Bridge Course of Mathematics

A mathematical prodigy, also known as the 'human computer', **Shakuntala Devi** (1929–2013) was known for her complex problem-solving skills without the aid of any mechanical device. During her early years, she shot to fame by mentally calculating one of the toughest mathematical multiplications 10 seconds before the fastest and the most efficient computer of the time.

Shakuntala Devi

She was able to answer all of his challenging mathematical problems, prompting him to call her a ''Mathematical Wizard''. Apart from being an unparalleled mathematician, Devi was also an astrologer, activist and a prolific writer, whose works went on to inspire millions of people.

> *''Just as a mountaineer climbs a mountain– because it is there, so a good mathematics student studies new material – because it is there''.*

– JAMES B. BRISTOL

Prepared by ***Dr. C. R. Mallick***
Department of Mathematics, P.M.E.C. Berhampur

MODULE – I

- INDICES
- LOGARITHMS
- BASIC CONCEPTS OF GEOMETRY
- THE EQUATION OF A LOCUS
- POLYNOMIALS
- QUADRATIC EQUATIONS

CHAPTER 1

INDICES

Index, Indices : A number indicating a typical characteristics of an expression. For example, in 2^3, 3 is the exponent or index or power and 2 is the base.

The Fundamental Law of Indices

This fundamental law of indices states that if m and n are positive integers then $a^m \times a^n = a^{m+n}$: the quantity a^m being the product of m factors each one being a. We can always extend this result to the product of $a^m \times a^n = a^p$ where m, n and p are each positive integers. Thus

$$a^m \times (a^n \times a^p) = a^m \times a^{n+p} = a^{m+n+p}$$

or $$(a^m \times (a^n) \times a^n = a^{m+n} \times a^p = a^{m+n+p.}$$

the same result being obtained whether we first associate a^n with a^p or a^m with a^n. All we have done here done here is use the associative law for multiplication. From the fundamental law we deduce the following results for positive integers m and n.

(i) $a^m \div a^n = a^{m-n}$ if $m > n$

(ii) $a^m \div a^n = \dfrac{1}{a^{n-m}}$ if $m < n$

(ii) $(a^m)^n + (a^n)^m = a^{mn} + a^{mn}$

We can prove these results by using the fundamental law as follows :

(i) Since $m - n$ is positive, $a^{m-n} \times a^n = a^{m-n+n} = a^m$. Q.E.D.

(ii) Since $n - m$ is positive then

$$\frac{1}{a^{n-m} \times a^m} = \frac{1}{a^{n-m+m}} = \frac{1}{a^n}$$

Now multiply both sides of the equation by a^m to get

$$\frac{a^m}{a^{n-m} \times a^m} = \frac{a^m}{a^n},$$

i.e. $a^m \div a^n = \dfrac{1}{a^{n-m}}$ Q.E.D.

(iii) $(a^m)^n = a^m \times a^m \times a^m \times a^m \ldots\ldots \times a^m$, a product of n factors. each one being a^m,

$= a^{2m} \times a^m \times a^m \ldots\ldots \times a^m$

$= a^{2m} \times$ (a product of $n - 2$ factors, each one being a^m)

$= a^{3m} \times$ (a product of $n - 3$ factors, each one being a^m)
$= a^{4m} \times$ (a product of $n - 4$ factors, each one being a^m)
$= a^{nm}$

Similarly $(a^n)^m = a^{mn} = a^{nm}$

Two further results which we need to be reminded of are

(*iv*) $(ab)^m = a^m b^m$ and (*v*) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Having identified these laws for positive indices m and n we assume that they are true for all rational numbers, i.e. any number of the form p/q where p and q are integers (positive or negative and $q \neq 0$. All that remains to be done is to find a meaning for some of the new expressions which are consistent with these laws.

Example – 1: To obtain a meaning for $a^{1/n}$, where n is a positive integer.

Solution : We start with some numerical examples $a^{1/2} \times a^{1/2} = a^1 = a$, but since we know that $\sqrt{a} \times \sqrt{a} = a$, but since we know that $\sqrt{a} \times \sqrt{a} = a$ it follows that $a^{1/2} = \sqrt{a}$, e.g. $9^{1/2} = 3$, $25^{1/2} = 5$.

Next consider $a^{1/3} \times a^{1/3} \times a^{1/3} \times a^{1/3} = a^{1/3} = a^{1/3 + 1/3 + 1/3} = a^1 = a$, but since we know that $\sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$ it follows that $a^{1/3} = \sqrt[3]{a}$ i.e. nth root of a (e.g. $8^{1/3} = 2$, $125^{1/3} = 5$).

For cases such as $17^{1/4}$ we are unable to simplify the results as we did for $32^{1/8}$ althrough we can find the fifth root of 17 by using logarithms as the reader possibly already knows. For a number such as $96^{1/5}$ we can go part of the way towards simplification by realising that since $96 = 32 \times 3$ the result (iv) above enables us to wirte

$96^{1/5} = (32 \times 3)^{1/5} = 32^{1/5} \times 3^{1/5}$

$2 \times 3^{1/5}$ or $2\sqrt[5]{3}$

Similarly

$250^{1/3} = (125 \times 2)^{1/3} = 125^{1/3} \times 2^{1/3} = 5 \times 2^{1/3} = 5\sqrt[3]{2}$

Example – 2 : To obtain a meaning for $a^{m/n}$, m and n being positive integers.

Solution: Now since $a^{m/n} = (a^{1/n})^m$ we may interrpret $a^{m/n}$ as an (nth root of a) taken or raised to the power of m, i.e. $\left(\sqrt[n]{a}\right)^m$. For example, $32^{3/5} = (32^{1/5})^3 = (2)^3 = 8$.

Another interpretation giving the same result is to consider $a^{m/n} = \left(a^m\right)^{\frac{1}{n}}$, in which case we now find an nth root of (a taken to the power m) that is $\sqrt[n]{a^m}$ Numerically this becomes $32^{3/5} = (32)^{1/5} = (32)^{1/5} = 8$, as before. Clearly the first method of evalution is the easier.

Example – 3 : To obtain a meaning for a^0, $a \neq 0$.

Solution: Consider the result $a^0 \times a^n = a^n \times a^0 = a^{n+0} = a^n$. Since multiplication by 1 is the only way to leave a^n unchanged, it follows that $a^0 = 1$.

Example – 4 : To obtain a meaning for a^{-n} when n is a positive integer.

