 : *Graphical representation of functions*

Graph of a function :

(i) Graph of a function denoted by G_f is $\{(x, f(x)) : x \in D_f\}$

(ii) No vertical line can meet the graph of a function in more than one point.

(iii) A function f is one-one if no horizontal line meet the graph of the function in more than one point.

(iv) Graph of f and f^{-1} are images of each other in the line y = x.

Types of functions :

Identity Function : The function f defined by $f(x) = x$ for $\forall\ x \in R$ is called the **identity function** on R.***(fig. 7.8)***

$D_f = R$; $R_f = R$. Its graph is a straight line passing through origin and is bisecting the angle between the axes.'

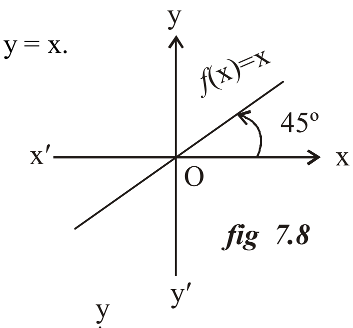


fig 7.8

Constant Function : The function f defined by $f(x) = c$ for all $x \in R$, where c is some real number is called a **constant function**, ***(fig .7.9)***

$D_f = R$, $R_f = \{c\}$. Its graph is a straight line parallel to x-axis at a distance c from it. If c > 0, then its graph is shown in the figure.

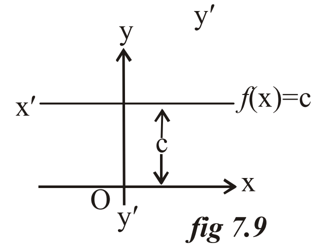


fig 7.9

Reciprocal Function : The function f defined by $f(x) = \frac{1}{x}$ is called **reciprocal function**. ***(fig 7.10)***

$D_f = R - \{0\}$; $R_f = R - \{0\}$. Its graph is as shown in the figure.

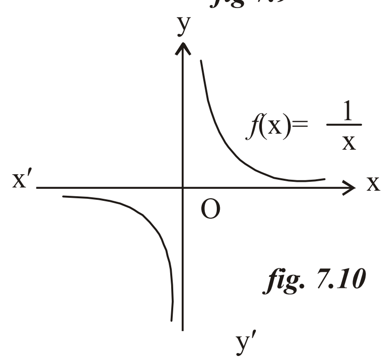


fig. 7.10

Modulus Function : The function f defined by $f(x)$

$$f(x) = |x| = \begin{cases} x \ \ when\, x \geq 0 \\ -x \ \ when\, x < 0 \end{cases}$$

is called the **modulus** or **absolute** value function.

$D_f = R$; $R_f = [0, \infty)$ Its graph is shown in the ***(fig. 7.11)***

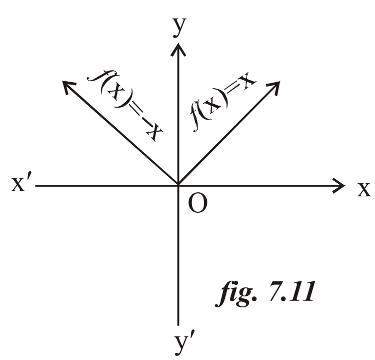


fig. 7.11

Greatest integer or Integral part of a real number : For every $x \in R$, $[x]$ is the greatest integer $\leq x$.

Let n be an integer.

Then $[x] = \begin{cases} n \text{ if } x = n \\ n-1 \text{ if } n-1 \le x < n \end{cases}$

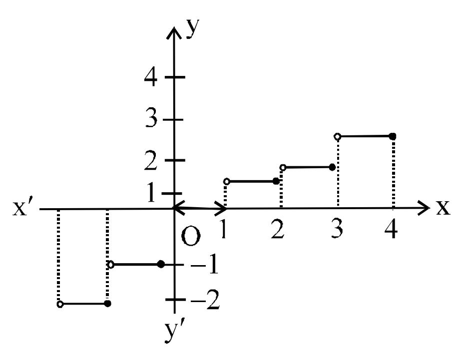
For example :

$[4] = 4$; $[-3] = -3$, $\left[\frac{19}{3}\right] = 6$, $[\sqrt{2}\,] = 1$; $[\sqrt{3}\,] = 1$, $[\pi] = 3$, $\left[-\frac{7}{2}\right] = -4$, $[-e] = -3$.

Greatest Integer Function : The function f defined by $f(x) = [x]$, for all $x \in$ R is called the **greatest integer** function.***(fig. 7.12)***

D_f = R; R_f = I. Its graph is constructed with the help of the following table : $f(x) = [x]$ = k (k ∈ I) iff $k \le x < k+1$.

x	...	$-2 \le x < -1$	$-1 \le x < 0$	$0 \le x < 1$	$1 \le x < 2$	$2 \le x < 3$
y	...	–2	–1	0	1	2



(fig 7.12)

Signum Function : The function f defined by

$$f(x) = \begin{cases} \frac{|x|}{x}, & x \ne 0 \\ 0, & \text{when } x = 0 \end{cases}$$

$$f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

is called the **signum** function

D_f = R; R_f = {–1, 0, 1}

Its graph is shown in the ***(fig. 7.13)***

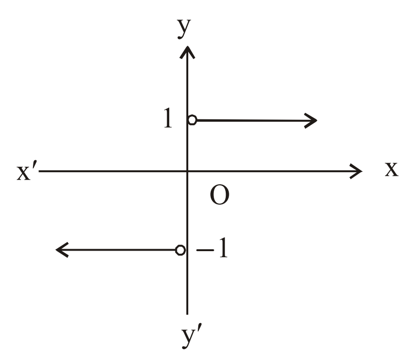


fig. 7.13

Logarithmic Function : The function f defined by $f(x) = \log_a x$, where $x > 0$, $a > 0$, ∀ x, a ∈R is called the **logarithmic** function.

$D_f = (0, \infty)$; $R_f = R$. Its graph is shown in the ***(fig. 7.14 (a) fig. (b))***

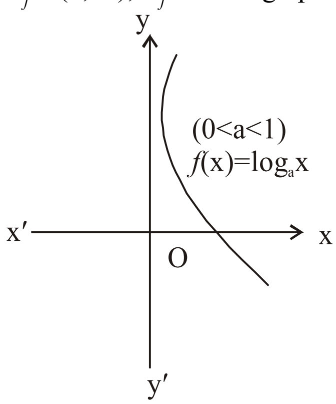


fig 7.14(a)

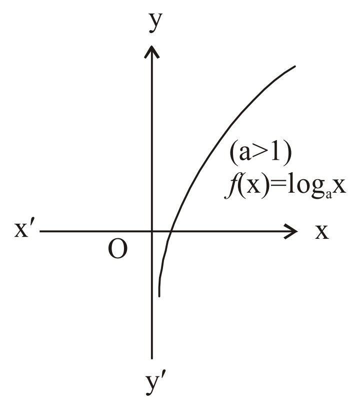


fig 7.14(b)

Exponential Function : The function f defined by $f(x) = a^x$, where $a > 0$ and $x \in R$ is called **exponential** function.

