PROBABILITY

$$P(A) = \frac{|A|}{|\Omega|}$$

$$0 \leq P(A) \leq 1$$

$$P(\phi) = 0 \ , \ P(\Omega) = 1$$

Ω = sample space.

Mutually Exclusive Event :

Occurence / Non-occurance of one doesn't effect the occurence / Non-oc... of the other.

eg → $\Omega = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 4, 6\}$

$B = \{1, 3, 5\}$

$A \cap B = \phi$

- Two events are said to be mutually exclusive if occurence or Non-occurence of 1st doesn't effect / impact the occurence / Non-occurence of other.
- In other words 2 events A and B are said to be mutually exclusive if $A \cap B = \phi$

$P(\phi) = 0$. ($\because$ 8 days in a week).

EQUALLY LIKELY :

- 2 outcomes are said to be equally likely if they have equal

Chance of occurence.

Independent Event:

Two events are said to be independent if the occurence of one doesn't effect the occurence of other.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

if $A \cap B = \phi$

$P(\phi) = 0$.

then:

$$P(A \cup B) = P(A) + P(B)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

For any $(A \cup B \cup C \cup \ldots \cup n)$

odd terms are added

even terms are subtracted.

eg, $P(A \cup B \cup C \cup D)$

$$= P(A) + P(B) + P(C) + P(D) - P(A \cap B) - P(A \cap C) - P(A \cap D) - P(B \cap C) - P(B \cap D) - P(C \cap D) + P(A \cap B \cap C) + P(B \cap C \cap D) + P(A \cap B \cap D) + P(A \cap C \cap D) - P(A \cap B \cap C \cap D)$$

eg> Suppose 3 coins are tossed, so what is the probability of getting

1> All heads
2> 2 heads
3> 1 head
4> At least 1 head
5> At most 1 head

S.S.

$\left\{ \begin{array}{ccc} H & H & H \\ H & H & T \\ H & T & H \\ H & T & T \\ T & H & H \\ T & H & T \\ T & T & H \\ T & T & T \end{array} \right\}$

1/8, 3/8, 3/8, 7/8, ~~3/8~~ 4/8

eg 2> Dice 2 dice are thrown simultaneously find the probability of getting an even no. as sum.

2> sum as a prime no.

3> Total of atleast 10.

4> same no. on both dies.

$\Omega = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6) \}$

$P(A) = 1/2$

$P(B) = 1/9$

$P(C) = 1/4$

$P(D) = 1/6$

Q3) Find the probability of a leap year that is selected at random will contain 53 Sundays

$P(A) = 2/7$

PROBABILITY BASED ON COMBINATIONS

eg 1) A bag contains 9 red, 7 white, 4 black balls. If 2 balls are drawn at random find the probability that both the balls are red

1) $\dfrac{{}^{9}C_2}{{}^{20}C_2}$

2) one ball is white. $\cancel{7/20}$ ${}^{7}C_1 \times {}^{13}C_1 / {}^{20}C_2 = \dfrac{{}^{7}C_1 \times {}^{13}C_1}{{}^{20}C_2}$

3) both are of same colour.

$\dfrac{{}^{9}C_2 + {}^{7}C_2 + {}^{4}C_2}{{}^{20}C_2}$

eg 2) Find the probability of getting an even no. on 1st die, or a total of 8 in a single throw of 2 dice.

$\Omega = 36$

$A = 18 : \{(2,1), \dots (2,6), (4,1) \dots (4,6), (6,1) \dots (6,6)\}$

$B = 5 \quad \{(3,5), (5,3), (4,4), (6,2), (2,6)\}$

~~$A \cap B =$~~

~~$P(A \cap B) = 3/36 = 1/12$~~

~~$A \cup B = 3$~~

~~$P(A \cup B) = 1/12$~~

~~$P(A \cap B) = 1 - 1/12$~~

~~$= 11/12$~~

$A \cap B = 3$.

$P(A \cap B) = 1/12$

$P(A) = 1/2$

$P(B) = 5/36$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 1/2 + 5/36 - 1/12$

failure of the apparatus.

Ans) $P[\text{failure}] = \cancel{0.07}\ 0.04$

$p^c = 1 - 0.04$

$= 0.96.$

As the apparatus contains 3 tubes all 3 tubes are operatives,

$P\left[\begin{smallmatrix}\text{all 3 tubes}\\ \text{operative}\end{smallmatrix}\right] = 0.96^3 = 0.88476$

$P[I]$, $p^c = 1 - 0.88476$

$= 0.115264.$

CONDITIONAL PROBABILITY

eg → I will take umbrella if it rains.
condition →

eg → Attendance must be 75% to attend BPUT examination.

