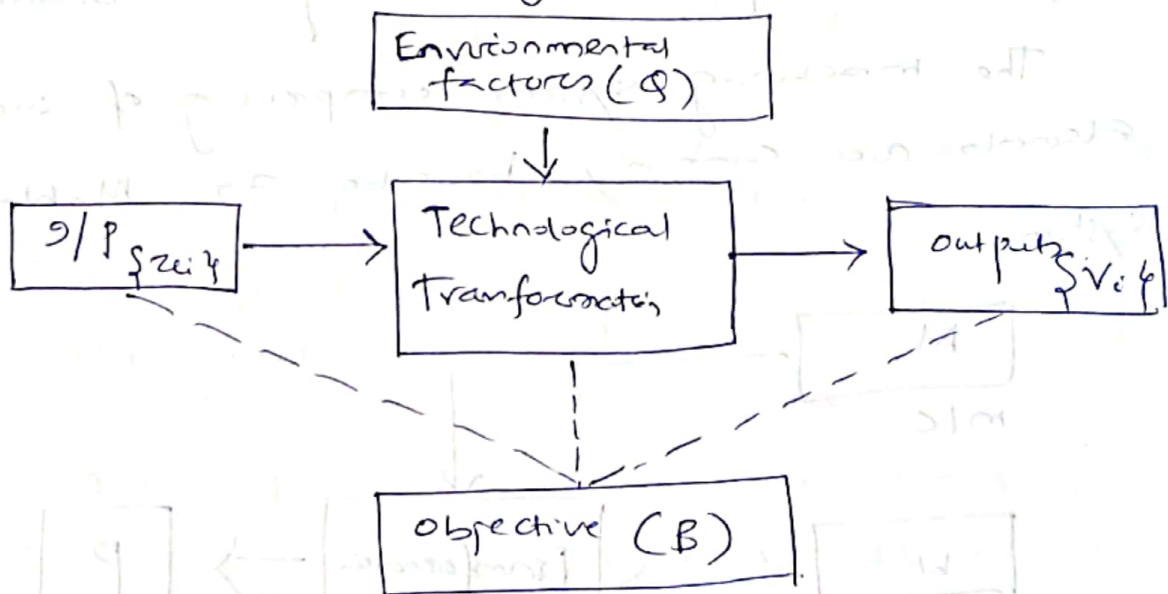


## Overall perspective

- (\*) Basic principles of machining by the special technique of metal cutting is a subsystem of a much larger system commonly called production engineering.
- (\*) Production is a process of transformation of a set of input elements into a set of desired output elements for increasing the utility of goods & services.
- (\*) Process may be defined as the engineering methods/techniques for performing technological transformation.



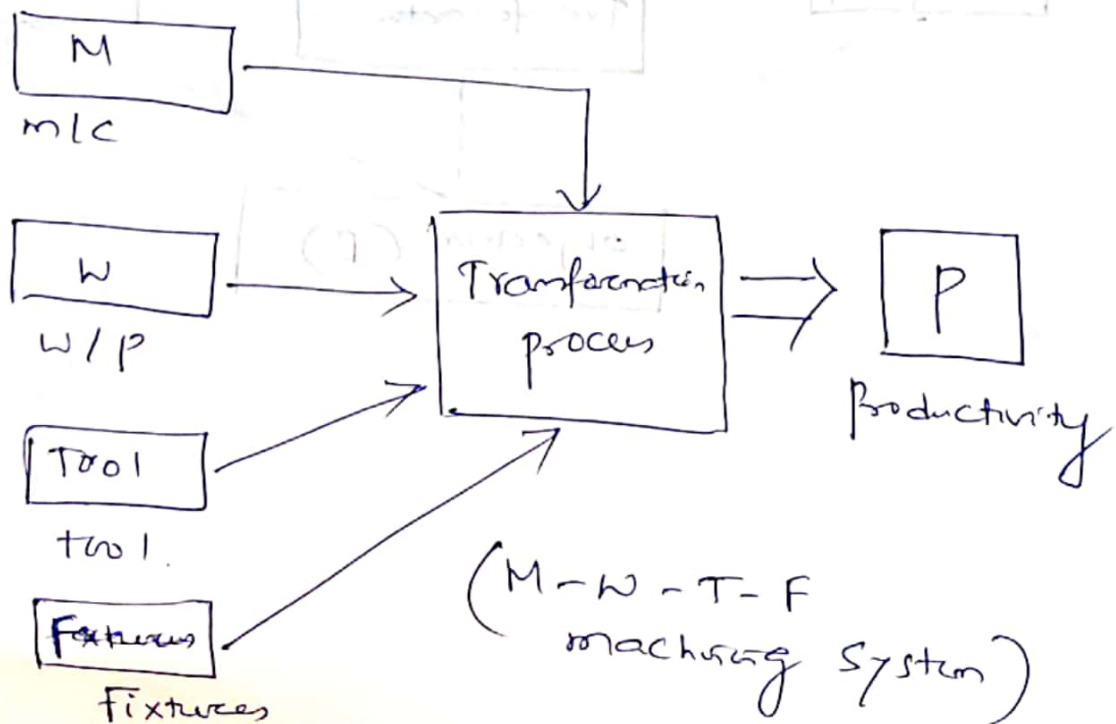
ON  
Manufacturing describes the entire sequence of work from time the raw material is brought into the production system until it is changed into the desired characteristics of a product.

(\*) A special class of manufacturing processes is the machining system.

Where a W/P (W) with the aid of a m/c tool (M) & cutting tool (T) is transformed into a product (P).

Both the W/P & cutting tool need fixtures (F) for proper holding, guidance in order that reg. size, shape & accuracy are obtainable.