$D_f = R$; $R_f = (0, \infty)$. Its graph is shown in the ***(fig. 4.15 (a) and (b))***

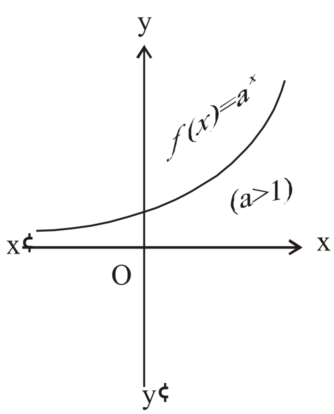


fig 7.15(a)

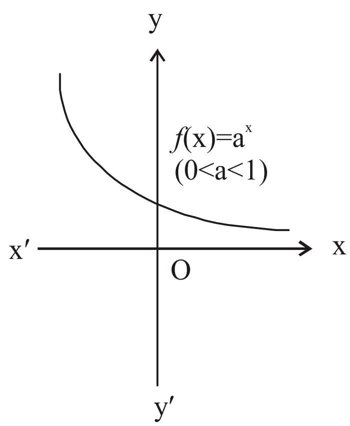


fig 7.15(b)

Square Root Function : The function f defined by $f(x) = \sqrt{x}$ is called the **square root** function,

$D_f = [0, \infty)$; $R_f = [0, \infty)$

Its graph is shown in the ***(fig 7.16)***

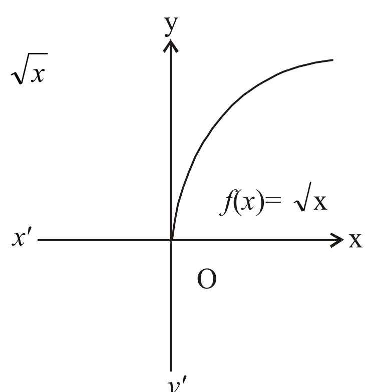


fig 7.16

Polynomial Function : The function f defined by $f(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$, $a_n \neq 0$. Where $a_0, a_1, a_2.....a_n$ are real numbers and $n \in N$ is called a polynomial function of degree n.

$D_f = R$; $R_f = R$

Polynomial of degree 1, 2, 3, 4 are respectively called linear, quadratic, cubic and biquadratic polynomials.

(i) $f(x) = ax + b$, $a \neq 0$ is called a linear polynomials.

(ii) $f(x) = ax^2 + bx + c$, $a \neq 0$ is called a quadratic polynomials.

(iii) $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$ is a cubic polynomial.

Rational Function : The function f defined by $f(x) = \dfrac{g(x)}{h(x)}$, where $g(x)$ and $h(x)$ are polynomial functions and $h(x) \neq 0$ is called a **rational function**.

The domain of f is the set of all real numbers except those values of x, for which $h(x) \neq 0$.

Trigonometric Functions : $\sin x$, $\cos x$, $\tan x$, $\cot x$, $\sec x$, $\mathrm{cosec}\, x$ are trigonometric functions. Also $-1 \leq \sin x \leq 1$, $-1 \leq \cos x \leq 1$, for all values of x. Here x is radian measure of an angle. The following table illustrating the domain and range of each trigonometric function also known as **Circular functions.**

Function	Domain	Range
$\sin x$	R	[–1, 1]
$\cos x$	R	[–1, 1]
$\tan x$	$R - \{(2n+1)\pi/2;\ n \in Z\}$	R
$\cot x$	$R - \{n\pi;\ n \in Z\}$	R
$\sec x$	$R - \{(2n+1)\pi/2;\ n \in Z$	$R - (-1,1)$
$\mathrm{cosec}\, x$	$R - \{n\pi\ ;\ n \in Z\}$	$R - (-1, 1)$

Trigonometrical Functions : (Sine Function) : The function that associates to each real number x to $\sin x$ is called the sine function. Here x is the radian measure of the angle, ***(fig 7.17)***

The domain of the sine function is R and the range is [–1, 1]

i.e. $-1 \leq \sin x \leq 1$

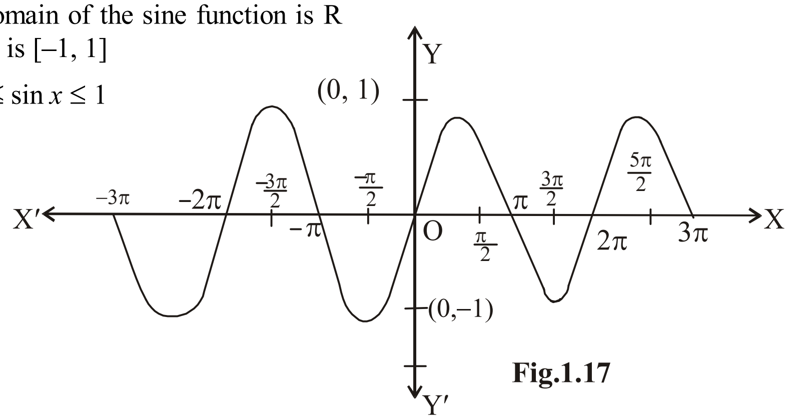
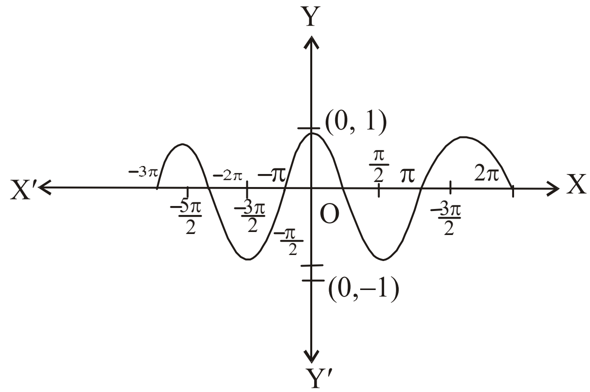


Fig.1.17

Cosine Function : The function that associates to each real number x to $\cos x$ is called the cosine function. Here x is the radian measure of the angle. ***(fig. 7.18)***

The domain of the cosine function is R and range is [–1, 1] i.e. $-1 \leq \cos x \leq 1$



(fig 1.18)

Tangent Function : We have know that tan $x = \dfrac{\sin x}{\cos x}$ is defined when cos $x \neq 0$. And cos x = 0 when x is an odd multiple of $\dfrac{\pi}{2}$ ***.(fig. 1.79)***

So tanx is not defined when x is an odd multiple of $\dfrac{\pi}{2}$.

