Binomial Distribution:

A series of independent trials which result in one of the two mutually exclusive possibilities, i.e success / failure; such that the probability of success ~~are~~ or failure, in each trial is constant, then such independent repeated trial is known as BERNOULI's TRIALS.

let p = probability of success of one trial.

q = probability of failure of one trial.

$q = 1 - p$

Let us consider the random variable X_i such that;

X_i if ith trial is a failure

$$X_i = \begin{cases} 0, & \text{if ith trial is a failure} \\ 1, & \text{if ith trial is a success} \end{cases}$$

Let $X_1, X_2, \ldots, X_n$ are the independent random variables.

Let X = total no. of success in 'n' trials

i.e $X = X_1 + X_2 + \ldots + X_n$

Range of X

$V_X = \{0, 1, 2, \ldots, n\}$

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Let $(X=K)$ denote event of getting
K success in 'n' trials and
'n-k' failures, so this can be done in

^nC_K or n_k mutually exclusive
ways, the probability of each way
is given by

$\boxed{p^K . q^{n-K}}$ ways

$$P(x=K) = \binom{n}{k} p^k q^{n-k}$$

$$= P(K) = B_k(n,p)$$

$$0 \leq K \leq n$$

which is the probability of K success
in 'n' rejected trials.
This is known as Binomial distribution
from the following distribution we
conclude That:

→ The no. of trial is finite & fixed
→ in every trial, there are only
two possible outcomes: success/
failure;

→ The trials are independent,
→ The outcome of one trial does not
effect the other trial;
→ p is the probability of success
from trial to trial ile fixed.

→ q is the probability of failure i.e. $1-p$, so this is same in each trial.

Mean μ:

$$\mu = E(X) = np$$

Variance:

$$\sigma^2 = npq$$

Std devn:

$$\sigma = \sqrt{npq}$$

eg) A fair coin is tossed 6 times ~~[illegible]~~ repeatedly:

1) find the probability of exactly two head, p

2) Probability of getting atleast 4 heads.

3) P of atleast one head.

(Biased coin)

$P = 1/6 , q = 5/6 ; n = 6$

1) $B_2(n, p) = B_2(6, 1/6)$

$= \binom{6}{2}(1/6)^2(5/6)^4$

2) $B_4(6, 1/6) + B_5(6, 1/6) + B_6(6, 1/6)$

$\binom{6}{4}(1/6)^4(5/6)^2 +$

$\binom{6}{5}(1/6)^5(5/6)^1 +$

$\binom{6}{6}(1/6)^6(5/6)^0$

$nC_r \cdot p^r \cdot q^{n-r}$

$6C_2$

3) $1 - P(X=0)$ $\quad P(X=k), nC_k p^k q^{n-k}$

$= 1 - B_0(n, p)$

$= 1 - {}^6C_0 (1/6)^0 (5/6)^{10}$

eg) Determine the B.D. whose mean is 3, $\sigma^2 = \sqrt{3/2}$ $\boxed{n=12}$

$\mu = np = 3$

$\sigma^2 = \sqrt{npq} = \sqrt{3/2}$ $\boxed{q = 3/2^2}$

$npq = 9/4$ $\boxed{p = 1/2^2}$

$$B.D = (p+q)^n$$

$$= \left(\frac{1}{4}+\frac{3}{4}\right)^{12}$$

$$= 1$$

Every time the B.D = 1

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eg) There are 2% defective nuts in a large bulk of nuts, write the probability distribution for getting defective nuts in a random sample of 10 nuts.

$$P(\text{Defective}) = \frac{2}{100} = \frac{1}{50} = p$$

$$P(\text{Good nuts}) = \frac{49}{50} = q$$

$$B.D = (p+q)^n$$

$$= \left(\frac{1}{50}+\frac{49}{50}\right)^{10}$$

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eg) A die is tossed thrice. A success is getting 1 or 6 on a toss, find the mean and variance of the success.

$p = 2/6 = 1/3$, $q = 2/3$

$n = 3$

$\mu = np = 1$

$\sigma^2 = npq = 2/3$

eg) 5 cards are drawn successively with replacement from a well shuffled deck of 52 cards, what is the probability that all the

1) 5 cards are spades.

2) Only 3 cards " "

3) None is a spade.