The machining system comprising of such elements are commonly describe as M-W-T-F system.



## Introduction to material removal processes:-

Study of metal cutting and machine tools is one of the most important study.

Machining of materials is adopted

- for higher surface finish

- close tolerances.

- complex geometric shapes, difficult to obtain in other way.

\* Material removal is perhaps the most expensive one in all manufacturing processes available.

Reason :- Substantial amount of material is removed in the form of chips, to achieve the final shape.

- lot of energy is expended in the process of material removal.

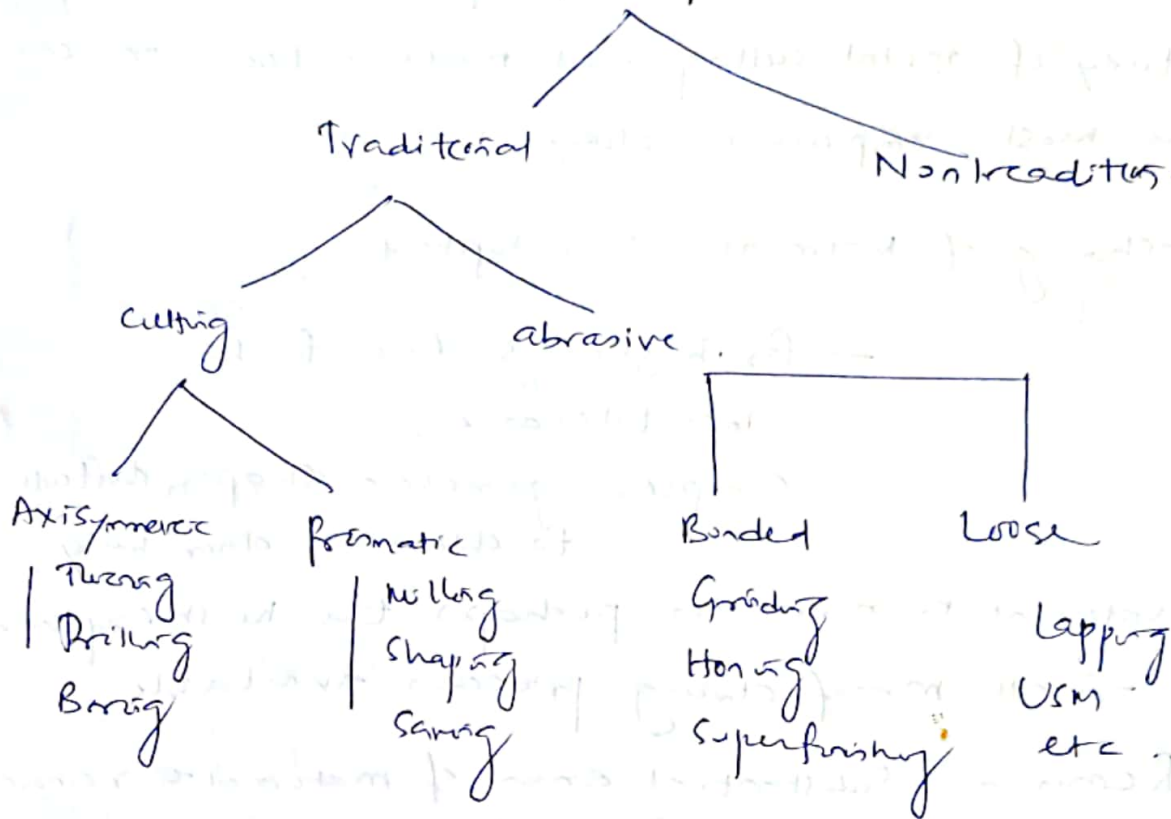
\* Invariably, all the component will be undergoing a MR operation at one point or other.

\* Machine tool  $\Rightarrow$  is defined as one which while holding the cutting tools would be able to remove metal from a w/p in order to generate the requisite job of given size, configuration & finish.

Machine and m/c tools are different.

Machine  $\Rightarrow$  Means of converting the source of power from one form to the other.

# Broad classification of M/R process





Machining is one of the manufacturing process, which generally refers to the material removal by relative motion betn workpiece & cutting tool (which is generally harder than W/P).

\* This definition may not hold good for non-conventional machining process.

Machining  $\begin{cases} \text{W/P} \\ \text{cutting tool} \end{cases}$

Metal cutting:

~~Basic shape of cutting tools: The Wedge~~

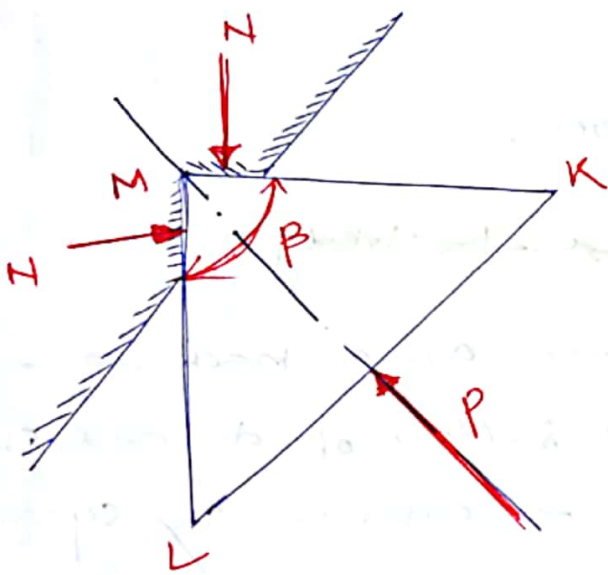
Metal cutting can be defined as a machining - process in which a finished surface of desired size, shape, accuracy or finish is obtained by separating the excess layer of material in the form of chips with the aid of an wedge shaped device called cutting tools.