$\therefore$ Domain = R – $\left\{(2k+1)\dfrac{\pi}{2} : k \in I\right\}$.

since tanx may assume any value, its Range = R

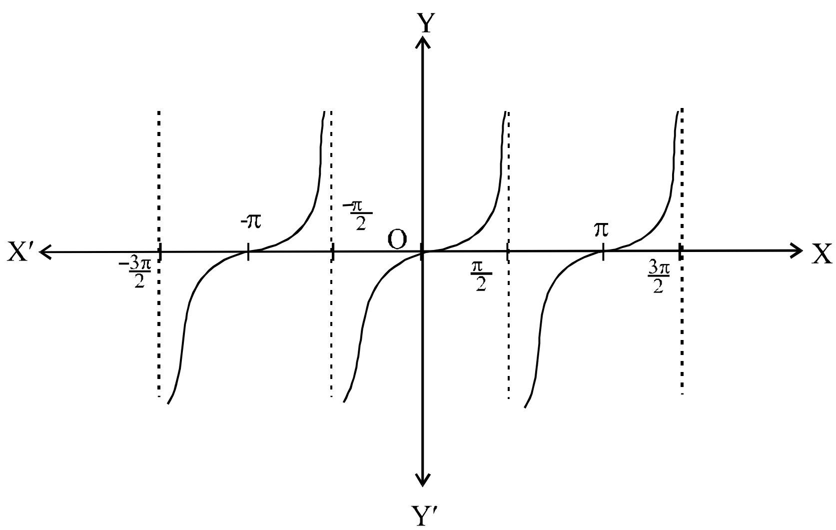


Fig. 7.19

Secant Function : We know that sec $x = \dfrac{1}{\cos x}$ is not defined when cos $x = 0$.

And cos $x \neq 0$ when x is an odd multiple of $\frac{\pi}{2}$.

$\therefore$ secx is not defined, when x is an odd multiple of $\frac{\pi}{2}$. So its domain

$= R - \left\{(2k+1)\frac{\pi}{2} : k \in I\right\}$

As the numerical value of sec x is never less than 1, it follows that its range = R – (–1, 1).

Cotangent Function : We know that

$\cot x = \frac{\cos x}{\sin x}$ is not defined when sinx = 0, $x = k\pi,\ k \in z$

$\therefore$ domain $= R - \{k\pi : k \in z\}$

clearly, its range = R

Cosecant Function : Clearly cosecx = $\frac{1}{\sin x}$ is not defined when sinx = 0

And $\sin x = 0 \Leftrightarrow x = k\pi,\ k \in z$

So its domain $R = \{k\pi, k \in z\}$

and its range = R – (–1, 1)

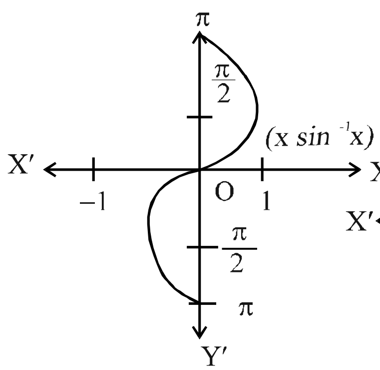
Inverse Trigonometric Functions : The inverse trigonometric functions are $\sin^{-1} x, \cos^{-1} x$, $\tan^{-1}x$, $\operatorname{cosec}^{-1}x$, $\sec^{-1}x$, $\cot^{-1}x$. These are real functions. The domain and range of (confining only principal values) of the inverse trigonometric functions are stated below :

Function	**Domain**	**Range**	**Definition of the function**
$\sin^{-1}x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$y = \sin^{-1}x$ $\Leftrightarrow x = \sin y$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$	$y = \cos^{-1} x$ $\Leftrightarrow x = \cos y$
$\tan^{-1}x$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	$y = \tan^{-1} x$ $\Leftrightarrow x = \tan y$
$\cot^{-1} x$	$(-\infty, \infty)$	$(0, \pi)$	$y = \cot^{-1} x$ $\Leftrightarrow x = \cot y$
$\sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$	$y = \sec^{-1} x$ $\Leftrightarrow x = \sec y$
$\operatorname{cosec}^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$	$y = \operatorname{cosec}^{-1} x$ $\Leftrightarrow x = \operatorname{cosec} y$

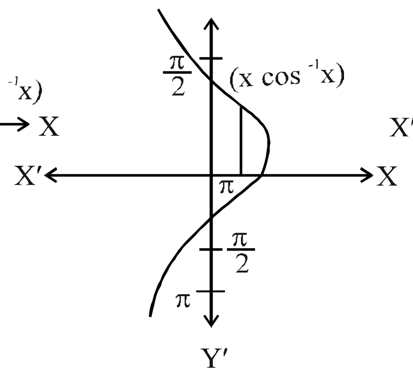
Graphs of Inverse Trigonometric Function :

The equation $y = \sin^{-1}x$ and $x = \sin y$ are some when $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Thus the graphs represented by these two equations are same. Tabulating the values of sines for various values of $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ we represent the graph of $x = \sin y$ as shown in ***(fig. 7.20)***

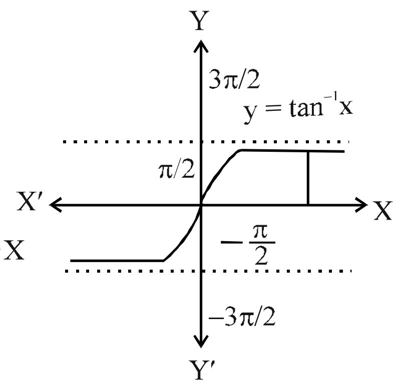
The graphs of $y = \cos^{-1}x$, $y = \tan^{-1}x$, $y = \cot^{-1}x$, $y = \sec^{-1}x$ and $y = \mathrm{cosec}^{-1}x$ are shown in the figures ***Fig 7.21, fig. 7.22, fig 7.23, fig 7.24*** and ***fig 7.25*** respectively.



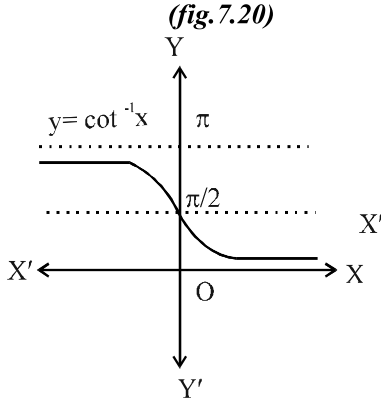
(fig.7.20)



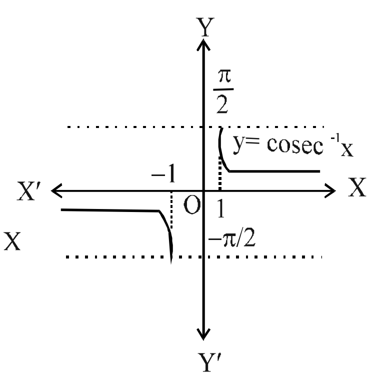
(fig. 7. 21)



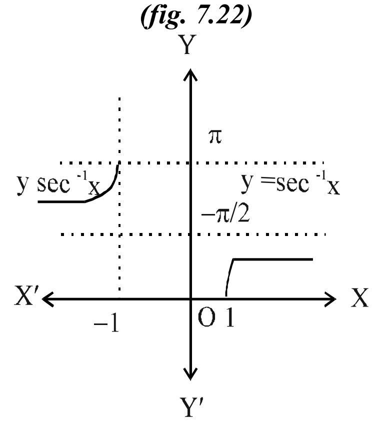
(fig. 7.22)



(fig 7.23)



(fig 7.24)



(fig 7. 25)

Even Function : A Function $f(x)$ is said to be an even function if $f(-x) = f(x)$, for all x. All even powers of x are even function.