$P = \frac{13}{52} = 1/4$, $q = 3/4$, $n = 5$

1) $\binom{5}{5}\left(3/4\right)^0 \left(1/4\right)^5 = \frac{1}{1024}$

2) $\binom{5}{3}\left(1/4\right)^3 \left(3/4\right)^2$

3) $\binom{5}{0}\left(1/4\right)^0 \left(3/4\right)^5$

eg) If the probability of hitting a target is 25%, and 4 shots are fired independently, what is the probability, the target is hit atleast once.

$P = 1/4$, $3/4 = q$, $n = 4$.

$\sum_{x=1}^{4} \binom{4}{x} (1/4)^x (3/4)^{4-x}$

POISSION DISTRIBUTION

→ Given by – simund-D– poisson from french
→ Limiting case of binomial distribution

from B.D: $B_k(n.p) = {}^{n}C_k\, p^k q^{n-k}$

if $n \to \infty$
$k \to 0$
$(n.p) \to \lambda$

- it is a limiting condn of B.D.
- In B.D. we know the no. of times an event does occure & also the times an event doesn't occure.
- But there are cases p is very small and n is very large. (P(success)), the calculation will be long
- so we have to use P.D.

eg) The no. of suicides / deaths by heart attack in time 't'.

eg2) The no of printing mistakes on each page of a book.

eg3) The no. of defective material for pkg manufactured by a good concern (-)

eg 4) No. of cars passing per minute.

eg 5) No. of telephone calls per minute.

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1) Conditions of B.D to be P.D :

a) No. of trials 'n' is indefinitely large, i.e $n \to \infty$

b) p its success approaches to '0'

$p \to 0$.

c) $(n, p) \to \lambda$ or $(n,p) = \lambda$

$p \to \lambda/n$

In BD :

$B_k(n\ p) = \binom{n}{k} p^k q^{n-k}$

$= \binom{n}{k} (\lambda/n)^k (1-\lambda/n)^{n-k}$

In P.D

$$P(X=k) = \frac{e^{-\lambda}\lambda^k}{k!} = P(k)$$

This is called P.D of the random variable P(k) and is also called poisson's probability funn, and poisson variant.

k	0	1	2
P(k)	$e^{-\lambda}$	$e^{-\lambda}\lambda$	

Mean

$\mu = \lambda$

Variance

$\sigma^2 = \lambda$

Standard deviation

$\sigma = \sqrt{\lambda}$

$\boxed{2 < e < 3}$

eg) A Book contains 100 misprints distributed randomly through out its 100 pages, what is the probability that a page observed at random contains atleast ② misprints

$n = 100$

Let k = no. of misprints in each page

$\lambda = \frac{100}{100} = \frac{n}{p}$ $n \times \frac{1}{p} = n \times p$

∴ 100 pages contains 100 misprints

$P(x \geq 2) = 1 - [P(0) + P(1)]$

$= 1 - P(x \leq 2)$

$= 1 - \left[\frac{e^{-1}(1)^0}{0!} + \frac{e^{-1} \times 1}{1!} \right]$

$\simeq 1 - \left[\frac{2}{e}\right]$

$\simeq 0.264$

2) It is known from a past experience that in a certain plant, there are on an average 4 industrial accidents per month. Find the probability that in a given year there will be less than 4 accidents.

$\boxed{\lambda = 4}$

$P(0) + P(1) + P(2) + P(3)$

$P(x < 4)$

3) If a ticket office can serve 4 customers per minute and the average no. of customers is 120/hr, what is the probability that during a given minute customers will have to wait.

$\lambda = \frac{120}{60} = 2$

$1 - [P(0) + P(1) + P(2) + P(3) + P(4)]$

5) If the probability that an individual suffers a bad reaction from a certain injection is 0.001, find the probability that out of 2,000 individuals
1) Exactly 3 individuals will suffer a bad reaction
2) None will suffer a bad reaction
3) More than 1 individual will suffer

1)

$$P(3) = \frac{e^{-2} \cdot 2^{3}}{3!} = 0.18$$

2) $P(0) = 0.13$

3) $P(2) + P(3) + P \ldots$

$$= 1 - [P(0) + P(1)]$$

$$= 1 - \frac{e^{-2} \cdot 2^{0}}{0!} + \frac{e^{-2} \cdot 2^{1}}{1!}$$

6) A certain factory turning out razor blades, is a small probability of 1/500 for any blade to be defective. Blades are supplied in pkts of 10. 80, use PoD to calculate approximate no. of packets containing no defective, 1 defective, 2 defective

Blades are in a bunch of 10k packets.