(\*) It is a chip forming machining process where the excess material is removed in the form of a chip to get a desired product with more accuracy.

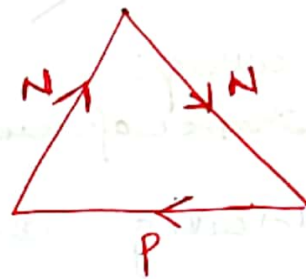
Cutting tools  $\begin{cases} \text{Single point cutting tool} \\ \text{Multi-point cutting tool} \end{cases}$

Ques  
Basic shape of cutting tools: The wedge

Almost all cutting tools used in metal-cutting operations consist of the basic form of an **WEDGE** which is defined as one form of inclined plane in the shape of **triangular prism**.



(Force acting on an indenting wedge)



(Force triangle at the wedge cheek)

Assume that a wedge under the action of force  $P$  is penetrating into another body at a constant speed.

The body resists the motion of the wedge. The reaction  $N, N$  appear at the cheeks of the wedge.

From equilibrium diagram, we get-

$$\frac{N}{P} = \frac{KM}{KL} = \frac{1}{\frac{KL}{KM}} = \frac{1}{2 \times \frac{KL}{2KM}}$$

$$= \frac{1}{2 \frac{(KL/2)}{KM}} = \frac{1}{2 \sin(\beta/2)}$$

$$\Rightarrow \boxed{\frac{N}{P} = \frac{1}{2 \sin(\beta/2)}}$$

Force depends on wedge angle ( $\beta$ ).

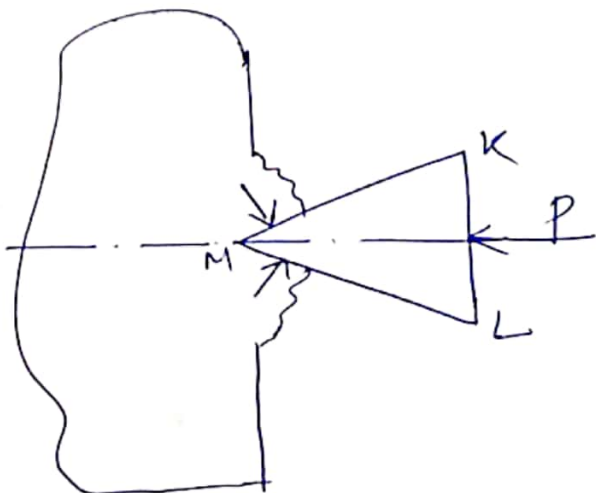
The smaller the angle of wedge the greater will be the gain in force.

In other words, the wedge angle ( $\beta$ ) determines the resisting force of cutting edge.

The cutting edge must be oriented at certain reqd angle with the work surface depending on the nature of operation to be performed.

### Parting operation

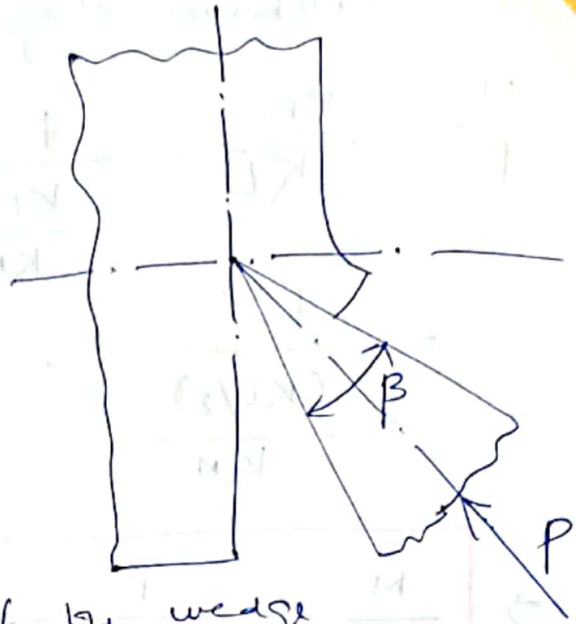
In parting operation the wedge must be set at right angle to the work surface.



(Orientation of wedge during parting or cutting off by the identification process).

## Cutting operation

During chipping, the wedge must be set at an angle relative to work surface.



(Orientation of the wedge along the separator of chips)

## Development of cutting tool.



Why rake angle is provided?

To facilitate machining work. less effort, less cutting force, less temp. requirement; less energy requirement

To avoid rubbing between the flank surface of the tool with the machined surface.

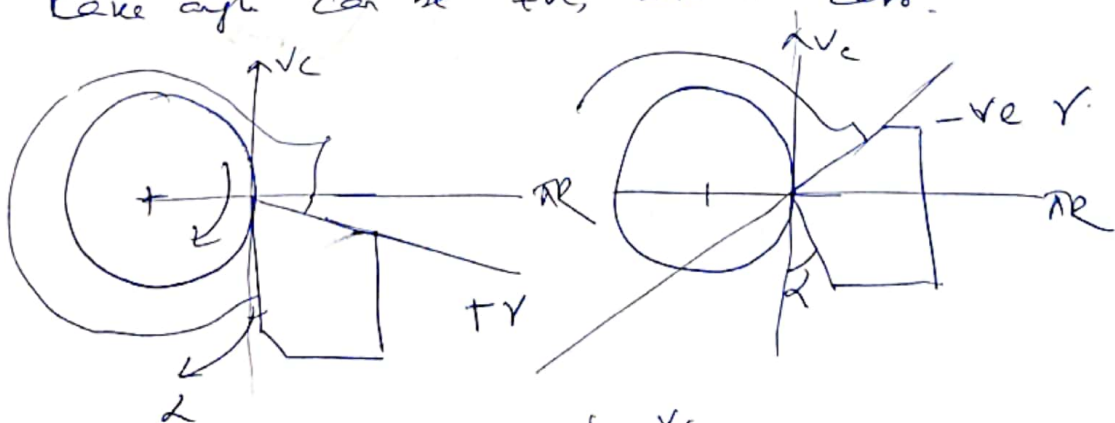
Rake angle: Angle of inclination of the rake surface of the tool from reference plane.