Example : $f(x) = \cos x$ is an even function as $f(-x) = \cos(-x) = \cos x = f(x)$.

Odd Function : A function $f(x)$ is said to be odd function if $f(-x) = -f(x)$, for all x. All odd powers of x are odd function.

Example : $f(x) = \sin x$ is an odd function as $f(-x) = \sin(-x) = -\sin x = -f(x)$.

The graph of an even function is always symmetrical about y-axis. The graph of an odd function is always symmetrical in opposite quadrants.

Prepared by ***Dr. C. R. Mallick***
Department of Mathematics, P.M.E.C. Berhampur

Explicitly Function : A function which is expressed directly in terms of independent variable is called an explicit function. e.g. $y = x^2 - 2x + 5$

Implicitly Function : If a function is not expressed directly in terms of independent variable, it is called an Implicit function. In such a case, the relation between dependent and independent variable is expressed in the form $f(x, y) = 0$ and either of the variable is said to be implicit function of the other. e.g. $y = x^2y + xy^2 = 3$

Single valued Function : A function $y = f(x)$ is said to be single valued function if there is one and only value of y corresponding to each value of x.

Periodic Function : A function $f(x)$ is said to be a periodic function if there exists a positive real constant T such that $f(x + T) = f(x)$, for all $x \in R$. If T is the smallest positive real number such that $f(x + T) = f(x)$, for all $x \in R$, then T is called the fundamental period of $f(x)$.

Example : $\sin x$ is periodic function and its period is 2π, whereas $\tan x$ is also a periodic function but its period is π.

Some Standard functions and their periods

Function	Period
$\sin x$	2π
$\cos x$	2π
$\tan x$	π
$\cot x$	π
$\sec x$	2π
$\text{cosec } x$	2π

Function	Period
$x - [x]$	1
$\sin^2 x$, $\cos^2 x$	π
$\lvert\sin x\rvert$, $\lvert\cos x\rvert$	π
$\sin^4 x + \text{cox}^4 x$	$\pi/2$

Some Important Results based on periodic function :

(i) If $f(x)$ is a periodic function with period T and $a, b \in R$ such that $a > 0$, then $f(ax + b)$ is periodic with period $\frac{T}{a}$.

(ii) If $f_1(x), f_2(x)$ and $f_3(x)$ are periodic functions with periods T_1, T_2, and T_3 respectively, then $a_1 f_1(x) + a_2 f_2(x) + a_3 f_3(x)$ is a periodic function with period equal to L.C.M. of T_1, T_2, T_3, where a_1 , a_2, a_3 are nonzero real numbers.

(iii) If $f(x)$ is periodic function with period T and $g(x)$ is any function such that domain of f is a proper subset of g then gof is periodic with period T.

Continuous and Discontinuous Functions : The function $y = f(x)$ is said to be a continuous function of x, if there is no break in its graph, if on the other hand, there is some break in the graph of $y = f(x)$, then it is called discontinuous function of x.

Undefined Functions or Indeterminate forms : Let $y = f(x)$ be a function of x i.e. $y = f(x)$ $= \frac{g(x)}{h(x)}$ (say). Let $x = a$ be any value of x such that.

Case I : $g(a) = 0$, $h(a) = 0$, then the value of y at $x = a$ is of the form $\frac{0}{0}$.

Case II : g(a) = b, h(a) = 0. Then the value of y at x = a is of the form b/0. The value of the form $\frac{0}{0}$ or $\frac{b}{0}$ as in case I and case II have no meaning. These are known as indeterminate forms.

Operations on Functions : Let f and g be two real valued functions with domains D_f and D_g respectively and let $D = D_f \cap D_g \neq 0$, then

(i) **Sum** of f and g denoted by f + g is the function defined by $(f+g)(x) = f(x) + g(x)$ with domain D.

(ii) **Difference** of f and g denoted by f – g is the function defined by $(f - g)(x) = f(x) - g(x)$ with domain D.

(iii) **Product** of f and g denoted by fg is the function defined by $(fg)(x) = f(x)g(x)$ with domain D.

(iv) **Quotient** of f and g denoted by f/g is the function defined by $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ with domain D′ where $D' = \{x : x \in D, g(x) \neq 0\}$.

(v) **Multiplicative inverse** of f written as $\left[\frac{1}{f}\right]$ is defined as $\frac{1}{f}(x) = \frac{1}{f(x)}$, for all $x \in D_f$ excluding those points where $f(x) = 0$.

(vi) **Scalar Multiple of** f : Let c be any real number, c f is called scalar multiple of f by 'c' and is defined as $(cf)(x) = cf(x)$ with domain = D_f.

Remark The above operations are defined here only for real functions. For general function from one set to another, these don't make sense.

Composition of functions : Let f and g be two real valued functions and let $D = \{x : x \in D_f$ and $f(x) \in D_g\} \neq \phi$, then the composite of f and g denoted by $(gof)(x) = g(f(x))$ with domain D. Composite of two functions is also called **resultant** of two functions or **function of a function.** Note that if $R_f \subset D$ then $D_{gof} = D_f$. Similarly $f$og is defined by $(fog)(x) = f[g(x)]$ with domain D. If $R_g \subset D_f$, then $D_{fog} = D_g$.

Example : If $f(x) = \sin x$, $g(x) = x^2$, then $(gof)(x) = g[f(x)] = g(\sin x) = \sin^2 x$, $(fog)(x) = f([g(x)] = f(x^2) = \sin x^2$.

❒ ☸ ❒

CHAPTER 8

TRIGONOMETRY

Definition of Trigonometry : The word trigonometry means the measurement of their sides, angles and various relationship that exists between them.

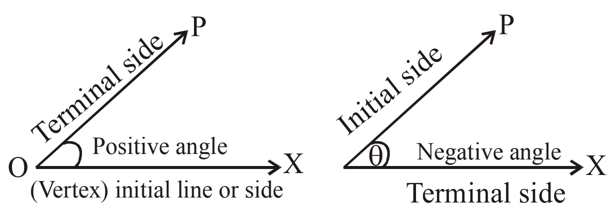
Angle. An angle is thought of as traced out by the rotation of a revoving line from the initial position OX to the terminal position OP.

The point O is called the **vertex**.

The line Ox is called the **initial line.**

OP is called **terminal side**.

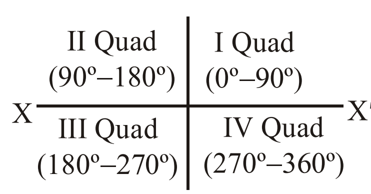
If the line side OX is called the initial line and OP is called revolving or **terminal side.** If the line side OX moves anticlockwise to the terminal side OP from the vertex O, then the angle XOP as shown in Figure (i) is called a positive angl. But if the initial side OX moves in the clockwise direction then the angle XOP is called a **negative angle**.