Ans) $P(D) = 1/500 = 0.002$

P be the probability of D. Blades

$n = 10$

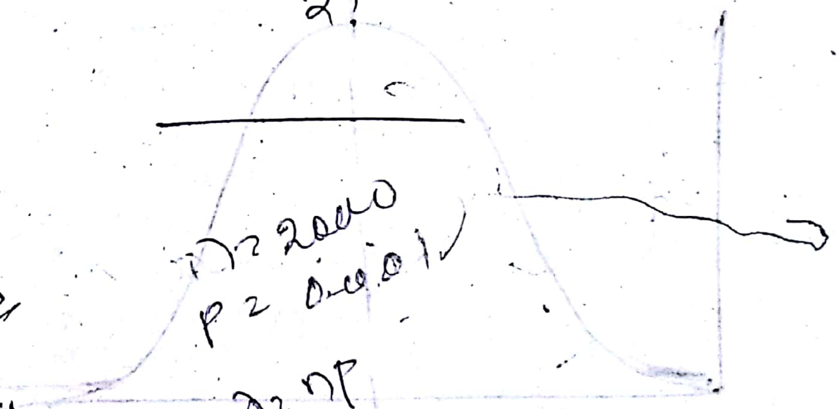
$\lambda = np = 0.02$

$f(K) = 10,000 \times P(K)$

$$f(0) = \frac{e^{-0.02}(0.02)^0}{0!} \times 10,000$$

$$f(1) = \frac{e^{-0.02} \times (0.002)^1 \times 10,000}{1!}$$

$$f(2) = \frac{e^{-0.02}(0.02)^2}{2!} \times 10,000$$



$$\frac{e^{-\lambda}\lambda^K}{K!}$$

$$\frac{e^{-\lambda}\lambda^x}{x!}$$

$$\frac{e^{-\lambda}\lambda^4}{4!}$$

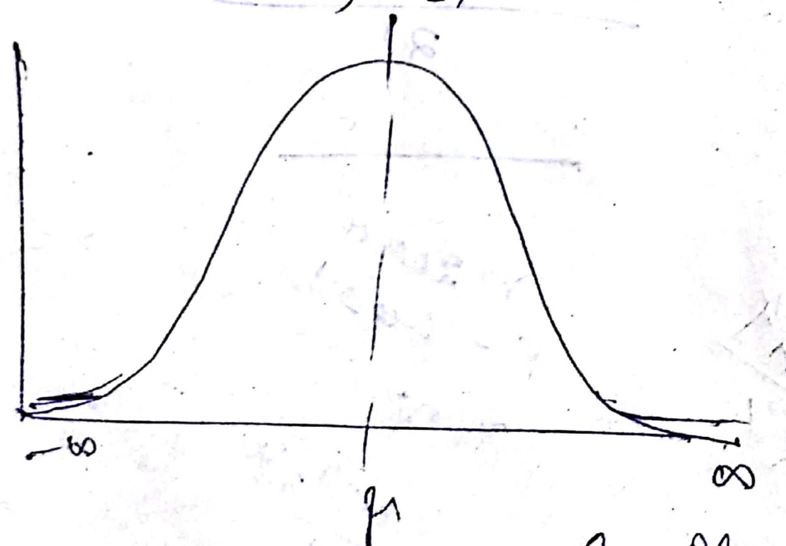
Normal Distribution:

→ Limiting case of B.D.

→ $B_k(n,p) = \binom{n}{k} p^k q^{n-k}$

N.D is a continuous ~~P.D~~ probability distribution in which the relative frequencies of a continuous variable are distributed according to the normal probability law.

In other words, it is a symmetrical distribution in which frequencies are distributed evenly about the mean of the distribution.



N.D is the limiting form of B.D under the following conditions:

1) n (no. of trials is v. large) $n \to \infty$

2). Neither p nor q is very small, p and q near equal.

3) 'X' (the Random variable) is said to have a N.D. with mean μ and S. devⁿ σ if its probability density funⁿ

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$-\infty < x < \infty$$

$$-\infty < \mu < \infty$$

$$\sigma > 0.$$

$$e = 2.7183$$

$$\sqrt{2\pi} = 2.5060$$

The P.D. funⁿ with $\mu = 0$ and Standard devⁿ σ is given by

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

$\therefore$ μ and σ are called parameters.

- The graph of the curve is bell shaped extending in both directions, and moving near and nearer to the horizontal axis but never touches it.