Solution: Consider $a^n \times a^{-n} = a^{-n+n} = a^0 = 1$

then $\dfrac{a^n \times a^{-n}}{a^n} = a^{-n} = \dfrac{1}{a^n}$

e.g. $2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$. In words, a^{-n} is the reciprocal of a^n

thus $a^{-1} = \dfrac{1}{a}$

Finally we consider $a^{-1/n}$ where n is a positive integer. We have

$(a^{-1/n})^n = a^{-n/n} = a^{-1} = 1/a.$

Taking the nth roots of both sides we get

$$\left(a^{-1/n}\right) = \left(\frac{1}{a}\right)^{1/n} = \frac{1}{a^{1/n}}$$

e.g. $8^{1/3} = \dfrac{1}{8^{1/3}} = \dfrac{1}{2}$

Rationalisation

A number such as $\sqrt{3}$, $\sqrt{2}$, $\sqrt{8}$ is called an irrational number because it cannot be expressed as a quotient of two integers like

$$7.7 = \frac{15}{2} \text{ and } -2\frac{1}{3} = \frac{-7}{3} or \frac{7}{-3}$$

Now suppose we have to simplify $\sqrt{\dfrac{7}{9}}$, i.e. $\left(\dfrac{7}{9}\right)^{1/2}$,since this is the same as $\dfrac{7^{1/2}}{9^{1/2}}$ we obtain the result $\dfrac{(7)^{1/2}}{3}$,and $\sqrt{7}$ will have to be either-left as it is or written as 2.65, after consulting tables of square roots. Thus $\dfrac{(7)^{1/2}}{3}$ is the simplification and $\dfrac{2.65}{3} = 0.88$ (2 decimal places) is the evaluated form. On the other hand, if we attempt to simplify $\left(\dfrac{9}{7}\right)^{1/2}$ we obtain $\dfrac{3}{\sqrt{7}}$, we consider that a further simplification is necessary by rationalising the denominator so as to make

$$\frac{3}{\sqrt{7}} = \frac{3 \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \frac{3\sqrt{7}}{7}$$

Using the result $x^2 - y^2 = (x - y)(x + y)$, so that $\frac{1}{(x+y)} = \frac{x-y}{x^2-y^2}$ *or* $\frac{1}{x-y} = \frac{x+y}{x^2-y^2}$, we now consider rationalising the denominator in the expression such as $\frac{1}{\sqrt{2}-1}$ by writing

$$\frac{1}{(\sqrt{2}-1)} = \frac{(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{\sqrt{2}+1}{2-1} = \frac{\sqrt{2}+1}{1} = \sqrt{2}+1$$

Similarly $\frac{1}{\sqrt{2}+1} = \sqrt{2}-1$

Example – 5 : Simplify $\frac{1}{x+\sqrt{(x^2+1)}}$

Solution: Rationalise the denominator

$$\frac{1}{x+\sqrt{(x^2+1)}} = \frac{1}{x+\sqrt{(x^2+1)}} \times \frac{x-\sqrt{(x^2+1)}}{x-\sqrt{(x^2+1)}} = \frac{x-\sqrt{(x^2+1)}}{x^2-\left(\sqrt{(x^2+1)}\right)^2} = \frac{x-\sqrt{x^2+1}}{x^2-x^2-1}$$

$$= \sqrt{(x^2+1)} - x$$

Example – 6 : Given that $\frac{a}{b}$ is a good approximation to $\sqrt{2}$ prove that $\frac{a+2b}{a+b}$ is a better approximation.

Solution: We are being asked to prove than $\left(\frac{a+2b}{a+b}\right)^2$ is closer to 2 than $\frac{a^2}{b^2}$. We can illustrate this numerically by taking $a = 7$, $b = 5$, in which case $\frac{a^2}{b^2} = \frac{49}{25}$ so that

$$2 - \frac{a^2}{b^2} = 2 - \frac{a^2}{b^2} = 2 - \frac{49}{25} = \frac{1}{25}$$

$\left(\frac{a+2b}{a+b}\right)^2 = 2 - \frac{289}{144} = -\frac{1}{144}$ so we see that $\frac{17}{12}$ is closer to $\sqrt{2}$ than $\frac{7}{5}$ because $\frac{1}{144}$ is smaller than $\frac{1}{25}$.

The closeness of $\frac{a^2}{b^2}$ of 2 is given by $2 - \frac{a^2}{b^2} = \frac{2b^2-a^2}{b^2}$. The closeness of $\left(\frac{a+2b}{a+b}\right)^2$ to 2 is given by

$$2-\frac{(a+2b)^2}{(a+b)^2}=2-\frac{a^2+4ab+4b^2}{a^2+2ab+b^2}$$

$$=\frac{2a^2+4ab+2b^2-\left(a^2+4ab+4b^2\right)}{a^2+2ab+b^2}=\frac{a^2-2b^2}{(a+b)^2}$$

Now $(a+b)^2 > b^2$ for $a > 0$, $b > 0$, so that $=\frac{a^2-2b^2}{(a+b)^2}$ is less than $\frac{2b^2-a^2}{b^2}$ in magnitude although of different sign. This means that if the approximation $\frac{a}{b}$ is less than $\sqrt{2}$ then the approximation $\frac{a+2b}{a+b}$ is greater than $\sqrt{2}$ (but still closer to $\sqrt{2}$).

It we return to the numerical case suggested after 7/5 we produced a closer or better approximation to $\sqrt{2}$ in 17/12. Now let us start again but this time with 17/12, i.e $a = 17$, $b = 12$. Thus

$$\frac{a+2b}{a+b}+\frac{17+24}{29}=\frac{41}{29}$$

so that

$$\left(\frac{a+2b}{a+b}\right)^2=\left(\frac{41}{29}\right)^2=\frac{1681}{841}$$

The closeness of this approximation is given by

$$2-\frac{1681}{841}=\frac{1}{841}$$

and since $\frac{1}{841}$ is less than $\frac{1}{144}$ we have $\frac{41}{29}$ as a better approximation to $\sqrt{2}$ than $\frac{17}{12}$.

Note that the errors in the approximations are $\frac{1}{25}=\frac{1}{5^2},\frac{1}{144}=\frac{1}{12^2},\frac{1}{841}=\frac{1}{29^2}$ so that a continuation of this process produces closer and closer approximations. This procedure produces a sequence of rational numbers which get as close as we please to (i.e. tend to) the irrational number $\sqrt{2}$.

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CHAPTER 2

LOGARITHMS

For any number 'a' and a positve integer 'n' we have

$a^n = a \times a \times a \times a$ .... (n terms)

In this the real number 'a' is known as base and 'n' is known as exponent of the n^{th} power of a.

Logarithms. For any positive number 'a' ($a \neq 1$), If $a^m = b$, then logarithm of 'b' to base 'a' is 'm' and it can be written as $\log a^b = m$

so we find that

$a^m = b$ and $\log a^b = m$ are equal.

(Note : log is the abbreviation to **logarithm)**

The logarithm of 'b' to base 'a' is defined as the number 'm' which has to be given as an exponent to obtain b.

Some Results

$\log 1 = 0$

$\log a = 1$

Properties of Logarithms

$\log_a x.y = \log_a x + \log_a y$

$\log_a \left(\frac{x}{y}\right) = \log_a x - \log y$

$\log_a x^n = n \log_a x$

$\log_a \sqrt[n]{x} = \frac{1}{n} \text{logax}$

(Note : $\log_a (b \pm c) = \log_a b \pm \log_a c$)

Decimal Logarithm and Natural Logarithm

Those logarithms whose base is 10 are known as **decimal logrithms.** They are indicate by 'Log' with no base.

Those logarithms which has a base '*e*' (e =2.71828) are known as **Natural or Naplerian logarithms.** They are indicated by *ln*.