Quadrants.

The two axes X′ OX and Y′OY divides the plane into four quadrants.

(i) In first quadrant, all trigonometric ratios are positive.



(ii) In second quadrant, only sinθ and cosecθ are positive.

(iii) In third quadrant, only tanθ and cotθ are positive.

(iv) In fourth quadrant, only cosθ and secθ are positive.

Units of Measurement of Angles

(1) Sexagesimal system (English system)

(1) 1 Right angle = 90 degrees (written as 90^o)

(2) 1 Degree = 60 minutes (written as $60'$).

(3) 1 minute = 60 seconds (written as $60''$).

(2) Centesimal system (French system)

1 right angle = 100 grades (written as 100g)

1 grade = 100 minutes (written as $100'$).

1 minute =100 seconds (written as $100''$).

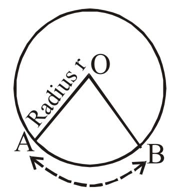
(3) Circular system

In this system the fundamental unit of measurement is a radian.

Radian. An angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.

π radian = 180^o = 2 rt. angles

$$1 \text{ radian} = \frac{180^o}{\pi} = 57.2958^o \text{ nearly}$$



[Radian measures of some commonangles,

Angle in degree 30^o 45^o 60^o 90^o 180^o 270^o 360^o

Angle in radians $\pi/6$ $\pi/4$ $\pi/3$ $\pi/2$ $\pi/3$ $\pi/2$ 2π

Relation between the three systems of measurement of an angle, π radians = 180^o = 200^o.]

Trigonometric Ratios or Functions

$$\sin\theta = \frac{Perpendicular}{Hypotenuse} \qquad \text{cosec}\theta = \frac{Hypotenuse}{Perpendicular}$$

$$\cos\theta = \frac{Base}{Hypotenuse} \qquad \sec\theta = \frac{Hypotenuse}{Base}$$

$$\tan\theta = \frac{Perpendicular}{Base} \qquad \cot\theta = \frac{Base}{Hypotenuse}$$

Trigonometric Ratios

Reciprocal of Six Trigonometric Ratios
Trigonometric Identites
Trigonometric Ratios of the angles $90^o - \theta$
Trigonometric Ratios of $90^o + \theta$
Trigonometric Ratios of $180^o - \theta$
Trigonometric Ratios of $180^o + \theta$
Trigonometric Ratios of Some Specific Angles

} More practice see Chapter – 6

Note : No mistake should be committed by regarding $\sin\theta$ as $(\sin \times \theta)$, $\sin\theta$ is correctly read as the sine of the angle.

$(\sin\theta)^n$ is wirtten as $\sin^n\theta$, if $n \neq -1$

$(\sin\theta)^2 = \sin^2\theta$, $(\cos\theta)^3 = \cos^3\theta$

But $(\sin\theta)^{-1} \neq \sin^{-1}\theta$, $\sin^{-1}\theta \neq \dfrac{1}{\sin\theta}$

1. $\sin^2\theta + \cos^2\theta = 1$; $\sin^2\theta = 1 - \cos^2\theta$; $\cos^2\theta = 1 - \sin^2\theta$

2. $1 + \tan^2\theta = \sec^2\theta$; $\tan^2\theta = \sec^2\theta - 1$

3. $1 + \cot^2\theta = \text{cosec}^2\theta$: $\cot^2\theta = \text{cosec}^2\theta - 1$

The signs of t– ratios can easily be remebered with the help of the rhymes. 'all – sin – tan – cos' or 'all silver tea cups'

4.

$\theta =$	0°	30°	45°	60°	90°
$\sin\theta$	$\sqrt{\frac{0}{4}} = 0$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$	$\sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}} = 1$
$\cos\theta$	$\sqrt{\frac{4}{4}} = 1$	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$	$\sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$	$\sqrt{\frac{0}{4}} = 0$

5. (A)

	$(-\theta)$	$(90^\circ \pm \theta)$	$(180^\circ \pm \theta)$	$(360^\circ \pm \theta)$
sin	$-\sin\theta$	$\cos\theta$	$\mp\sin\theta$	$\pm\sin\theta$
cos	$\cos\theta$	$\mp\sin\theta$	$-\cos\theta$	$\cos\theta$
tan	$-\tan\theta$	$\mp\cot\theta$	$\pm\tan\theta$	$\pm\tan\theta$
cot	$-\cot\theta$	$\mp\tan\theta$	$\pm\cot\theta$	$\pm\cot\theta$
sec	$\sec\theta$	$\mp\text{cosec}\,\theta$	$-\sec\theta$	$\sec\theta$
cosec	$-\text{cosec}\,\theta$	$\sec\theta$	$\mp\text{cosec}\,\theta$	$\pm\text{cosec}\,\theta$

(B) When n is any integer , $n \in z$

$\sin(n\pi + \theta) = (-1)^n \sin\theta$

$\cos(n\pi + \theta) = (-1)^n \cos\theta$

$\tan(n\pi + \theta) = \tan\theta$

When n is odd

$$\sin\left(\frac{n\pi}{2} + \theta\right) = (-1)^{\frac{n-1}{2}} \cos\theta$$

$$\cos\left(\frac{n\pi}{2} + \theta\right) = (-1)^{\frac{n+1}{2}} \sin\theta$$

$$\tan\left(\frac{n\pi}{2} + \theta\right) = -\cot\theta$$

6. Addition Formulae :

(i) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

(ii) cos (A+B) = cos A cos B – sin A sin B

(iii) $\tan (A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(iv) $\tan (A + B + C) = \dfrac{\tan A + \tan B + \tan C - \tan A . \tan B . \tan C}{1 - \tan A . \tan B - \tan B . \tan C - \tan C . \tan A}$

7. Subtraction Formulae :

(i) sin (A–B) = sin A cos B – cos A sin B

(ii) cos (A–B) = cos A cos B + sin A sin B

(iii) $\tan (A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

8. $\sin (A+B) \sin (A - B) = \sin^2 A - \sin^2 B$

9. $\cos (A+B) \cos (A - B) = \cos^2 A - \sin^2 B$

10. (i) $\sin 75^o = \dfrac{\sqrt{3}+1}{2\sqrt{2}} = \cos 15°$ (ii) $\sin 15^o = \dfrac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^o$

(iii) $\tan 75^o = 2 + \sqrt{3} = \cot 15^o$ (iv) $\tan 15^o = 2 - \sqrt{3} = \cot 75^o$

11. The products of sine -cosine ratios as their sum and difference :

(i) 2 sin A cos B = sin (A+B) + sin (A–B)

(ii) 2 cos A sin B = sin (A+B) – sin (A–B)

(iii) 2 cos A cos B = cos (A+B) + cos (A–B)