- The line $x = \mu$ divides the total area under the curve which is equal to 1 in to 2 equal parts area to the right = left = 0.5

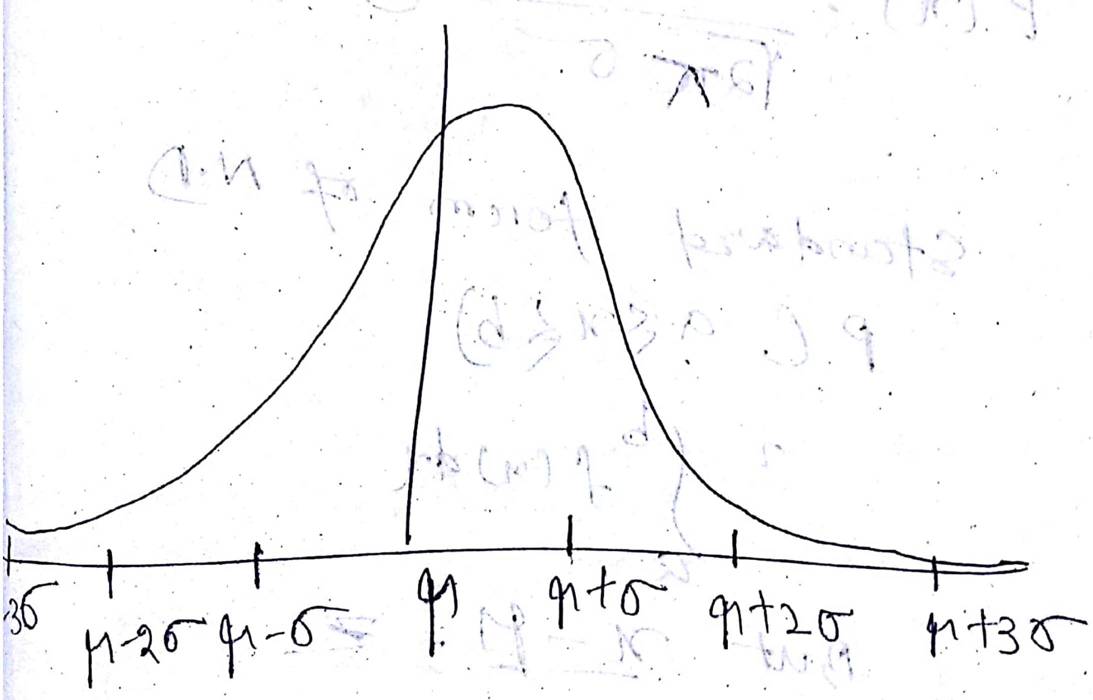
- $\mu = \mu$ (mean)
- σ^2 = varriance
- $s.d = \sigma$

Properties of Normal curve:

→ The normal probability curve, with mean $= \mu$ and $s.d = \sigma$ have the following properties:

The equn: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ is bell shaped

- The curve is symmetrical about the line, $x = \mu$.
- mean, median & mode coincides
- The point of ~~inflation~~ inflexion (1st order derivative $= 0$) of the curve are at $(x = \mu + \sigma)$ and $(x = \mu - \sigma)$
- and the curve changes from concave to convex at $x = \mu + \sigma$ to $x = \mu - \sigma$.



- The total area under the normal curve is equal to unity and the % distribution of area under the normal curve are as follows :-

a) About 68% of area falls b/w $(\mu-\sigma)$ to $(\mu+\sigma)$

b) About 95.5% of area falls b/w $(\mu-2\sigma)$ to $(\mu+2\sigma)$

c) About 99.7% of area falls b/w $(\mu-3\sigma)$ to $(\mu+3\sigma)$

Deductions :

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Standard form of N.D

$$P(a \le x < b) = \int_a^b f(x)\,dx$$

But $\frac{x-\mu}{\sigma} = z$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

The random variable Z is called standard normal vari let $z_1 = \frac{a-\mu}{\sigma}$

$z_2 = \frac{b-\mu}{\sigma}$

$P(a < x < b)$

$P(z_1 \leq z < z_2)$

$$= \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-\frac{z^2}{2}} dz \quad \boxed{\int_{z_1}^{z_2} e^{-\frac{z^2}{2}} \cdot dz}$$

- The integral will represent area bounded by the curve b/w the 'z' axis and line $z = z_1$ and $z = z_2$

$$P(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-z^2/2} \cdot dz$$

- $f(z)$ is a probability density fun^n for normal distribution.