Def^n →

let A and B be 2 events associated with sample space Ω, then the probability of occurence of the event 'A' when 'B' is occured is called conditional probability of 'A'.

i.e $P(A/B) = \frac{P(A \cap B)}{P(B)}$, $P(B) \neq 0$

similarily

$P(B/A) = \frac{P(A \cap B)}{P(A)}$, $P(A) \neq 0$

i.e if the condition is rain happens then I will take umbrella.

eg1) Consider all families with 2 children, assume that the boys and girls are equally likely, if a family is chosen at random, and found to have a boy, what is the probability that it has another boy.

$S = [BB, BG, GB, GG]$, $B = [BB, BG, GB]$

$P[B] = 3/4$

$A = \{(B, B)\}$

$A \cap B = \{B, B\}$

$P(A/B) = \frac{1/4}{3/4}$

$= 1/3$

---

eg 2) Consider all families with 2 children a child is chosen at random from that family and found to have a boy, what is the probability that the family has also a boy, assume that boys and girls are equal likely.

$\Omega = \{b_b, bg, g_b, g_g\}$

$B = \{b_b, bg\}$

$A = \{b_b\}$

$A \cap B = \{b_b\}$

$P(A) = 1/4$

$P(B) = 1/2$

$P(A \cap B) = 1/4$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/2}$

$= 1/2$

BAYE'S THEOREM :

Proportion 1:

for any Arbitrary event $A_1, A_2, \dots A_n$ of sample space Ω,

$$P(A_1 A_2 \dots A_n) = P(A_1) \cdot P(A_2/A_1) \cdot P(A_3/A_1 A_2) \dots P(A_n/A_1 A_2 \dots A_{n-1})$$

Provided

$$P(A_1 A_2 \dots A_{n-1}) \neq 0$$

Proportion 2:

Let $A_1, A_2, \dots A_n$ are ~~sample~~ partition of sample space Ω, i.e

$$\Omega = \sum_{i=1}^{n} A_i = A_1 + A_2 + \dots A_n$$

and 'B' be any subset of Ω then

$$P(B) = \sum P(A_n) \cdot P(B/A_n)$$

$$= P(A_1) \cdot P(B/A_1) + P(A_2) P(B/A_2) + \dots + P(A_n) \cdot P(B/A_n).$$

Statement :

Let B be any non-empty subset of the sample space Ω, and $A_1, A_2, \dots A_n$ be the partition of Ω, then

$$P(A_n/B) = \frac{P(A_n) \cdot P(B/A_n)}{\sum P(A_n) \cdot P(B/A_n)}$$

eg:7 6 dies are rolled once, what is the probability of getting 6 different faces.

S.S, $\Omega = \{6^6\}$

$= \frac{6!}{6^6} = \frac{6\times5\times4\times3\times2\times1}{6^6}$

$= 0.0154$

$= 1.54\%$.

Ans7

let A_1 denote any face shown by die 1

A_2 denote die 2, showing different faces other die 1.

A_3 denotes die 3, showing different faces other than die 1, 2

A_4 " " 4, " " " " " die 1, 2, 3

A_5 " " 5 " " " " " " 1, 2, 3, 4,

A_6 denotes die 6, showing " " " " " 1, 2, 3, 4, 5

Event of getting 6 different faces is :

$$P(A_1 A_2 A_3 A_4 A_5 A_6)$$

$$= P(A_1)\, P(A_2/A_1) \cdot P(A_3/A_1 A_2)$$

$$\cdot\, P(A_4/A_3 A_2 A_1) \cdot P(A_5/A_1 A_2 A_3 A_4)$$

$$\cdot\, P(A_6/A_1 A_2 A_3 A_4 A_5)$$

$P(A_1) = S.S, \ \Omega = 6,$

$P(A_1) = 6/6$ (as, any face)

$$= \frac{6}{6} \cdot \frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} \cdot \frac{2}{6} \cdot \frac{1}{6}$$

$$= \frac{6!}{6^6}$$

RANDOM VARIABLES

~~Def^n : A random variable which is defined as a finite sample space~~

A random variable is a real valued function defined over a sample space Ω, i.e a variable whose value is a no. determined by the sample point of a sample space is called a random variable.

Random variable 'x' is a fun^n whose values are real no. which depends on chance.