Reference plane: Plane  $\perp$  to the velocity vector.

clearance angle: Angle of inclination of the flank surface from the machined surface or clearance surface.

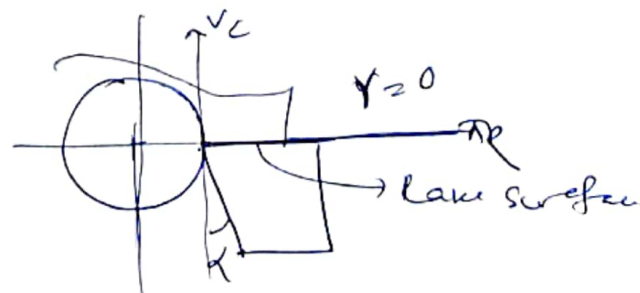
Rake sign of rake & clearance angle

Rake angle can be +ve, -ve or zero.



$\alpha$  cannot be -ve or zero.

$\alpha$  can vary from  $4^\circ$  to  $16^\circ$ .



Tool geometry can be described in different systems.  
Reference systems of description of tool geometry.

(1) Tool in hand system.

wrt  
to WC

(2) Machine reference system - ASA System.

American  
Standard  
Association

(3) Tool reference system.

wrt to  
tool

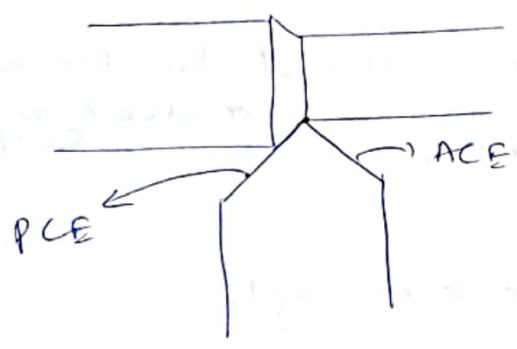
• Orthogonal rake system (ORS)

• Normal rake system (NRS)

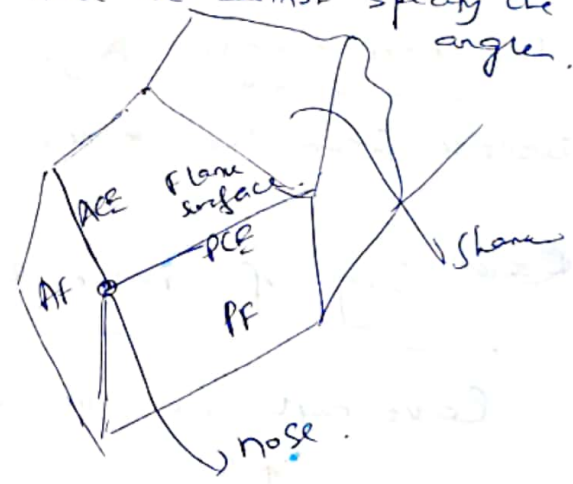
(4) Work reference system (WRS)

Configuration  
of the w.r. to the cutting tool  
together as reference

Tool in hand system:



Identify some of the features  
Here we cannot specify the angle.



# ASA system

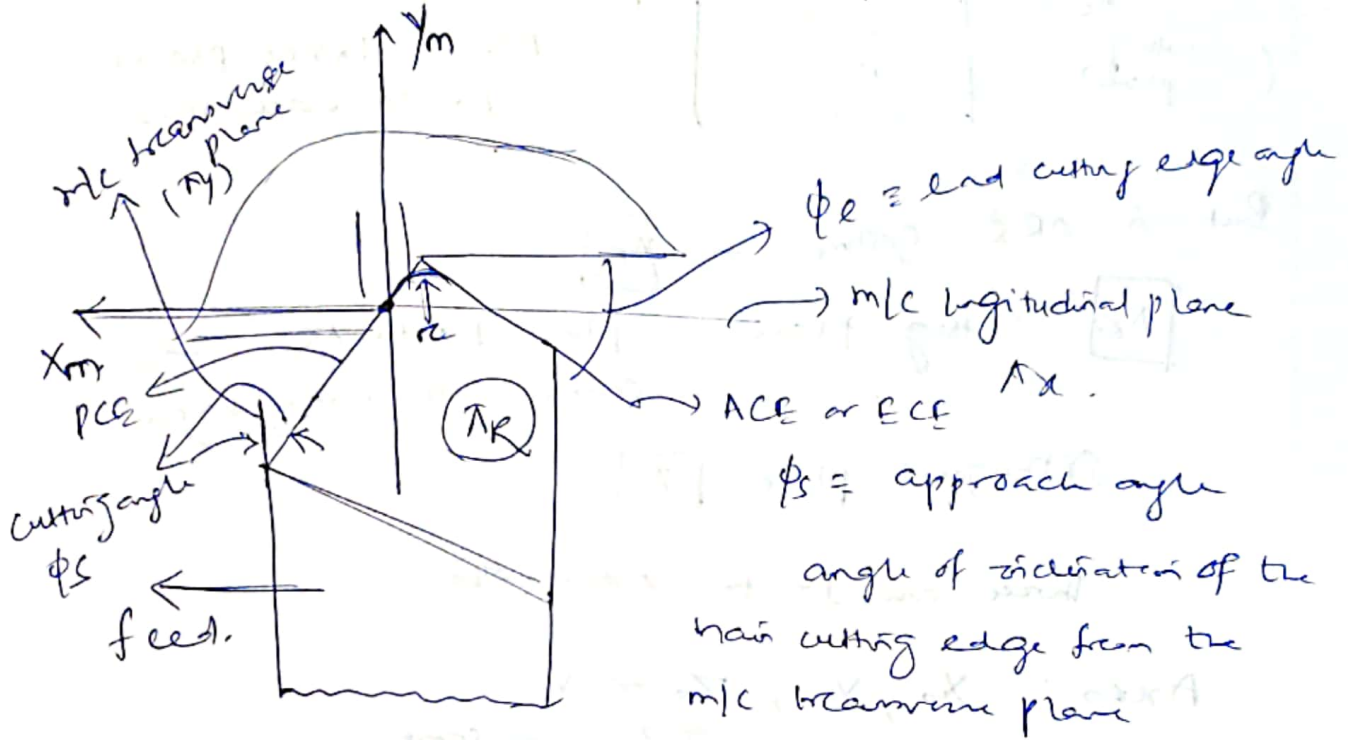
Inclination of feature wrt to certain planes