The following formula can be used to change from decimal logarithm to natural logarithm or vice-versa.

$$\left.\begin{aligned} Logb &= In.b\,.Log\,e \\ Inb &= \frac{Log\,b}{Log\,e} \end{aligned}\right] Log\,e = 0.434294$$

EQUALITY IN LOGARITHM

$$log_a x = \log_a y \Leftrightarrow x = y$$

INEQUALITY IN LOGARITHM

(i) If $0 < a < 1$, then

$$\left.\begin{aligned} \log_a x < 0 \; when \; x > 1 \\ \log_a x > 0 \; when \; 0 < x < 1 \end{aligned}\right\}$$

(ii) If $a > 1$, then

$$\left.\begin{aligned} \log_a x > 0, \; when \; x > 1 \\ \log_a x < 0, \; when \; 0 < x < 1 \end{aligned}\right\}$$

(iii) $\log_a x > \log_a y$

$$\Rightarrow \begin{cases} x \geq y \; if \; a > 0 \; and \; a > 1 \\ x \leq y \; if \; 0 < a < 1 \end{cases}$$

(iv) If $a > 1$, then

$$\log_a x < \log_a y \Leftrightarrow 0 < x < y$$

(v) If $0 < a < 1$ then

$$\log_a x > \log_a y \Leftrightarrow 0 < x < y$$

(vi) $a > 1, x > 1 \Rightarrow \log_a x > 0$

(vii) $0 < a < 1, 0 < x < 1 \Rightarrow \log_a x > 0$

(viii) $0 < a < 1, x > 1 \Rightarrow \log_a x < 0$

(ix) $a > 1, 0 < x < 1 \Rightarrow \log_a x < 0$

(x) $a > 1, x > 1$ and $x > a \Rightarrow \log_a x > 1$

(xi) $a > 1, x > 1, x < a \Rightarrow 0 < \log_a x < 1$

(xii) $0 < a < 1, 0 < x < 1, x > a \Rightarrow 0 < \log_a x < 1$

(xiii) $0 < a < 1, 0 < x < 1, x < a \Rightarrow \log_a x > 1$

(xiv) If $a > 1$, then $\log_a x < b \Rightarrow 0 < x < a^b$

(xv) If $a > 1$, then $\log_a x > b \Rightarrow x > a^b$

(xvi) If $0 < a < 1$, then $\log_a x < b \Rightarrow x > a^b$

(xvii) If $0 < a < 1$, then $\log_a x > b \Rightarrow 0 < x < a^b$.

Standard Form of a Number

If m is a positive real number, it may be written as $m = n \times 10^p$, where $p \in I$ and $I < n < 10$. This form is called the standard form e.g. 8634 can be written as 8.634×10^3 standard form).

Characteristics and Mantissa

We know, $\log m = \log n + p$

where p is an integer and as $1 \le n < 10$ so $0 \le \log n < 1$ *i.e.* $\log n$ lies between 0 and 1. When $\log m$ has been has been expressed as $p + \log n$ where p is an integer and $0 \le \log n < 1$, we say the p is **Characteristic** of $\log m$ and that $\log n$ is the **mantissa.**

Points to Remember

1. It is possible to express all positive decimals in the form $m = n\ 10^o$
   where p is an integer (positive, zero or negative) and $1 \le n < 10$
2. The characteristic of the logarithm of a given number can be any integer, positive zero or negative but mantissa

How to find the characteristic

Following rules may be applied to find the characteristic.

For a positive number m, the characteristic of $\log m$ is found by using following rules.

(i) If $m > 1$, count the number of digits in the integral part of m and subtract 1 from it to get characteristic e.g. In 563.4 is $3 - 1 = 2$

[Converse is also true i.e. If the characteristic of $\log m$ is 2, then number of digits in integral part of m is $2 + 1 = 3$

(ii) If $m < 1$, count the number of zeroes immediately after the decimal point and add 1 to it. The number so obtained with a negative sign gives the characteristic. *e.g.* if $m = 0.0321$ the number of zeroes immediately after the decimal point is one, therefore characterterstic of log 0.0321 is $1 + 1 = 2$ with negative sign and it is written as $\bar{2}$.

Converse is also true i.e.. If characteristic of log m is $\bar{2}$ then the number of zeros immediately after decimal point in m is $\bar{2}$ $-1 = 1$.

How to find the mantissa of the log of a Number

To find **mantissa** we have to consult the log lables. following procedure is adopted to find the mantissa.

1. Locate first two digits of the given number in the left hand column, of the log tables, headed by N. If there is only one digit in the given number then we put a zero to the right of the number to make it a two digit number.
2. Locate the entry in the row obtained earlier in the column headed by third digit in earlier in the number whose logarithm has to be found.
3. Now locate the entry in the same row, and in the column headed by the fourth digit in the given number in the table of proportional parts, and add the entry now obtained to the entry obtained earlier.

How to find the antilogarithm of a Given Number

To find the antilogartithim of a given number following rules may be used.

1. To find antilogarithm only **mantissa** is to be considered.
2. In the antilogarithm table, locate the first two digits in the number in the first column.
3. Locate the entry in this row and the column headed by third digit in the number.
4. In the same row and column headed by fourth digit in the number locate the entry in the table of proportional parts.
5. Add this to the entry obtained earlier and insert the decimal point at a suitable place following the rules of characteristics.

Applications of Logarithms

Logarithms are quite useful in carrying out complicated calculations. e.g.

(i) In calculations of compound interest.

(ii) In Calculations of population growth.

(iii) In Calculations of depreciation value (depreciation or decay is negative growth).

(iv) In Calculations of mensurations.

(v) In calculations of least square method.

Example – 1: Find the value of $7^{2\log_7 5}$.

Solution : $7^{2\log_7 5} = 7^{\log_7 25} = 25$ $\quad [\because a^{\log_a M} = M]$

Example – 2: Find the number of soluions of the equation $\log_2(x^2 + 2x - 1) = 1$

Solution : $\log_2(x^2 + 2x - 1) = 1$

$\Rightarrow x^2 + 2x - 1 = 2^1 = 2$

$\Rightarrow x^2 + 2x - 3 = 0$

$\Rightarrow (x + 3)(x - 1) = 0$

$\Rightarrow x = -3, 1$

Number of solutions is 2.

Example – 3 : If $\log_2(9^{x-1} + 7) - \log_2(3^{x-1} + 1) = 2$, then, find the values of x.

Solution : $$\log_2\left(\frac{9^{x-1}+7}{3^{x-1}+1}\right) = 2$$

$$\Rightarrow \frac{9^{x-1}+7}{3^{x-1}+1} = 2^2 = 4$$

$$\Rightarrow \frac{y^2+7}{y+1} = 4 \qquad (\text{Taking } y = 3^{x-1})$$

$\Rightarrow y^2 - 4y + 3 = 0$

$\Rightarrow y = 1, 3$

$\Rightarrow 3^{x-1} = 3^0$ or 3^1

$\Rightarrow x = 1, 2$

Example – 4 : Solve for x : $\log_x (8x-3) - \log_x 4 = 2$.