A random variable is of 2 types
- → Discrete
- → Continuous

eg) Let 'X' be a Random variable which is no. of heads obtained in 2 tosses of a coin.

$\Omega = \{HH, HT, TH, TT\}$

$X(HH) = 2$

$X(HT) = 1$

$X(TH) = 1$

$X(TT) = 0$

$X = \{2, 1, 1, 0\}$ (Terms are not repeated)

$= \{0, 1, 2\}$

eg 2) let 'X' be a random variable which is the no. of heads obtained when 3 coins are tossed.

$S = \Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$X(HHH) = 3$

$X(HHT) = 2$

$X(HTH) = 2$

$X(HTT) = 1$

$X(THH) = 2$

$X(THT) = 1$

$X(TTH) = 1$

$X(TTT) = 0$

$X = \{0, 1, 2, 3\}$

<u>Discrete</u>: A random variable is said to be discrete if it assumes only finite or infinite no. of values, such as $\{0, 1, 2, 3\}$ or $\{0, 1, 2, 3, \ldots\}$

Scanned by CamScanner

Continuous: If a R.V. takes on all values within a certain interval then the R.V is called continuous R. variable.

Probability distribution fun of a random variable:

A random variable 'X' has value $x_1, x_2, \dots x_n$ with probabilities $P_1, P_2, \dots P_n$.

Then different values of a random variable together with their probabilities form a probability distribution.

$$X = x_1, x_2 \dots x_n$$
$$P(X) - P(x_1)\ P(x_2) \dots P(x_n)$$

Mean and variance of a Random variable:

Mean: μ (P_i)

$$\mu = \frac{\sum X_i\ f_i}{\sum P_i} = \sum P_i = 1$$

$$= \boxed{\sum x_i P_i}$$

Scanned by CamScanner

Variance :

$$\sigma^2 = \sum (x_i - \mu)^2 p_i$$

Standard deviation :

Positive square root of variance is known as standard deviation.

---

eg) A die is rolled twice, a
1) success is getting an odd no. on each toss, find the probability distribution of no. of successes.

sln) let X be the random variable that denotes no. of success obtained in 2 dies in 2 tosses of die.

X takes values $\{0, 1, 2\}$

$p(\text{success}) = 1/2 = p$

$p^c = 1/2 = q$ $\quad p^c = 1 - p(\text{succ}) = 1 - p = q = 1 - 1/2$

$P(X=0) = qq$
$= 1/2 \times 1/2 = 1/4$

$P(X=1) = pq + qp$
$= 1/4 + 1/4 = 1/2$

$P(X=2) = pp$
$= 1/2 \times 1/2 = 1/4.$

Scanned by CamScanner

X	0	1	2
P(X)	1/4	1/2	1/4

7. 1) Find the probability distribution of no. of 6's in 3 tosses of a die

$X = \{0, 1, 2, 3\}$

$P(S) = 1/6$ $\quad 1 - \frac{1}{6} = \frac{6-1}{6} = \frac{5}{6}$

$q = 5/6$

$P(X=0) = 5/6 \cdot 5/6 \cdot 5/6 = 0.578$

$P(X=1) = (5/6 \cdot 1/6 \cdot 5/6)\,3$

$P(X=2)\ (1/6 \cdot 1/6 \cdot 5/6)\,3$

$P(X=3) = 1/6 \cdot 1/6 \cdot 1/6$

X	0	1	2	3
P(X)	0.578	0.347	0.069	4.628×10^{-3}

Here — 3 tosses is Random variable
No. of 6's is success.

3) A bag contains 4 white balls and 3 red balls, find the probability distribution of no. of red balls in 3 drawn.

soln) $X = \{0, 1, 2, 3\}$

Total Balls = 7

$P(\text{Redballs}) = 3/7 = p$

$P(f) = 4/7 = q$

$P(X=0) = (4/7 \cdot 4/7 \cdot 4/7)$

$P(X=1) = (4/7 \cdot 4/7 \cdot 3/7) \times 3$

$P(X=2) = (4/7 \cdot 3/7 \cdot 3/7) \times 3$

$P(X=3) = (3/7 \cdot 3/7 \cdot 3/7)$

---

4) 3 cards are drawn successively from a well shuffled deck of 52 cards, a random variable x denotes no. of spades in the 3 cards, find the probability distribution of x.