Turning Top View of

$V_c$

$\pi_R \perp V_c$

$\pi_x \parallel$  to the direction of feed



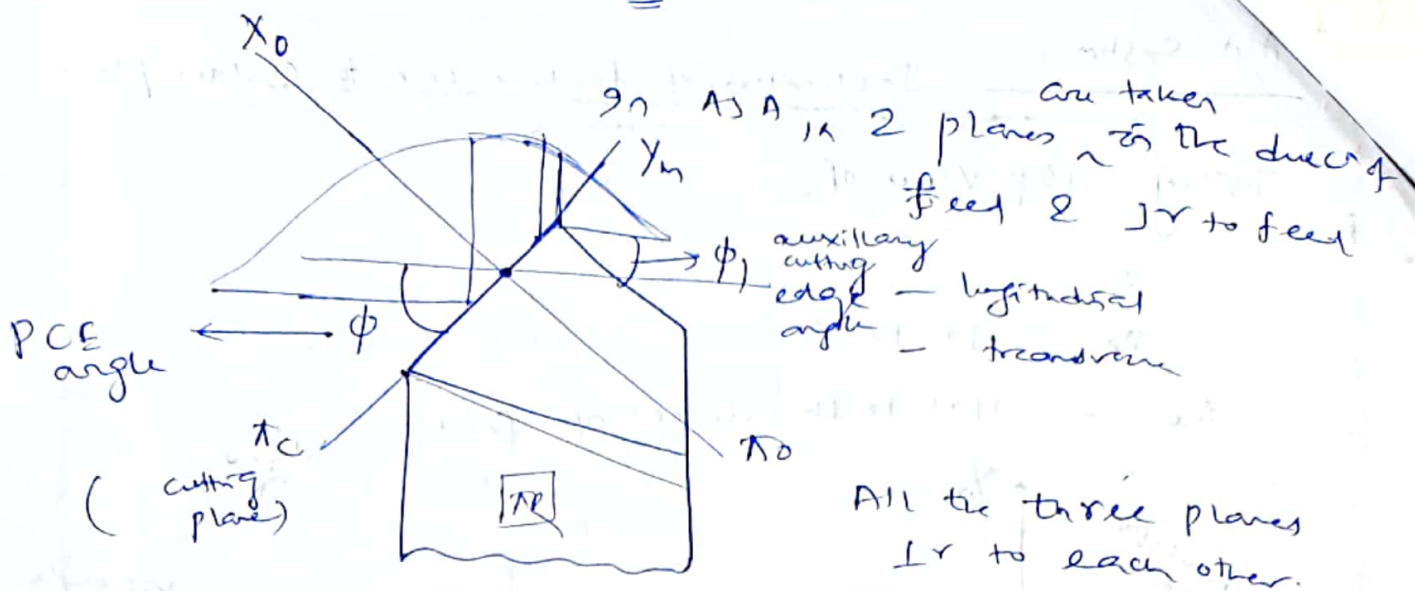
end cutting angle  $\phi_e \equiv$  angle of inclination of the auxiliary cutting edge from  $\pi_x$  plane.

$r \equiv$  radius of curvature

## Tool reference system (Study of tool geometry in tool reference system)

Here the configuration of the cutting tool is taken as the reference for the tool.

Tool reference system  $\rightarrow$  ORS or ISO-old

Om

But in ORS system -  $\boxed{K}$

$\boxed{\pi_G}$  Cutting plane — plane  $\perp$  to  $\pi_R$  and contains the cutting edge

Orthogonal plane  $\boxed{\pi_0}$

Three are 1r to each other

Axes:  $x_0, y_0, z_0 \sim v_c$   
or  $z_m$  are same.