Solution : We have $\log_x \frac{8x-3}{4} = 2$

$\therefore\ x^2 = \frac{8x-3}{4}$, or $4x^2 = 8x - 3$, or $4x^2 - 8x + 3 = 0$,

or, $4x^2 - 6x - 2x + 3 = 0$, or, $2x(2x-3) - 1\,(2x-3)$

$= 0$, or, $(2x-3)(2x-1) = 0$.

$\therefore$ either $2x - 3 = 0$ or $2x - 1 = 0$.

$\therefore\ x = 3/2$ or $x = 1/2$

Example – 5 : Show that

$$23\log\frac{16}{15} + 17\log\frac{25}{24} + 10\log\frac{81}{80} = 1 \text{ (Base 10).}$$

Solution : $23\log\frac{16}{15} = 23\,[\log 16 - \log 15]$

$= 23[\log 2^4 - 10g(3\times 5)]$

$= 23[4\log 2 - \log 3 - \log 5]$

$= 92\log 2 - 23\log 3 - 23\log 5$

$17\log\frac{25}{24} = 17\,[\log 25 - \log 24]$

$= 17\,[\log 5^2 - \log(2^3 \times 3)]$

$= 17\,[2\log 5 - 3\log 2 - \log 3]$

$= 34\log 5 - 51\log 2 - 17\log 3$

$10\log\frac{81}{80} = 10[\log 81 - \log 80]$

$= 10\,[\log 3^4 - \log(2^4 \times 5)]$

$= 10[4\log 3 - 4\log 2 - \log 5]$

$= 40\log 3 - 40\log 2 - 10\log 5$

$\therefore$ L.H.S of the given relation becomes

$= \log 2\,(92 - 51 - 40) + \log 3\,(-23 - 17 + 40) + \log 5\,(-23 + 34 - 10)$

$= \log 2 + \log 5 = \log 10 = 1$ ($\because$ the base is 10, $\log_{10} 10 = 1$).

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Prepared by ***Dr. C. R. Mallick***
Department of Mathematics, P.M.E.C. Berhampur

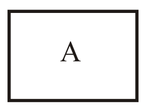
CHAPTER 3

BASIC CONCEPTS OF GEOMETRY

Geometry. The branch of mathematics which deals with lines, curves, solids, surfaces and points in space.

Point : In Geomertry a point has a position only and is represented by a dot. A point has no length, width or thickness. It is designated by capital letters next to the dot as A,B,C,D,E,F,X,Z, etc.

Here A is as point



Line: A line has length but no thickness or width. A straight line may be created by any method, extended indefinitely in both directions. Lines are denoted by small letters *i.e.* *a*, *b*, *c* etc.

A geometrical line is a set of points and extends endlessly in both are

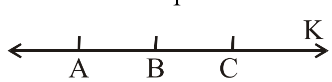
Plane : A plane is a flat surface such that a straight line joining any two of its plane lies whoolly in the surface is called a plane (surface) or a plane. It is also a set of points and it has no thickness. Planes, lines and points are related to one another in following manner.

(1) A line is terminated by points or lines meet in points.

(2) A plane is bounded by lines or the planes meets in line.

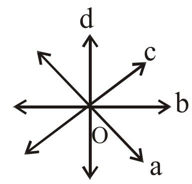
Line Segment : The position of a line with end points are called line segment A & B A, B.

Collinear Point : Three or more than three points are said to be collinear they



all lie on same straight line.

Concurrent Lines : Three or more than three are said to be concurrent if there is a point that lies pon all of them.

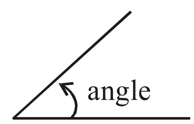


In it, lines a, b, c d are concurrent because point o lines on all tthe lines a, b, c, and d.

Intersecting Lines : Two lines whose intersection is not empty are known as intersecting lines. The common point is called the point of inter-section.

Parallel Lines : Two Coplanar line whose intersection is empty are known as **parallel lines.**

Angle : An angle is the union of two



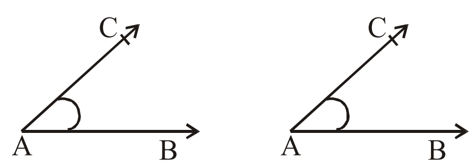
rays with a common initial part *i.e.*

Vertex : The common initial point is known as the vertex of the angle.

Arm : The two rays constituting the angle are called the **arms** (of angle). The magnitude of an angle is the amount of rotation in which one arm must be rotated about the vertex, so as to coincide with the other arm.

The unit for the measurement of an angle is called **Degree.**

The angle formed by AB and AC is written as $\angle$ BAC or $\angle$ CAB. In naming the angle, the vertex is always written in the middle. But sometimes.

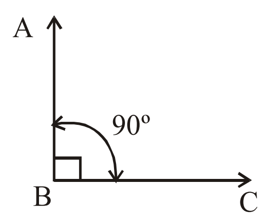


for convenience, angles are denoted by θ, α 1, 2 etc.

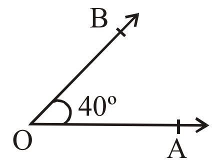
Congruent or equal angles : Two angle are congruent if they cover each other completely and exactly.

Types of Angles :

Right angles : A right angle is an angle if its measure is 90º $\angle$ABC is a right angle.

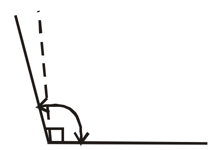


Acute angle : An acute angles is less than a right. Its measure lies between

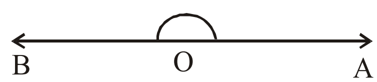


0º and 90º. In figure ∠ AOB is an acute angle.

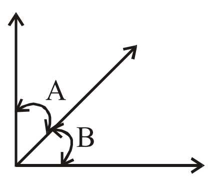
Obtuse angle : An obtuse angle (measure lies between 90 and 180º as shown is diagram.



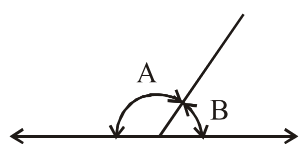
Straight Angle : An angle of 180º is called a straight angle.



Complementary Angles : Two angles are complementary if their sum measure 90º. Here A + B = 90º, therefore A and B are the example of Complement-ary angles.



Reflex Angle : A reflex angle is an angle if its measure lies between 180º and 360º as shown in figure.



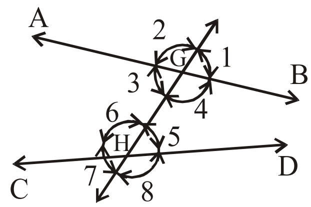
Complete Angle : An angle of 360º is called a complete angle.

Supplementary Angles : Two angles are said to be supplementary if their measure sum is of 180º as in diagram shown below.

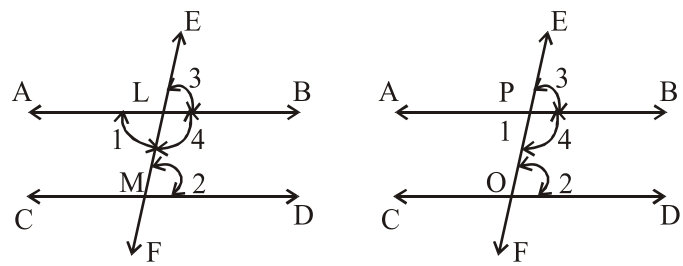
A + B = 180º, therefore A and B are supplementary angles.