$X = \{0, 1, 2, 3\}$

$p = 1/4, \quad q = 3/4$

$P(X=0) = 3/4 \cdot 3/4 \cdot 3/4$

$P(X=1) = (3/4 \cdot 3/4 \cdot 1/4)\, 3$

$P(X=2) = (3/4 \cdot 1/4 \cdot 1/4)\, 3$

$P(X=3)$
$1/4 \times 1/4 \times 1/4$

1) Mean, $\mu = \sum p_i x_i$

$= 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4}$

$= 1$

Variance $= \sigma^2$

$= \sum (x_i - \mu)^2 p_i$

$= (0-1)^2 \times \frac{1}{4} + (1-1)^2 \times \frac{1}{2}$

$+ (2-1)^2 \cdot \frac{1}{4}$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

2)

X	0	1	2	3
P(X)	0.578	0.347	0.069	4.62×10^{-3}

$\mu = 0.347 + 0.138 + 0.013$

$= 0.49886$

$\sigma^2 = (0-0.499)^2 \times 0.578 +$

$(1-0.499)^2 \times 0.347 +$

$(2-0.499)^2 \times 0.069 +$

$(3-0.499)^2 \times 4.62 \times 10^{-3}$

Def: Distributio... variable ... defined ...

Properties

Distrib... i.e,

Distr... i.e

DISTRIBUTION FUN^n

Def^n:
Distribution fun^n 'F' for random variable 'X' is a real valued fun^n defined by:

$F_X : R \rightarrow R$ such that

$F_X(x) = P(X \leq x)$, $-\infty < x < \infty$

Properties:

$P(a < X \leq b) = F_X(b) - F_X(a)$

$P(a \leq X \leq b) = P(X = a) + F_X(b) - F_X(a)$

$P(a \leq X < b) = P(X = a) - P(X = b) + F_X(b) - F_X(a)$

$P(a < X < b) = F_X(b) - F_X(a) - P(X = b)$

- Distribution fun^n is bounded in 0, 1
  i.e,
  $0 \leq F(X) \leq 1$
- Distribution fun^n is non decreasing i.e increasing
- D.F 'F' has a limit 0 and 1 at $-\infty$ and $+\infty$.
  $F(-\infty) = 0$, $F(\infty) = 1$

Discrete probability distribution:

~~A fun~~ If for each value of x_i of a discrete Random variable 'x', we assign a real no. $P(x_i)$ such that:

(a) $P(x_i) \geq 0$

(b) $\sum P(x_i) = 1$

then the fun^n $p(x)$ is called probability fun^{n}s.

The set x_i associated with $P(x_i)$ is called discrete probability distribution of discrete random variable 'X'.

The fun^n $P(x)$ is called probability density fun^n (PDF) or probability Mass fun^n (PMF).

Mean and variance of D. P. Distribution

Mean = μ

$$\mu = \sum x_i P(x_i)$$

Variance = σ^2

$$\sigma^2 = \sum (x_i - \mu)^2 P(x_i)$$

eg) Show that the following distribution represents Discrete P.D., find μ and σ^2 :

x	10	20	30	40
$P(x)$	$1/8$	$3/8$	$3/8$	$1/8$

sol) Proof :

i) $P(x_i) \geq 0$

ii) $\sum P x_i = 1$ $\left[\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8} = \frac{8}{8} = 1\right]$

$\mu = \sum x_i P(x_i)$

$10/8 + 20 \times \frac{3}{8} + \frac{90}{8} + \frac{40}{8}$

$= 200/8 = 25$

$\sigma^2 = \sum (x_i - \mu)^2 \, P(x_i)$

$= (10-25)^2 \times \frac{1}{8} + (20-25)^2 \times 3/8$

$+ (30-25)^2 \times \frac{3}{8} + (40-25)^2 \times 1/8$

$= 28.125 + 9.375 + 9.375 + 28.125$

$= 56.25 + 18.75$

$= 75.$

eg) Find the value of 'K' such that the following represents finite probability distribution find μ and σ^2

also find, $P(X \le 1)$, $P(X > 1)$

$P(-1 < x < 2)$

X	-3	-2	-1	0	1	2	3
$P(X)$	k	$2k$	$3k$	$4k$	$3k$	$2k$	k

$k + 2k + 3k + 4k + 3k + 2k + k = 16$

$k = 1/16 = 0.0625$

$\mu = \sum x_i P(x_i)$

X	-3	-2	-1	0	1	2	3
$P(X)$	$1/16$	$2/16$	$3/16$	$4/16$	$3/16$	$2/16$	$1/16$

$\mu = -\frac{3}{16} - \frac{4}{16} - \frac{3}{16} + 0 + \frac{3}{16} + \frac{4}{16} + \frac{3}{16}$