(iv) 2 sin A sin B = cos (A–B) – cos (A+B)

12. Product Formulae :

(i) $\sin C + \sin D = 2 \sin \left(\dfrac{C+D}{2}\right) \cos\left(\dfrac{C-D}{2}\right)$

(ii) $\sin C - \sin D = 2 \cos \left(\dfrac{C+D}{2}\right) \sin\left(\dfrac{C-D}{2}\right)$

(iii) $\cos C + \cos D = 2 \cos \left(\dfrac{C+D}{2}\right) \cos\left(\dfrac{C-D}{2}\right)$

(iv) $\cos C - \cos D = 2 \sin \left(\dfrac{C+D}{2}\right) \sin\left(\dfrac{D-C}{2}\right)$

13. Trigonometrical Ratios of Multiple Angles :

(i) sin 2A = 2 sin A cos A

(ii) $\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$

(iii) $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$ (iv) $\sin 2A = \dfrac{2 \tan A}{1 + \tan^2 A}$

(v) $\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$ (vi) $\sin A = 2\sin\dfrac{A}{2}\cos\dfrac{A}{2}$

(vii) $\cos A = \cos^2\dfrac{A}{2} - \sin^2\dfrac{A}{2} = 2\cos^2\dfrac{A}{2} - 1 = 1 - 2\sin^2\dfrac{A}{2}$

(viii) $\tan A = \dfrac{2\tan A/2}{1-\tan^2 A/2}$ (ix) $\sin A = \dfrac{2\tan A/2}{1+\tan^2 A/2}$

(x) $\cos A = \dfrac{1-\tan^2 A/2}{1+\tan^2 A/2}$ (xi) $\sin 3A = 3\sin A - 4\sin^3 A$

(xii) $\cos 3A = 4\cos^3 A - 3\cos A$ (xiii) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

14. (i) $\sin\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{2}}$, (ii) $\cos\dfrac{A}{2} = \pm\sqrt{\dfrac{1+\cos A}{2}}$

(iii) $\tan\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{1+\cos A}}$, (iv) $\sin\dfrac{A}{2} + \cos\dfrac{A}{2} = \pm\sqrt{1+\sin A}$

(v) $\sin\dfrac{A}{2} - \cos\dfrac{A}{2} = \pm\sqrt{1-\sin A}$

15. (i) $\sin 22\tfrac{1}{2}^\circ = \sqrt{\dfrac{2-\sqrt{2}}{2}}$, (ii) $\cos 22\tfrac{1}{2}^\circ = \sqrt{\dfrac{2+\sqrt{2}}{2}}$

(iii) $\tan 22\tfrac{1}{2}^\circ = \sqrt{2} - 1$

16. (i) $\sin 18^\circ = \dfrac{\sqrt{5}-1}{4} = \cos 72^\circ$, (ii) $\cos 18^\circ = \dfrac{1}{4}\sqrt{10+2\sqrt{5}} = \sin 72^\circ$

17. (i) $\cos 36^\circ = \dfrac{\sqrt{5}+1}{4} = \sin 72^\circ$, (ii) $\sin 36^\circ = \dfrac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54^\circ$

18. Trigonometrical Equations :

(i) If $\sin\theta = 0$, then $\theta = n\pi,\ n \in z$

(ii) If $\cos\theta = 0$, then $\theta = (2n+1)\dfrac{\pi}{2},\ n \in z$

(iii) If $\tan\theta = 0$, then $\theta = n\pi,\ n \in z$

(iv) If $\sin\theta = \sin\alpha$, then $\theta = n\pi + (-1)^n\alpha,\ n \in z$

(v) If $\cos\theta = \cos\alpha$, then $\theta = 2n\pi \pm \alpha,\ n \in z$

(vi) If $\tan\theta = \tan\alpha$, then $\theta = n\pi + \alpha,\ n \in z$

(vii) If $\sin^2\theta = \sin^2\alpha$; or $\cos^2\theta = \cos^2\alpha$; or $\tan^2\theta = \tan^2\alpha$, then $\theta = n\pi \pm \alpha,\ n \in z$

19. Sine Formulae :

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

20. Cosine Formulae :

(i) $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, (ii) $\cos B = \dfrac{c^2 + a^2 - b^2}{2ca}$

(iii) $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

21. Napier's Analogies :

(i) $\tan\left(\dfrac{A-B}{2}\right) = \dfrac{a-b}{a+b}\cot\dfrac{C}{2}$ (ii) $\tan\left(\dfrac{B-C}{2}\right) = \dfrac{b-c}{b+c}\cot\dfrac{A}{2}$

(iii) $\tan\left(\dfrac{C-A}{2}\right) = \dfrac{c-a}{c+a}\cot\dfrac{B}{2}$

22. Sines of half the angles in terms of the sides :

(i) $\sin\dfrac{A}{2} = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$, i.e. $s = \dfrac{a+b+c}{2}$

(ii) $\sin\dfrac{B}{2} = \sqrt{\dfrac{(s-c)(s-a)}{ca}}$ (iii) $\sin\dfrac{C}{2} = \sqrt{\dfrac{(s-a)(s-b)}{ab}}$

23. Cosines of half of the angles in terms of the sides :

(i) $\cos\dfrac{A}{2} = \sqrt{\dfrac{s(s-a)}{bc}}$ (ii) $\cos\dfrac{B}{2} = \sqrt{\dfrac{s(s-b)}{ca}}$

(iii) $\cos\dfrac{C}{2} = \sqrt{\dfrac{s(s-c)}{ab}}$

24. Tangents of half the angles in terms of the sides ;

(i) $\tan\dfrac{A}{2} = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$, (ii) $\tan\dfrac{B}{2} = \sqrt{\dfrac{(s-c)(s-a)}{s(s-b)}}$

(iii) $\tan\dfrac{C}{2} = \sqrt{\dfrac{(s-a)(s-b)}{s(s-c)}}$

25. Properties of triangles;

(i) $\Delta = \frac{1}{2}ab \sin C = \frac{1}{2}bc \sin A = \frac{1}{2} ca \sin B$

(ii) $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ (iii) $R = \dfrac{a}{2\sin A} = \dfrac{b}{2\sin B} = \dfrac{c}{2\sin C}$

(iv) $R = \dfrac{abc}{4\Delta}$

(v) $r = \dfrac{\Delta}{s} = (s-a)\tan\dfrac{A}{2} = (s-b)\tan\dfrac{B}{2} = (s-c)\tan\dfrac{C}{2}$

(vi) $r_1 = \dfrac{\Delta}{s-a}$

$r_2 = \dfrac{\Delta}{s-b}$

$r_3 = \dfrac{\Delta}{s-c}$

(vii) $r_1 = s\tan\dfrac{A}{2}$

$r_2 = s\tan\dfrac{B}{2}$

$r_3 = s\tan\dfrac{C}{2}$

26. Trigonometrical Inverse Functions :

(i) $\sin^{-1} x = \operatorname{cosec}^{-1}\dfrac{1}{x}$; $\operatorname{cosec}^{-1} x = \sin^{-1}\dfrac{1}{x}$