Main cutting edge will be part of the same surface

(\*) angle of incidence of the wave surface from ~~cutting edge~~ reference plane

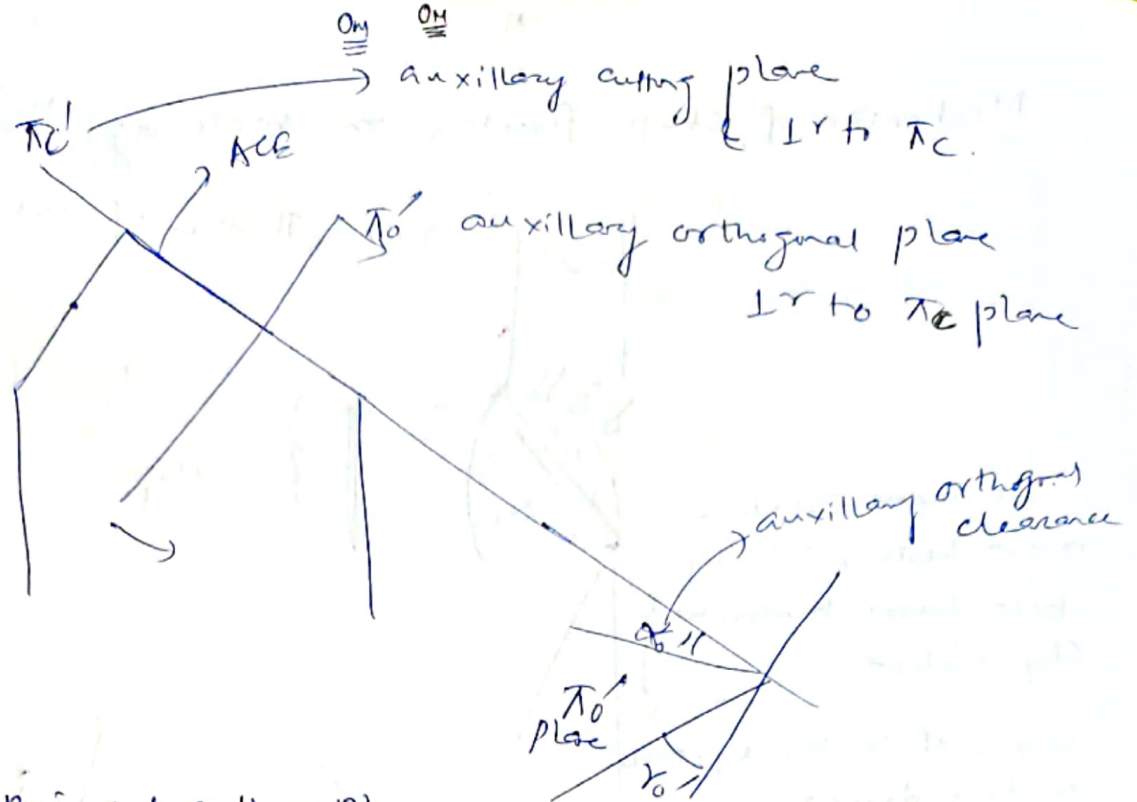
⑦  $\equiv$  special name

$\lambda = \text{inclination angle}$

$$\phi_1 \equiv$$

In case of ASA it is  $\phi$





$\pi_c \equiv$  principal cutting plane

$\pi_o \equiv$  auxiliary principal orthogonal plane

Mechanics of chip formation  $\Rightarrow$

Machining  $\Rightarrow$  is characterised by gradual material removal in the form of chips.

The form of chips is an important index of machining as it indicates:

Nature & behavior of the work material under machining.

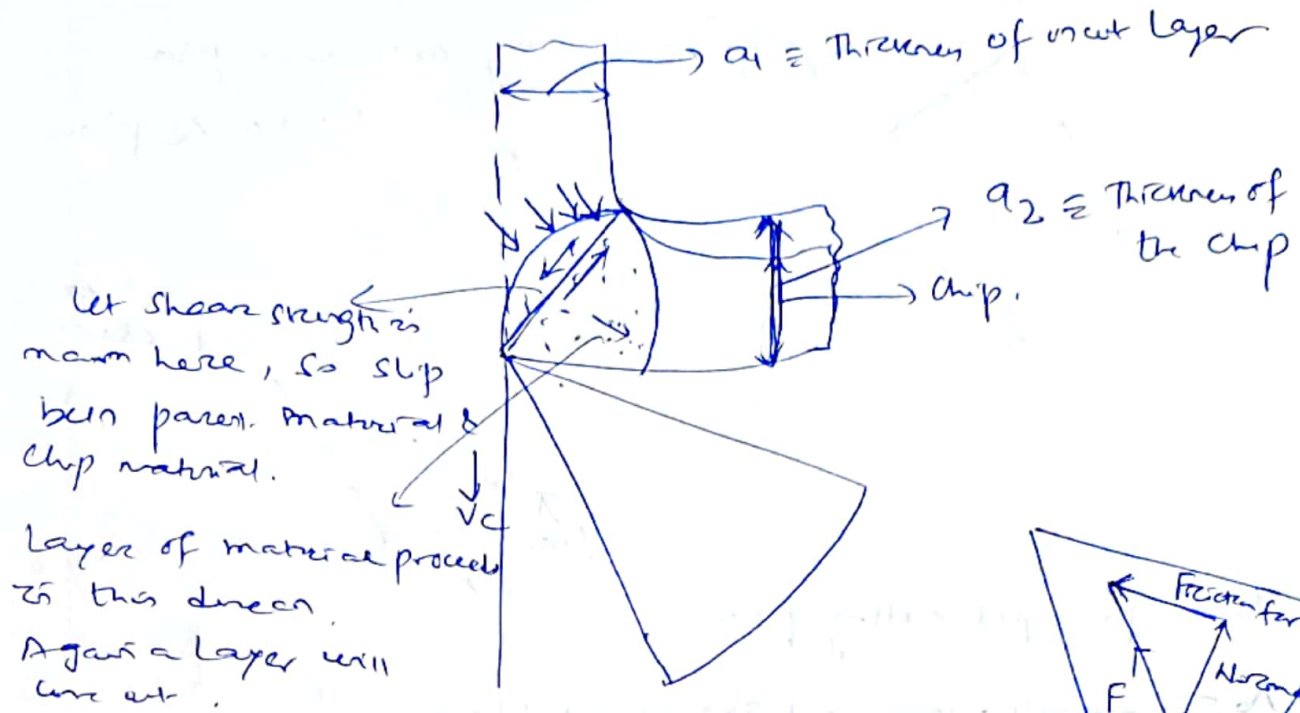
Specific energy required to m/c.