If two angles are supplementary, then each is called the **supplement** of the other. Thus angles 120º and 60º are supplementary and angle of 60º is the supplement of an angle 120º.

Transversal : A line that intersect two or more lines at distinct points is called a transversal. In figure AB and CD are two lines and EF is a transversal cutting at G and H. The angles 1, 2, 7,8 are called **exterior angles.** Angles 3, 4, 5, 6 are called **interior angles.**



The pairs of angles 1 and 5, 4 and 8,2 and 6, 3 and 7 are called **corresponding angles**. The pairs of angles 3 and 5, 4 and 6 are called **alternate interior angles**.



The pair of angles 1 and 3, 2, and 4, 5 and 7, 6 and 8 are called **vertically opposite angles.**

Adjacent Angles : Two angles are said to be adjacent if they have the same vertex, common arm and the other arm of one angle is on side of the common arm and that of the other is on the opposite side.

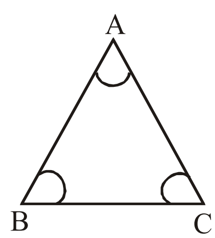
Vertically Opposite Angles : Anytwo angles that are formed by two intersecting lines and are not adjacent are known as vertically opposite angles.

Traingles :

Traingle A figure that has three sides and three angles is known as a traingle and it is denoted by Δ delta, a Greek letter.

Sides Three line segment AB, BC, CA are called sides of angle.

Angles $\angle$ ABC, $\angle$ BAC and $\angle$ ACB are called its angles. The angle of the traingle in short can be written as $\angle$ A, $\angle$ B and $\angle$ C.



Vertices The points ABC are called as the vertices of the traingle ABC.

Types of triangles "

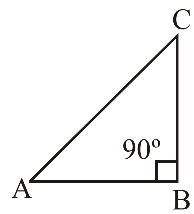
A traingle, whose two sides are unequal is called a **scalene triangle.**

A traingle, whose two sides are equal is called an **isosceles traingle**.

an equilateral triangle. If in a triangle, all the angles are equal then it is called equilateral triangle.

If in a traingle, all the angles are acute (less than 90º) then it is called an **Acute angled traingle**.

If in a traingle an angle is a right angle, then it is called a **Right angled traingle** and the side opposite the right angle is called the **Hypotenuse.**



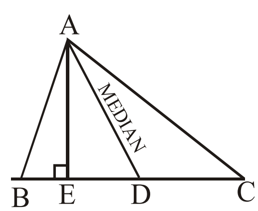
If in a triangle an angle is obtuse (greater than 90º) then it is called an **Obtuse angled traingle.**

Regular Polygon : A polygon is called regular if all its sides are equal and all its angles are equal.

Each angle of polygon $= \dfrac{(n-2)\times 180^{\circ}}{n}$

where n is the no. of sides of polygon.

Median : In a traingle the straight line joining the middle point of any side to the opposite vertex is called a **median**.



Altitude : In a traingle a straight line is drawn, from one angular point, to the opposite side, the straight line is called the **altitude** or **height** with respect to that side.

Theorem : A sum of interior angles of a quadrilateral is 360º.

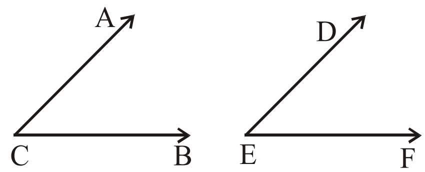
Here is a quadrilateral ABCD

then $\angle A + \angle B + \angle C + \angle D = 360^{\circ}$

Fig

Congruent

Two line segments are congruent if and only if their lengths are equal and without bending, stretching or twisting one figure can be superimposed on the other so that both of them can be brought into coincidence as shown in figure.



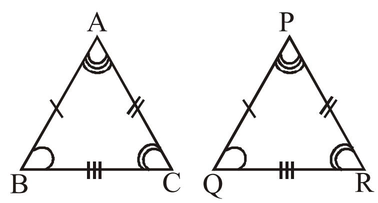
Fig

Congruence of two angles

Two angles are congruent if and only if their measure are equal to each other as shown here.

Congruence of two triangle

Two traingles are congruent if and only if there is a correspondence between their vertices such that the corresponding sides and corresponding angles of two traingles are equal as shown in diagram.



Fig

Congruent traingles are similar but the converse is not always true *i.e.*, the similar traingles are not congruent always.

Congruent triangles coincide by superposition.

In congruent traingles, corresponding sides lie opposite to equal side.

If ΔABC is congruent to ΔPQR and the correspondence ABC « PQR makes the six pairs of corresponding parts of the two traingles congruent then it is written as $\Delta ABC \cong \Delta PQR$.

The symbol "c.p.c.t." is used to indicate **correspondong parts of two congruent triangles.**

Criteria for congruent of two triangles

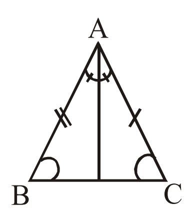
(1) SAS criteria – Side-Angle-Side

(2) ASA criteria – Angle – Side – Angle

(3) SSS criteria – 3 sides

Theorem : Angle opposite to two equal sies of a Δ are equal as shown here

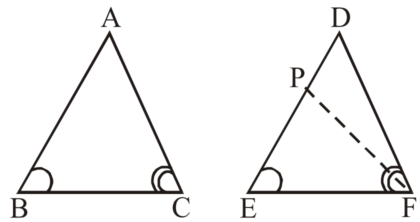
ΔABC in which AB = AC, then n ∠ C = ∠ B.



Fig

Theorem : Two triangles are **congruent** if two angles and the included side of one traingle are equal to the corresponding two angles and the included side of the other triangle.

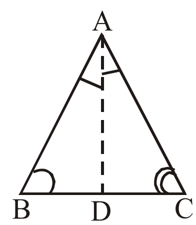
Here in ΔABC & ΔDEF. $\angle B = \angle E$, $BC = EF$ and $\angle C = \angle F$ then $\Delta ABC \cong \Delta DEF$.



Fig

And if two angles of a traingle qare equal, then the sides opposite to them are also equal.

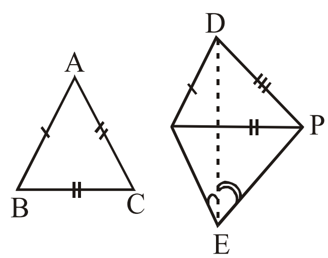
Here in diagram ΔABC in which $\angle C = \angle B$ then $AB = AC$.



Fig

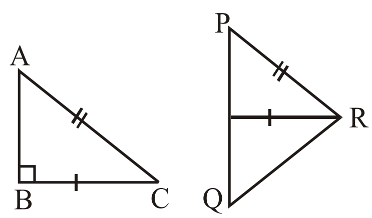
Theorem : Two triangle are congruent if the three sides of one triangle are equal to the corresponding three sides of the other triangle.

Here in diagram ΔABC & ΔDEF, $AB = DE$, $BC = EF$ and $AC = DF$ then $\Delta ABC \cong \Delta DEF$.