$\cos^{-1}x = \sec^{-1}\dfrac{1}{x}$; $\sec^{-1} x = \cos^{-1}\dfrac{1}{x}$

$\tan^{-1} x = \cot^{-1}\dfrac{1}{x}$; $\cot^{-1} x = \tan^{-1}\dfrac{1}{x}$

(ii) $\sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}$

$\tan^{-1}x + \cot^{-1}x = \dfrac{\pi}{2}$

$\operatorname{cosec}^{-1}x + \sec^{-1}x = \dfrac{\pi}{2}$

(iii) $\sin^{-1}(\sin\theta) = \theta$, $\cos^{-1}(\cos\theta) = \theta$, $\tan^{-1}(\tan\theta) = \theta$
$\operatorname{cosec}^{-1}(\operatorname{cosec}\theta) = \theta$, $\sec^{-1}(\sec\theta) = \theta$, $\cot^{-1}(\cot\theta) = \theta$

(iv) $\tan^{-1}x + \tan^{-1} y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$

(v) $\tan^{-1}x - \tan^{-1} y = \tan^{-1}\left(\dfrac{x-y}{1+xy}\right)$

(vi) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\dfrac{x+y+z-xyz}{1-xy-yz-zx}\right)$

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CHAPTER 9

LINEAR EQUATIONS IN TWO VARIABLE

(1) The system of linear equation

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

is (a) consistent, if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

(b) inconsistent, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(c) dependent, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(2) The solution of the system of linear equation

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

is given by

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

$$y = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

(3) A number of two digits is given by

10 (digit at ten's place) + (dight at units place)

(4) Time = $\frac{\text{Distance}}{\text{Speed}}$

(5) Speed = $\frac{\text{Distance}}{\text{Time}}$

Linear Equations : An equation in-volving two any unkown in the first degree is called a linear equation.

HOW TO SOLVE A SET OF LINEAR EQUATIONS GRAPHICALLY

Draw the graph of the first eqwuation then first on the same axis, draw the graph of the second equation and to find the solution read the coordinates of the points of intersection of the two graphs.

Points to note

1. If the lines meet in a point, there is only one solution and the given system of equation is known as consistent.

2. If the lines of the equation are coincident then the equation is called dependent.

Method of solving two simutaneous linear equation in two unkonw.

Algebraic methods of solving systems of solving systems of eqsuations.

1. The method of Substitution

Step I. Find the value of one of the variable in terms of the variable in terms of the other from any one of the given equation.

Step II. Substitue the values of the variables so obtained in the other equation.

Step III. Solve the simple equation thus got and find the value of one of the variables.

Step IV. Put the value of the variable so obtained in any one of the given equation and find the value of the order variable.

The Method of Equalising Coefficients

Multiply the equations so as to make the coefficients of the variable to the eliminated equal.

Add the equations if the terms having the same coefficient are opposite signs and subtract if they are of the same sign.

Solve the simple equation thus obtained.

Substitute the value found in any one of the given equations and find the value of the one variable.

To solve a word problem, the following points should not be avoided.

(1) Make assumptions, using two variables say x and y.

(2) Express the conditions of the problems in symbolical language and form two equations in the terms of x and y.

(3) Solve the equations simultaneously and verify the results.

G.C.D. and L.C.M. of Polynomials

(1) H.C.F. of two or more polynomials is the ‘common factor of highest degree’.

(2) L.C.M. of two or more polynomials is the

(3) $\text{L.C.M.} = \dfrac{\text{Ist Polynomial} \times \text{IInd Poynomial}}{\text{H.C.F.}}$

(4) L.C.M. × H.C.F. = Product of two polynomials.

(5) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

(6) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$x^3 + y^3 + z^3 = (x + y)(x^2 - xy + y^2)$

(7) $(x^2 + y^4 + z^2 - xy - yz - zx)$

(8) $x^4 + y^4 = (x - y)(x + y)(x^2 + y^2)$

(9) If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

Highest Common factor

Following methods should be followed.

(1) By using factorisation :

(a) Find the factors of the given polynomials.

(b) Find the set of their common factors.

(c) The common factor with the highest degree or power is the H.C.F. of the given polynomial.

(2) By using successive division :

(a) Take out the common factor, if any from the given polynomials and find their H.C.F.

(b) Write the polynomials in descending order of their powers or degree.

(c) By taking the polynomial of higher degree as dividend and one with lower degree as divisor perform division.

(d) If there is any remainder then take it as the divisor and the divisor of the previous step as dividend. Again perform division.

(e) Continue this process till the remainder is zero.

The last divisor is the H.C.F. of the given polynomial.

e.g. H.C.F. of 480, 360, 600 can be found as under :

$480 = 2^5 \times 3 \times 5$

$360 = 2^3 \times 3^2 \times 5$

$600 = 2^3 \times 3 \times 5^2$

Thus H.C.F. = 23 × 3 ×5 = 8 × 3 × 5 = 120

Point to note

(1) If at any stage, the remainder contains common factor, take it out.

(2) If the first turn of the remainder is negative then take out (–1) as common factor.

(3) If at any stage of division, the quotient-exists as a fraction, then multiply the dividend by a suitable number to avoid fractional quotients.

Least Common Factor

(1) About the following methods

By using factorization

(a) Find the factors of the given polynomials.

(b) Write all different factors from the given polynomial taking a factor once only.

(c) If numerical coefficients of polynomial exist, then take their L.C.M. separately. The product of all the factors [from (a) & (b) is the L.C.M. of the given polynomial.

(2) By using division method

(i) Find the H.C.F. of the given polynomials.

(ii) Use the following formula for finding L.C.M.

$$L.C.M. = \frac{\text{Product of two polynomial}}{\text{H.C.F of two polynomials}}$$

e.g. the L.C.M. of 480, 360, 600 can be found as under :

$480 = 2^5 \times 3 \times 5$

$360 = 2^3 \times 3^2 \times 5$

$600 = 2^3 \times 3 \times 5^2$

CHAPTER 10

ELEMENTARY ALGEBRA

Formulae :

1. $(a + b)^2 = \mathrm{a}^2 + 2ab + \mathrm{b}^2$
2. $(a - b)^2 = \mathrm{a}^2 - 2ab + \mathrm{b}^2$
3. $(a + b)^2 = (a - b)^2 + 4ab$
4. $(a - b)^2 = (a + b)^2 - 4ab$
5. $(a - b)(a + b) = \mathrm{a}^2 - \mathrm{b}^2$
6. $(a + b)^3 = \mathrm{a}^2 + b^3 + 3ab\ (a + b) = a^3 + b^3 + 3a2b + 3a\mathrm{b}^2$
7. $(a - b)^3 = a^3 + b^3 + 3ab\ (a - b) = a^3 - b^3 - 3a^2b + 3ab^2$
8. $a^3 + b^3 = (a + b)3 - 3ab\ (a + b)$
9. $a^3 - \mathrm{b}^3 = (a + b)^3 - 3ab\ (a + b)$
10. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Factorisation