Nature & degree of interactions at the work-tool interfaces.

The form & color of chips depend upon

- ① Work material
- ② cutting tool - material & geometry.
- ③ Level of the process parameters.
- ④ appln of cutting fluids

# Mechanism of chip formation in machining ductile material



all sided compression.

Shear stress simultaneously develops at different directions & in different magnitude.

\* When shear stress <sup>developed</sup> that reaches or exceeds the shear strength of the w/p material, failure will start (i.e. separation will take place from chip material from parent material).

layer by layer because of shearing ~~ag~~ action & yielding.

Ductile material is deformed by yielding  
or by shear

Brittle material is deformed by brittle fracture

Chip formation in machining brittle materials

Normal mechanism during machining  $\Rightarrow$

A small crack is developed at the end of the cutting tool.

At the end of the <sup>crack</sup> lot of stress concn.

During ductile  $\Rightarrow$  stress exceeds the yield strength of the material.

lot of stress concn at the tip of the crack which exceeds the strength of the material & where resistance is more the crack propagates & discontinue chips forms (random)

Quickly separate the chip from the parent body.

In brittle material: lot of voids & cracks are there.

Geometry characteristics of continuous chips

chip reduction coefficient ( $\lambda$ ) & cutting ratio ( $r$ )

$$\lambda = \frac{a_2}{a_1}$$

$$a_2 > a_1$$

Because of compression the chip becomes thicker ahead of the cutting tool.

Significance of  $\lambda$

Kroenerbergs model: relating  $\lambda$  with  $\gamma$  &  $\mu$ .

$\lambda = \frac{a_2}{a_1}$  i.e. degree of thickness of the chip

More thickness means more energy investment.



ie } should be as much as possible,  
(not at the cost of productivity)

less energy will be reqd to get the work done.

$$\zeta = e^{\mu(\frac{\pi}{2} - \gamma_0)}$$

(f<sup>n</sup> of rake angle)  $\Rightarrow$  Kronenberg model,

if  $\gamma_0 \uparrow$ ,  $\zeta \downarrow$

desirable

desirable  $\zeta$  should be as low as possible

$\gamma_0 \downarrow$ ,  $\zeta \uparrow$

if  $\gamma_0 \rightarrow -ve$ ,  $\zeta$  is very very large

$\Rightarrow$  unfavorable

undesirable

$\mu$  = coefficient of friction at the chip tool interface

if  $\mu \uparrow$ ,

$\zeta \uparrow$

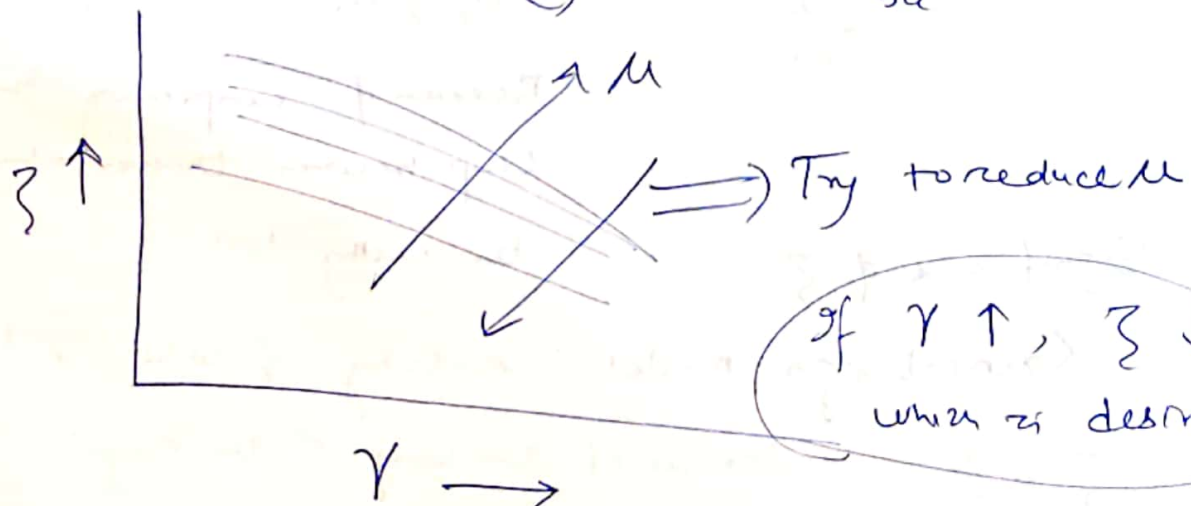
$\Rightarrow$  undesirable

if  $\mu \downarrow$ ,

$\zeta \downarrow$



$\Rightarrow$  is desirable





On

---

Group-A

Q. 8

$$L = 350 \text{ mm}$$

$$D = 50 \text{ mm}$$

$$V_c = 35 \text{ m/min}$$

$$f = 0.5 \text{ mm/rev.}$$

$$t = ?$$

$$V_c = \frac{\pi D N}{1000} = \frac{\pi \times 50 \times N}{1000}$$

$$\Rightarrow 35 = \frac{\pi \times 50 \times N}{1000} \Rightarrow N = 222.5$$

$\therefore$  No. of revolutions reqd for complete cut

$$= \frac{350}{0.5} = 700$$

$\therefore$  Time reqd for complete cut

$$= \frac{700}{222.5} = 3.14 \text{ min}$$

Q. 9

$$\text{Feed} = 1.5 \text{ mm/rev}$$

chip thickness after cutting is 2mm.