Fig

Theorem : RHS (Right angle Hypotenuse-Side)



Fig

Prepared by ***Dr. C. R. Mallick***
Department of Mathematics, P.M.E.C. Berhampur

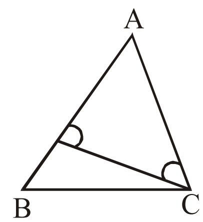
Two right triangles are congruent if the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and the corresponding side of the other triangle.

as in ΔABC & PQR, $\angle B = \angle Q = 90^\circ$ $AC = PR$ & $BC = QR$ then $\Delta ABC\ \Delta PQR$..

Some Inequality Relations

Theorem : If two sides of any triangle are unequal then the longer side has greater angle opposite to it.

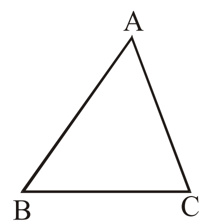
as in ΔABC, $AB > AC$ then $\angle ACB > \angle ABC$



Fig

And if in a traingle, the greater angle has the longer side opposite to it.

as in $\Delta\ ABC\ \angle\ ACB > \angle\ ABC$ then $AB > AC$



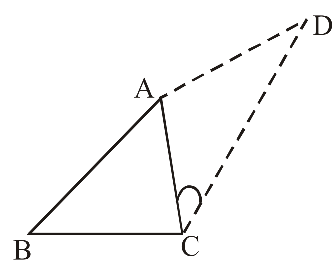
Fig

Theorem : The sum of any two sides of a traingle is greater than the third side *i.e.*

$AB + AC > BC$

as in ΔABC then $AC + BC > AB$

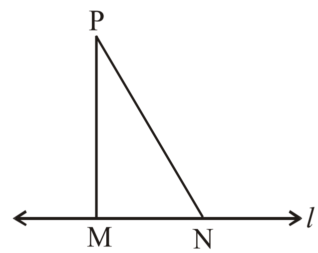
$AB + BC > AC$



Fig

Theorem : Of all the lines segments that can be drawn to a given line from a point which is not lying on it then the perpendicular line segment is the shortest.

as $\leftrightarrow$ is line and P is a point not lying on it, $Pm \leftrightarrow$ N is any point on $\leftrightarrow$ then M then $PM < PN$.



Fig

Parallelogram

A parallelogam is a quadrilateral whose opposite sides are parallel and equal (Abbreviated ‖ gm.)

A rhombus is a parallelogram which has two adjacent sides equal.

Rectangle is a parallelogram which has one of its angles a right angle.

Square is a rectangle which has two adjacent equal sides.

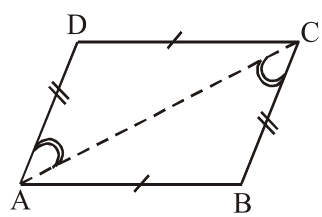
A square is also a quadrilateral all of whose sides are equal and all of whose angles are right angles.

A Kite is a quandrilqateral which has two pairs of adjacent sides equal.

Trapezium is a quadrilateral which has a pair of opposite sides parallel but the other two sides are not non-parallel and is called a *trapezium*.

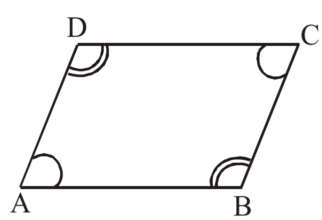
Isosceles trapezium . A trapezium in which the sides which are not parallel but equal to one another is known as an isosceles trapezium.

Theorem : A quadrilateral is a parallelogram, if opposite sides are of equal lengths.



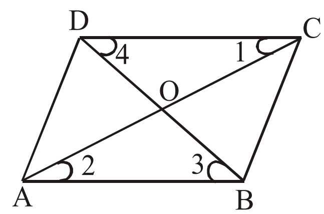
Fig

Theorem : The two diagonals of a parallelogram bisect each other



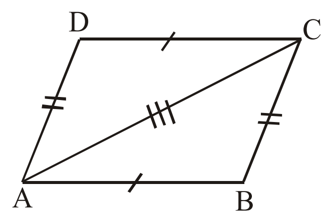
Fig

Theorem : The opposite angles of a parallelogram are equal.



Fig

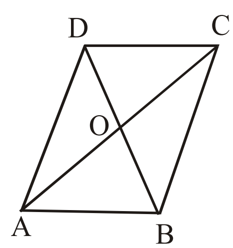
Theorem : A quadrilateral is a parallelogram if and only if a pair of opposite sides is parallel and of equal length.



Fig

Theorem : A parallelogram is a rhombus if and only if its diagonals are perpendicular to each other.

Here *ABCD* is a rhombus. Its diagonals *AC* & *BD* intersect at O, then $\angle BOC = 90^{o}$.

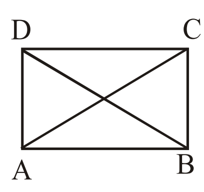


Fig

Theorem : A quadrilateral is a parallelogram if and only if the diagonals bisect each other.

Theorem :A parallelogram is *a* rectangle if any only if its diagonals have an equal length.

Here *ABCD* is a rectangle. *AC* and *BD* are its diagonals then $AC = BD$.



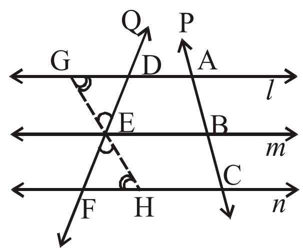
Fig

Theorem : A parallelogram is a square if and only if its diagonals are equal and are at the right angles.

Theorem : The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

Theorem : If there are three parallel lines, and the intercepts made by them on one transversal are equal, then the intercepts on any other transversal are also equal.

Here in figure three parallel lines I, m, n are cut by a transversal at A, B, C, such that $AB = BC$, r is any other transversal intersecting I, m, n at D, E, F, then $DE = EF$.



Fig

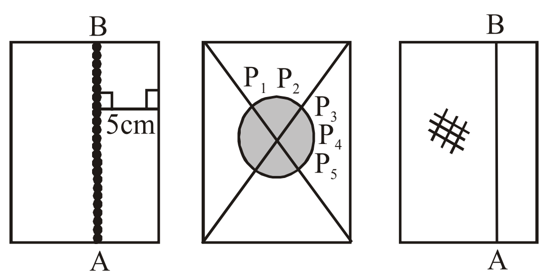
CHAPTER 4

THE EQUATION OF A LOCUS

Locus :

When a point moves so as to comply. with certain conditions, the path it traces out is called the locus of the point under these conditions. Thus a point which is completed to move on this page of paper so that it is always 5 cm from the right hand edge of the page can only move on a line which is parallel to the edge of the page and 5 cm away.

The conditions placed upon the point are : (*i*) the point must be on the page; (*ii*) the point must be on the pag ; (*ii*) the point must always be 5 cm from the right-handed edge. The locus of the point is the line segment *AB* given in Fig. We would arrive at the answer by plotting one or two possible positions, then a few more, until observing the answer as the line segment *AB*.