$= 0$

$\sigma^2 = \sum (x_i - \mu)^2 P(x_i)$

$= 9 \times \frac{1}{16} + 4 \times \frac{2}{16}$

$+ 1 \times \frac{3}{16} + 0 + 1 \times \frac{3}{16} + 4 \times \frac{2}{16}$

$+ 9 \times \frac{1}{16}$

$= 5/2 = 2.5$

$\sigma = 1.58$

Scanned by CamScanner

$$P(X \le 1) = P(1) + P(0) + P(-1) + P(-2) + P(-3)$$

$$= 3/16 + 4/16 + 3/16 + 2/16 + 1/16$$

$$= 13/16$$

$$P(X > 1) = P(2) + P(3)$$

$$= 2/16 + 1/16$$

$$= 3/16$$

$$P(-1 < x < 2) = P(0) + P(1)$$

$$= 4/16 + 3/16$$

$$= 7/16.$$

Continuous Probability Distribution

for every 'x' belonging to the range of a continuous random variable 'X', so we assign a real valued fun^n, i.e. $f(x)$ which satisfies following conditions:

1) $f(x) \geq 0$

2) $\int_{-\infty}^{\infty} f(x) \cdot dx = 1$

then $f(x)$ is called continuous P.D. or probability density fun^n PDF.

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

Cumulative distribution fun^n:

If 'X' is a continuous P.D. with density fun^n $f(x)$, then the fun^n $F(x)$ is defined by $P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$$

$$\frac{d}{dx} F(x) = f(x)$$

Mean / Expectation :

$$\mu = \int_{-\infty}^{\infty} x f(x)\,dx = E(X)$$

Variance :

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

Exponential distributions :

The continuous probability distribution having the probability density func $f(x)$ given by

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & x > 0 \\ 0 & \text{otherwise} \end{cases} \qquad \boxed{\alpha > 0}$$

it's known as exponential distribution.

Mean :

$$\mu = \frac{1}{\alpha}$$

Variance :

$$\sigma^2 = 1/\alpha^2$$

Standard deviation :

$$\sigma = 1/\alpha$$

eg) find the const. 'K' such that

$$f(x) = \begin{cases} kx^2, & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

verify it is c.p.D func
also find

1) $P(1 < x < 2)$

2) $P(x \leq 1)$

verifn:
$$\int_0^3 kx^2 \cdot dx = 1$$

$$= k\frac{x^3}{3}\Big]_0^3 = 1$$

$$= 9k = 1$$

$$\boxed{k = 1/9}$$

Process
$$\int_{-\infty}^{0} f(x) \cdot dx + \int_0^3 f(x) \cdot dx + \int_3^{\infty} f(x) \cdot dx$$

$$= \int_{-\infty}^{0} \cancel{0 \cdot dx} + \int_0^3 kx^2 dx + \int_3^{\infty} \cancel{0 \cdot dx}$$

1) $P(1 < x < 2) = \int_1^2 \frac{x^2}{9} \cdot dx$

2) $P(x \leq 1) = \int_0^1 \frac{x^2}{9} dx$

$$= \frac{x^3}{27}\Big]_1^2 =$$

mean :

$$\mu = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_0^3 x \cdot \frac{x^2}{9}\, dx$$

$$= \int_0^3 \frac{x^3}{9}\, dx$$

Variance

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

$$= \int_0^3 (x-\mu)^2 \times \frac{x^2}{9}\, dx$$

$$\frac{x^3}{27}\Big]_1^2 \quad \frac{8}{27}$$

Scanned by CamScanner

eg2) A fun^n defined by

$$f(x) = \begin{cases} 0 & x < 1 \\ 2/7(x+2) & 1 < x < 2 \\ 0 & x > 2 \end{cases}$$

1) Verify it is a C.P.D.
2) Find μ and σ^2
3) find the distribution fun^n

Sol^n) $f(x) \geq 0$ we know that

$\frac{2}{7}(x+2)$ is > 0

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$= \int_{-\infty}^{1} 0 \cdot dx + \int_{1}^{2} 2/7\,(x+2)\,dx + \int_{2}^{\infty} 0 \cdot dx$$

$$= \cancel{\frac{2}{7}\frac{x^2}{2} + \frac{2}{7} 2x \Big]_1^2}$$

$$\cancel{= \frac{8}{7} - \frac{1}{7} + \frac{8}{7} - \frac{4}{7}}$$

$$= 2/7 \left[\frac{x^2}{2} + 2x\right]_1^2$$

$$= 2 \times \frac{3}{7} + \frac{8}{7} - \left[\frac{1}{7} + \frac{4}{7}\right]$$

$$= \frac{2}{7} \times 7/2 = 1$$

---

3) $F(x) = \int_1^x \frac{2}{7}(x+2)\,dx$

Distribution fun^n is also a fun^n in terms of x.