1. $a^2 - b^2 = (a - b)\ (a + b)$
2. $a^3 + b^3 = (a + b)\ (a^2 - ab + b^2)$
3. $a^3 - b^3 = (a - b)\ (a + ab + b^2)$
4. $a^4 + a^2b^2 + \mathrm{b}4 = (a^2 + ab + b^2)\ (a^2 - ab + b^2)$
5. $a^{\mathrm{n}} - b^{\mathrm{n}} = (a - b)\ (a^{n-1} - a^{n-2}\ b + a^{n-3}\ b^2 + \ldots + b^{n-1})$; where $n \in N$
6. $a^n + b^n = (a + b)\ (a^{n-1} - a^{n-2}\ b + a^{n-3}\ b^2 + \ldots + b^{n-1})$; where n is an odd integer

Cyclic Factors

1. $\Sigma a\ (b^2 - c^2) = (a - b)\ (b + c)\ (c - a)$
2. $\Sigma a^2\ (b - c) = -(a - b)\ (b - c)\ (c - a)$
3. $a^3 - b^3 = (a - b)\ (a + ab + b^2)$
4. $\Sigma ab\ (b - c) = -(a - b)\ (b - c)\ (c - a)\ (a + b + c)$
5. $a^3 + b^3 + c^3 - 3abc = [a + b + c] \times (a^2 + b^2 + c^2 - bc - ca - ab)$
6. If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

Cyclic Expressions

1. $\Sigma a\ (b - c) = 0$
2. $\Sigma a^2\ (b^2 - c^2) = 0$
3. $\Sigma a^3\ (b^3 - c^3) = 0$
4. $\Sigma a^n\ (b^n - c^n) = 0$

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CHAPTER 11

NEWTON'S BINOMIAL

Binomials can also be obtained by using Newton's formula

$$(A + B)^n = \binom{n}{0}A^n + \binom{n}{1}A^{n-1}B + ... \binom{n}{n-1}AB^{n-1} + \binom{n}{n}B^n$$

Note :

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$ binomial Ceofficients in whcih n and k ε N

n ! = 1, 2, 3 ..... (n –1), n, It is called n factorial.
k ! = 1, 2, 3 .... (k –1), k
1 ! = 1
0 ! = 1

$$\binom{n}{0} = \binom{n}{n} = 1, \binom{n}{k} = \binom{n}{n-k}$$

Remainder Theorem : If a polynomial $p(x)$ of degree ≥ 1 be divided by $x - a$, then the remainder is $f(a)$, where a is a real number.

Proof : Let $q(x)$ be the quotient and $r(x)$, the remainder when $p(x)$ is divided by $x - a$.

Then by division algorithm, we have

$$p(x) = (x - a)\, q(x) + r(x)$$

where $r(x) = 0$ or degree $r(x) <$ degree $(x - a)$

since degree of $(x - a)$ is 1, either

$r(x) = 0$ degree of $(x - a)$ is 1, either

$r(x) = 0$ or degree $r(x) = 0$ (<1)

So $r(x)$ is a constant, say r.

Factor Theorem : Let $p(x)$ be a polynomial and a be a real number. Then $(x - a)$ is a factor of $f(x)$ if and only if $p(a) = 0$

i.e. (i) If $(x - a)$ is a factor of $p(x)$, the $n\, p(a) = 0$

(ii) If $p(a) = 0$ then $(x - a)$ is a factor of $p(x)$.

Proof : By division algorithm, we have,

$p(x) = (x - a)\ q\ (x) + r$ ...(i)

where $q(x)$ is the quotient and r, the remainder.

By remainder theorem $r = p(a)$

(i) If $(x - a)$ is a factor of $p(x)$, then $r = 0$ ...(ii)

$\Rightarrow p(a) = 0$ [from 2]

(ii) If $p(a) = 0$, then $r = 0$

So that $p(x) = (x - a)\ q(x)$

Hence $(x - a)$ is a factor of $p(x)$

A Binomial Expression Simplified into Factors

$A^2 - B^2 = (A - B)\ (A + B)$

$A^3 - B^3 = (A - B)\ (A2 + AB + B2)$

$A^4 - B^4 = (A - B)\ (A^3 + A^2B + AB^2 + B^3)$

$A^n - B^n = (A - B)\ (A^{n-1} + A^{n-2}B + A^{n-3})$

$B^2 + ... + AB^{n-2} + B^{n-1})$

$A^2 - B^2 = (A + B)\ (A - B)$

$A^4 - B^{2n} = (A + B)\ (A^3 + A^2B + AB^2 + B^3)$

$A^{2n} - B^{2n} = (A + B)\ (A^{2n-1} - A^{2n-1}.B + A^{2n-3}\ B^2 + ... + AB^{2n-2} - B^{2n-1})$

(**Note :** Alternate signs or minus sign if the exponent of B is odd)

$A^3 + B^3 = (A + B)\ (A^2 + AB + B^2)$

$A^5 + B^5 = (A + B)\ (A^4 + A^3B + A^2B^2 + AB^3 + B^4)$

$A^{2n+1} + B^{2n+1} = (A + B)\ (A^{2n} - A^{2n-1}B + AB^2B^{2n+1} + B^{2n})$

(Note : Alternate sign or minus sign if exponent of B is odd**)**

It may be remembered that it is not possible to simplify $(A^{2n} + B^{2n})$

Linear Polynomial : A polynomial of degree 1 is called linear polynomial.

e.g. $3x + 7$, $2x - 5$ etc.

Quadratic Polynomial : A polynomial of degree 2 is called a quadratic polynomial *e.g.*,

$2x^2 + 3x + 1$; $x^2 - 5x + 6$ etc.

Cubic Polynomial : Polynomial of degree 4 is called biquadratic polynomial.

Points to remember

(1) Finding factors of a polynomial means finding factors of positive degree, that is of degree $n \geq 1$.

(2) To factorse a polynomial means to obtain two or more polynomials whose product be the given polynomial.

Linear Equations in one Variable

Linear Equation : An equation involving only linear polynomials is called is linear equation.

e.g. (i) $5x + 2 = 12$

(ii) $12x + 4 + 4x + 28$

Solution of a Linear Equation : The value of variable or unknown that satisfy the equation is known as the solution of the equation.

Consider the linear equation.

$5x - 2 = 18$

If we substitute $x = 4$, we get

L.H.S. $= 5x - 2$

$= 5 \times 4 - 2 = 20 - 2 = 18$

R.H.S. $= 18$

$\therefore$ L.H.S. = R.H.S.

Thus 4 is a solution of $5x - 2 = 18$

Solving an Equation : Solving a linear equation means finding a value of variable which satisfy the equation.

By any of the following operations : an equation remains uneffected.

(1) Adding the same quantity to both sides of the equality.

(2) Subtracting the same quantity to both sides of the equality.

(3) Multiplying both side of an equality by the same non-zero number.

(4) Dividing both sides of an equality by the same non-zero number.