(a) (b) (c)

Fig

Another example would be to find the locus of a point which moved on this page so that it was always 5 cm from the centre *C* of this page. Again we note that the conditions to be complied with are : (*i*) the point must be on this page; (*ii*) the point must always be 5 cm from *C*. We try one or two points and then some more, as shown in Fig. and as we move from P_1 to P_2 to P_3, etc, we realise that we are plotting points on the circumference of a circle centre *C* and deduce that the locus is the circumference of this circle *i.e.* the set of all points which make up the circumference. If we relax the condition (*i*) that the point be on this page then we have to use a little imagination and 'see' that the locus is the surface of a sphere of radius 5 cm and centre *C*. This of course does imply that we always think of a sphere in this manner, *i.e*. that every point on its surface is the same distance from the centre.

As another example, consider finding the locus of a point which moves on this page so that it is equidistant from the two point A and B in above Fig. (a). As usual we start by identifying the conditions to be satisfied as: (i) the point must always be on this page ; (ii) wherever it is, it must be the same distance from A and B. We begin by plotting one or two trial points; the midpoint M of AB must be one point on the locus (assuming that there are others!) and, as illustrated in Fig. (c) the point N looks about right – certainly the point K looks wrong because it is nearer to A than B. Recalling the result of Fig.(b) we try a pair of compasses opened to a radius greater than AM and with centres alternately A and B we draw circles with the same radius and where they intersect will be a point which is the same distance from A as it is from B. Repeating this several times we eventually 'see' that the locus is a straight line though M perpendicular to AB. Thus each time, unless we 'know' the answer, we must build up the locus point by point until we identify its final form.

The Equation of a Locus

We describe any point P in the coordinate plane as $P\,(x, y)$ but, as soon as we impose conditions on its position, then it will only be able to move according to these conditions in what we have called a 'locus'. Imposing conditions on P means interpreting their influence of x and y. Thus we have seen that if P must move, so that it is only 3 units from the x axis, then the y co-ordinate of P must always be ±3 and so the conditions mean $y = 3$ and $y = -3$. The locus is therefore the two lines $y = 3$ and $y = -3$.

Similarly $x^2 + y^2 = 25$ is an equation of a locus which we determine in the usual way by finding as many points as necessary to deduce the complete curve. Thus (3, 4), (–3, 4), (–3, –4), (3, –4) are four points satisfying the equation but hardly enough to deduce that the locus will be a circle centre, the origin and radius 5 units. Indeed it will be very tedious to proceed in this way to deduce the locus. However, the reader should appreciate that any equation x and y which shall be dealing with can be represented by a curve or a straight line in the coordinate plane. What we need to do is to build a store of standard results which we can subsequently describe as 'well known'.

Consider how we can produce a straight line through a point. Firstly it must have the same gradient everywhere and secondly it will have to pass through the point. The basic problem of producing a particular line is thus expressed by the following example.

Example – 1. Find the equation of the straight line which has a gradient of –1 and passes through the point C (5, 2). Find also the point in which this line intersects the line $y = 3x - 5$.

Solution : We again use above Fig. and, as before, we suggest that the point $Q\,(x, y)$ lies on the line. Although we have not found the line yet, we do know that it slopes in the direction suggested because the gradient is negative.

We now calculate the gradient of the line by moving from C (5, 2) to $Q\,(x, y)$.

$$\therefore \text{ the gradient is } \frac{\text{change in } y}{\text{change in } x} = \frac{y-2}{x-5} = -1$$

$\therefore\ y - 2 = -1\,(x - 5)$

$i.e.$ $y = -x + 7$ is the required equation to the straight line.

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CHAPTER 5

POLYNOMIALS

Polynomials means an algebric expression consisting of many terms involving powers of the variable.

Definition : A function $p(x)$ of the from $p(x) = a_0 + a_1x + a_2x_2 + ... + a_nx^n$ where a_0a_1, a_2...., a_n are real number and n is a non-negative integer is called a polynomial in x, over reals.

Standard Form of a Polynomial : A polynomial is said to be in standard form if the powers of x are either in increasing or in decreasing order.

Some special types of Polynomials.

(1) Zero-polynomial : If in a polynomial all the coefficients are zero, then it is called the zero polynomial.

e.g. $0x^2 - 0x + 0$ is a zero polynomial

(2) Monomial : After combining like powers of x, if a polynomial consists pof a single term, it is called a monomial.

e.g., $3 -2x^2$, $7x^3$ etc. are called monomials.

(3) Binomial : If a polynomial consists of two terms, it is called a binomial.

e.g. $5x + 7$, $2x^3 + 3$ etc. are binomial consists of three terms, called a trainomial.

Powers of Polynomial

Notable Products

$(A + B)(A - B) = A^2 - B^2$

$(A + B + C)^2 = A^2 + B^2 + C^2 + 2AB + 2BC + 2CA$

Some Important Relations

$(A + B)^0 = 1$

$(A - B)^0 = 1$

$(A + B)^2 = A^2 + 2AB + B^2$

$(A - B)^2 = A^2 - 2AB + B^2$

$(A + B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$

$(A - B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$

$(A + B)^4 = A^4 + 4A^3B + 6A^2B^2 + {}^4AB^3 + B^4$

$(A - B)^4 = A^4 - 4A^3B + 6A^2B^2 - {}^4AB^3 + B^4$

$(A \pm B)^5 = A^5 \pm 5A^4B + 10A^3B^2 \pm 10A^2B^3 + 5AB^4 \pm B^5$

Standard form of a Polynomial : A polynomial $p(x)$ is said to be in standard form if the the powers of 'x' are either in increasing or decreasing order.

The power of a binomial can be calculated using the rules given below :

(i) The number of addendi of the development of the power of order n is ($x^0 = 1$).

(ii) The exponent of the letter x start from n and decrease by a single unit to zero.

For a binomial of the type.

$A^n \,.\, B^0 + A^{n-1}\, B + A^{n-2}\, B^2 + \ldots.\, AB^{n-1} + A^0B^n$

The exponents of letter A decrease from n to zero and those of letter B start from zero and increase by one. Hence the first addendum is A^nB^0 and the last one is A^0B^n.

(iii) The coefficients of terms equidistant from the ends are equal.

(iv) The sign of coefficients are always positive if it is a sum and alternately positive and negative if it is a difference.

(v) To obtain the coefficients, make use of PASCAL'S Traingle

		1								$n \to 0$
		1	1							1
	1	2	1							2
	1	3	3	1						3
	1	4	6	4	1					4
	1	5	10	10	5	1				5
1	6	15	20	15	6	1				6
1	7	21	35	35	21	7	1			7
1	8	28	56	70	56	28	8	1		8

Note :

(i) In Pascal's traingle we can observe that every line starts and ends with 1.

(ii) Every other number is obtained as a sum of two numbers immediately above it.

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CHAPTER 6

QUADRATIC EQUATIONS

An equation of the form $ax^2 + bx + c = 0$ where a, b, c are real and $a \neq 0$ is known as quadratic equation.

The two roots of a quadratic equation are given by the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The quantity $D = b^2 - 4ac$ is known the discriminant of the equation.