---

eg 3) Find C.D.F from following P.D.F

$$f(x) = \begin{cases} 6x - 6x^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\int_0^x 6x - 6x^2 \, dx$$

$$= 3x^2 - 2x^3 \Big]_0^x$$

$$= 3x^3 - 2x^4.$$

eg) A C.R.V where the D.F

$$F(x) = \begin{cases} 0 & x \le 1 \\ c(x-1)^4 & 1 \le x \le 3 \\ 1 & x > 3 \end{cases}$$

find c and also PDF.

Sol^n)

$$f(x) = \begin{cases} 0 & x \le 1 \\ 4c(x-1)^3 & 1 \le x \le 3 \\ 0 & x \ge 3 \end{cases}$$

$$\int_1^3 4c(x-1)^3 \cdot dx = 1$$

$$= \cancel{4}c \frac{(x-1)^4}{\cancel{4}} \Big]_1^3 = 1$$

$$4c \frac{(x-1)^4}{4} \Big]_1^3 = 1$$

31

$$= A\left(\int_1^3 x^3 - 3x^2 - 3x + 1 \cdot d\right)$$

$$= A(X \; A =)$$

$$C = 1/16$$

Q4) If 'X' is an exponential variant with mean = 3, $\mu = 3$, find:

1) $P(x>1)$

2) $P(x<3)$

$$\mu = \frac{1}{\alpha} \Rightarrow 3 = \frac{1}{\alpha} \Rightarrow \boxed{\alpha = 1/3}$$

$$\because \alpha > 0$$

$$f(x) = \begin{cases} \frac{1}{3} e^{-x/3} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

1) $P(x>1) = \int_1^{\infty} \frac{1}{3} e^{-x/3} \, dx$

$$P(x \geq 1) = 1 - P(x \leq 1)$$

$$= 1 - \int_0^1 \frac{1}{3} e^{-x/3} \, dx$$

2) $P(x<3) = \int_0^3 \frac{1}{3} e^{-x/3} \, dx$

eg) find K to the probability fun^n :

$$f(x) = K\,{}^3C_x \quad , \quad x = 0, 1, 2, 3$$

and sketch $f(x)$ $K\cdot{}^3C_1 + K{}^3C_2 + K{}^3C_3 + K = 1$

find $F(x)$.

$$\sum_{x=0}^{3} K\,{}^3C_x = 1$$

$$K\left[{}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3\right] = 1$$

$$\Rightarrow K \times 8 = 1$$

$$\Rightarrow K = 1/8$$

$$f(x) = \frac{1}{8}\,{}^3C_x$$

$$F(x) \cancel{\sum_{x=0}^{3} \frac{1}{8}}$$

$$F(0) = \sum_{x \le 0} \frac{1}{8}\,{}^3C_x$$

$$= \sum_{x=0} 1/8\,{}^3C_0$$

$$= 1/8$$

Scanned by CamScanner

$$F(1) = \sum_{x \le 1} \frac{1}{8}\, {}^3C_x$$

$$= \sum_{x=0} \frac{1}{8}\, {}^3C_0 + \frac{1}{8}\, {}^3C_1$$

$$= 1/2$$

$$F(2) = \sum_{x \le 2} \frac{1}{8}\, {}^3C_x$$

$$= f(0) + f(1) + f(2)$$

$$= \frac{1}{8}\left({}^3C_0 + {}^3C_1 + {}^3C_2\right)$$

$$= 7/8$$

$$F(3) = \sum_{x \le 3} \frac{1}{8}\, {}^3C_x$$

$$= f(0) + f(1) + f(2) + f(3)$$

$$= \frac{1}{8}\left({}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3\right)$$

$$= 1$$

$$F(X) = \cancel{P(x)} \begin{cases} 0 & x < 0 \\ 1/8 & 0 \le x < 1 \\ 1/2 & 1 \le x < 2 \\ 7/8 & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$$