$f(z) =$

$$P(z_1 \leq z \leq z_2) = \int^{z_2} f(z)\, dz$$

$$f(z) = \int_{\infty}^{z} f(z)\, dz \quad z_1$$

$$= \phi(z)\, P(Z \leq z)$$

(1) $$\int_{-\infty}^{\infty} \phi(z)\,dz = 1$$

(2) $$\int_{-\infty}^{0} \phi(z)\,dz = \int_{0}^{\infty} \phi(z)\,dz = 1/2$$

(3) $P(-\infty \leq z \leq \infty) = 1$

(4) $P(-\infty \leq z \leq 0) = 1/2$

(5) $P(0 \leq z \leq \infty) = 1/2$

or $P(z \geq 0)$

(6) $P(-\infty < z < z_1) =$

$P(-\infty < z \leq 0)$

$+ P(0 \leq z < z_1)$

If $E = \frac{1}{2} + \phi$

$\Rightarrow P(z < z_1) = 0.5 + \phi(z_1)$

$\Rightarrow P(z > z_2) = P(z \geq 0)$

$- P(0 \leq z < z_2)$

$= 0.5 - \phi(z_2)$

1) Evaluate the following probabilities with the help of normal probability tables

a) $P(z \geq 0.85)$

b) $P(-1.64 \leq z \leq -0.88)$

c) $P(z \leq -2.43)$

d) $P(|z| \leq 1.94)$

a) $P(z \geq 0) - P(z \leq 0.85)$

$= 0.5 - \phi(0.85)$

$= 0.5 - 0.3023$

$= 0.1977$

b) $P(-1.64 \leq z \leq -0.88)$

$= P(0.88 \leq z \leq 1.64)$

$= P(0 \leq z \leq 1.64) - P(0 \leq z \leq 0.88)$

$= 0.5 + \phi(1.64) - 0.5 - \phi(0.88)$

3) $P(z \le -2.43)$

$= P(2.43 \le z)$

$= P(z \ge 0) - P(0 \le z \le 2.43)$

$= 0.5 - \phi(2.43)$

$= 0.5 - 0.4925$

$= 0$

4) $P(|z| \le 1.94)$

$P(-1.94 \le z \le 1.94)$

$= P(-1.94 \le z \le 0) +$

$P(0 \le z \le 1.94)$

$= P(0 \le z \le 1.94) +$

$P(0 \le z \le 1.94)$

$= 2P(0 \le z \le 1.94)$

$= 2P \phi(1.94)$

$= 2P \times 0.4738$

eg2> If x is normally distributed with $\mu = 12$ and $\sigma = 4$, find $P(x \geq 20)$, $P(x \leq 20)$

$\mu = 12, \sigma = 4$

$$Z = \frac{x-\mu}{\sigma}$$

$$= \frac{x-12}{4} = \frac{x}{4} - 3$$

when $x = 20$

$z = 2$

so, to find $P(Z \geq 2)$ and $P(Z \leq 2)$.

1> $= 0.5 - \phi(2)$

2> $P(Z \leq 2) =$

$P(-\infty \leq z \leq 0) +$

$P(0 \leq Z \leq 2)$

~~0.0~~

$P(Z \geq 2)$

$P(Z \geq 0) - P(0 \leq z \leq 2)$

$= 0.5 - \phi(2)$

eg 3) In a N.D, 31% of items are under 45 and 81% of items are over 64. find the μ and σ

$P(x<45) = 31\% = 0.31$

$P(x>64) = 0.81$

$= z = \frac{x-\mu}{\sigma}$

= when $x = 45$

$\boxed{\frac{45-\mu}{\sigma} = z_1}$ (1)

$= 45-\mu = 0.31\sigma$

$= 0.31\sigma + \mu - 45 = 0$ (2)

when $x = 64$

$= 0.81 = \boxed{\frac{64-\mu}{\sigma} = z_2}$

$P(z<z_1) = 0.31$

and $P(z>z_2) = 0.81$

$0.5 + \phi(z_1) = 0.31$

and $0.5 - \phi(z_2) = 0.81$

$$\phi(z_1) = -0.19$$
$$\phi(z_2) = -0.31$$

$$z_1 = 0.1915 \approx 0.19$$

$$\phi(0.75)$$
$$\phi(z_1) = -\phi(0.5)$$
$$\phi(z_2) = -\phi(8.8)$$

$$\boxed{\begin{aligned} z_1 &= -0.5 \\ z_2 &= -8.8 \end{aligned}}$$

substituting in eq 1 and eq 2, get the values of μ and σ

$$\boxed{\sigma = 50, \mu = 20}$$

0.31 0.81