Sum and Product of the roots. If α and β are the two roots of the equation $ax^2 + bx + c =$ then

$$\alpha + \beta = -\frac{b}{a} = \frac{Coeff\ of\ x}{Coeff\ of\ x^2} \text{ and } \alpha\beta = -\frac{c}{a} = \frac{\text{Contant } term}{Coeff\ of\ x^2}$$

Nature of roots, For a given quadratic equation

if $D > 0$ then roots are real and unequal

if $D = 0$ then roots are real and equal and if $D < 0$ then roots are unreal and unequal.

Formation of quadratic equation, if its roots are given:

If α, β are the roots of a quadratic equation then the equation will be

$x^2 - (\alpha + \beta) + \alpha\beta = 0$

Some Important Hints.

(i) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

(ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

(iii) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

(iv) $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

Equations Reducible to Quadratic Form

There are some equations which after simplification or suitable substitutions are reduce to quadratic are as under

(1) Type $ax^2 + bx^2 + c = 0$

Hint. Substitute $x^n = y$

(2) Type $ay + \frac{b}{y} = c$, where y may involve radicals

For example : $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = \frac{13}{6}$ $\left(\frac{2x+3}{x+1}\right) + 6\left(\frac{x+1}{2x+3}\right) = 7$

Note. Put $\sqrt{\frac{x}{1-x}} = y$ *or* $\frac{2x+3}{x+1} = y$ respectively

(3) Type $\sqrt{ax+b} + \sqrt{cx+d} = \sqrt{ex+f}$ such as $\sqrt{2x+1} + \sqrt{3x+2} = \sqrt{5x+3}$

(4) Type $(x + a)(x + b)(x + c)(x + d) = K$ such as $(x + 1)(x + 2)(x + 3)(x + 4) + 1 = 0$

Note. Multiply $(x + 1)(x + 4)$ and $(x + 2)(x + 3)$ and then put $x^2 + 5x = y$.

Graph of the quadratic function

$ax^2 + bx + c = 0 \ (a \neq 0)$

Let $f(x) = ax^2 + bx + c$

Method. (i) Make a table of corresponding values for x and the function.

(ii) Plot these points, on a pair of $x, f(x)$ axis.

(iii) Draw a smooth line joining the plotted points.

A. *Complex Numbers*

(i) The set of complex numbers is denoted by C.
$C = \{a + ib;\ a \in R\}$, where $i^2 = -1$.

(ii) Given two complex numbers $a + ib$ and $c + id$, their sum is defined as the complex number $(a + c) + i\,(b + d)$.

(iii) Given two complex numbers $a + ib$ and $c + id$, their product is defined as the complex numbers $(ac - bd) + i\,(ad + bc)$.

(iv) For any complex number $a + ib$, $a - ib$ is called its **conjugate** complex number.

(v) Any complex number $a + ib$ can be represented by the ordered pair (a, b).

(vi) Properties of complex numbers
  (a) The set of complex numbers is a field.
  (b) The set of complex numbers is not an ordered field.

(vii) $x + iy = r(\cos\theta + i\sin\theta)$, where r is the modulus and θ is the amplitude of the complex number $x + iy$.

(viii) $\forall z_1, z_2 \in C,\ |z_1 z_2| = |z_1| \cdot |z_2|$

(ix) $\forall z_1, z_2 \in C,\ |z_1 + z_2| \leq |z_1| + |z_2|$

(x) De Moivre's Theorem
$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, where $\theta \in R$, $n \in N$.

(xi) Every non-zero complex number $a + ib$ has exactly two square roots.

B. *Permutation and Combination*

(i) Factorial Notation $n! = 1.2.3.4 \ldots\ldots..(n-1)n$

$n! = n(n-1)!,\ (n+1)! = (n+1)\,n!$

(ii) **Permutation.** An arrangement or ordering of the elements of a set is called a permutation of that set of elements.

(1) $^nP_r = n(n-1)(n-2) \ldots\ldots\ldots(n-r+1)$

$= \dfrac{n!}{(n-r)!}$, where $r \in N$ and $r \le n$

(2) The number of permutations of n different elements, taken r at a time, repetitions being permissible is n^r.

(3) If there are n elements in a set such that p are alike and of one kind, q are alike and of second kind, and r are alike and of third kind, then the number of different permutations is

$$\frac{n!}{p!q!r!}$$

(iii) **Combination.** A selection of elements from a set of given elements in which ordering does not matter is called a combination.

(1) $^nC_r = \dfrac{n(n-1)(n-2)\ldots\ldots.(n-r+1)}{1.2.3.4\ldots\ldots\ldots r}$

(2) $^nC_n = 1,\ ^nC_0 = 1,\ ^nC_1 = n$

(3) $^nC_r = {}^nC_{n-r}$

(4) If $^nC_x = {}^nC_y$ then $x = y$ or $x + y = n$

(5) $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

C. *Binomial Theorem*

(i) Binomial Expansion

$(x+a)^n = x^n + {}^nC_1x^{n-1}a + {}^nC_2x^{n-2}a^2 + \ldots\ldots.. + {}^nC_rx^{n-r}a^r + \ldots\ldots.. + {}^nC_na^n$

$\forall x, a \in C$ and $\forall\ n \in N$

The number of terms in the expansion $= n + 1$

The coefficient of $(r+1)^{th}$ term $= {}^nC_r$

(ii) Binomial Coefficients

(1) Sum of the binomial coefficients $= 2^n$

(2) $C_0+C_1+C_2+C_3+\ldots\ldots\ldots C_n = 2^n$

(3) $C_1+C_2+C_3+\ldots\ldots C_n = 2^n-1$

(4) Sum of the coefficients of odd terms

= Sum of the coefficients of even terms $= 2^n-1$

(iii) The sum of the coefficients in the expansion of $(1-x)^n$ is zero.

Prepared by ***Dr. C. R. Mallick***
Department of Mathematics, P.M.E.C. Berhampur

D. *Partial Fraction*

Form of the rational function	***Form of partial fraction***
(1) $\dfrac{px+q}{(x-a)(x-b)}$	$\dfrac{A}{x-a}+\dfrac{B}{x-b}$
(2) $\dfrac{px+q}{(x-a)^2}$	$\dfrac{A}{(x-a)}+\dfrac{B}{(x-a)^2}$
(3) $\dfrac{px+q}{x^2+ax+b}$ (where $x^2 + ax + b$ cannot be factorised)	No reduction keep it as it is
(4) $\dfrac{px^2+qx+r}{(x-a)(x-b)(x-c)}$ where a, b, c are distinct	$\dfrac{A}{x-a}+\dfrac{B}{x-b}+\dfrac{C}{x-c}$
(5) $\dfrac{px^2+qx+r}{(x-a)^2(x-b)}$ $(a \neq b)$	$\dfrac{A}{x-a}+\dfrac{B}{(x-a)^2}+\dfrac{C}{(x-b)}$
(6) $\dfrac{px^2+qx+r}{(x-a)^3}$	$\dfrac{A}{x-a}+\dfrac{B}{(x-a)^2}+\dfrac{C}{(x-a)^3}$
(7) $\dfrac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\dfrac{Bx+C}{x^2+bx+c}+\dfrac{A}{x-a}$

where $x^2 + bx + c$ can not factorised.

Note that there are as many constants to be determined as the degree of the denominator.

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Prepared by ***Dr. C. R. Mallick***
Department of Mathematics, P.M.E.C. Berhampur