$$\gamma = 10^\circ \quad V_c = 30 \text{ m/min} \quad \zeta = ? \quad \kappa = ? \quad \beta = ?$$

chip thickness before ~~cutting~~ cutting ~~is~~  $a_1 = 1.5 \text{ mm}$ .

$$a_2 = 2 \text{ mm} \quad \zeta = a_2/a_1 = 1.34$$

$$\zeta = \frac{a_2}{a_1} \Rightarrow \text{chip thickness ratio } (\kappa) = \frac{a_1}{a_2} = \frac{1.5}{2} = 0.75$$

$$\tan \beta = \frac{\cos \gamma_0}{\zeta - \sin \gamma_0} = \frac{\cos 10^\circ}{\left(\frac{2}{1.5}\right) - \sin 10^\circ} = 0.85$$

$$\Rightarrow \beta = \tan^{-1}(0.85)$$

$$\Rightarrow \beta = 40.33^\circ$$

$$= 40^\circ 22'$$

Q.8

$$D = 25 \text{ mm.}$$

$$V_c = 50 \text{ m/min}$$

$$\text{rpm} = ?$$

$$V_c = \frac{\pi D N}{1000} \text{ m/min}$$

$$50 = \frac{\pi \times 25 \times N}{1000}$$

$$\Rightarrow N = \frac{50 \times 1000}{\pi \times 25} = 637 \text{ rpm.}$$

Q.9

$$\gamma = 5^\circ$$

$$V = 400 \text{ rpm}$$

$$\text{Axial feed} = 0.4 \text{ mm/min}$$

$$\text{radial depth of cut} = 5 \text{ mm}$$

$$\text{chip thickness} = 3 \text{ mm,}$$

$$\text{Shear angle } \phi (\text{degree}) = ?$$

## Learning of SPTT geometry

↳ single pt turning tool

Material & geometry of tools - equally important.

These play important roles on effectiveness, eff., economics & integ.

Tool geometry substantially affects:

- mechanism & mechanics of chip formation
- cutting temp & wear.
- tool life & produces accuracy & finish.

Cutting tools

- Single pt. - turning, shaping, boring etc
- Double pt - drills.
- Multiple - milling cutters, hubs etc.

## Geometry of SPTT

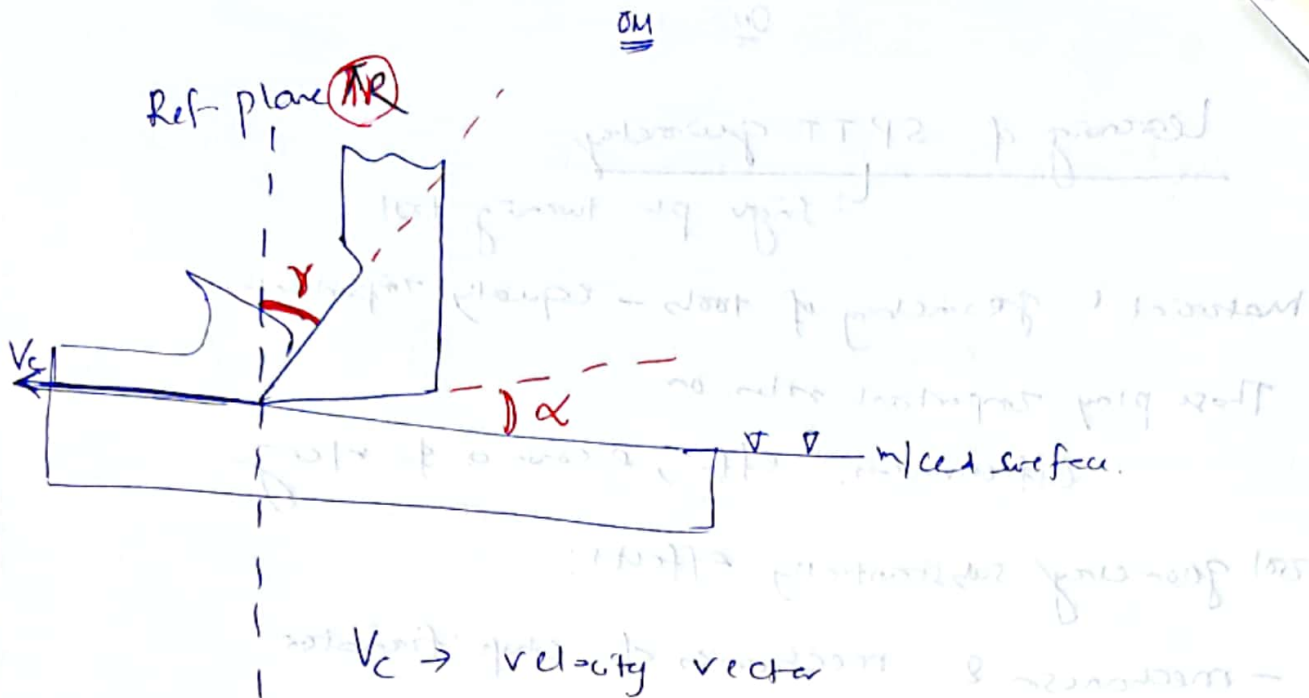
### (i) Concept of rake and clearance angles

⊗ Tool geometry refers to some specific angles or slopes of its salient faces & edges.

⊗ Rake angle & clearance angle - most important

⊗ Illustration of rake & clearance of SPTT.



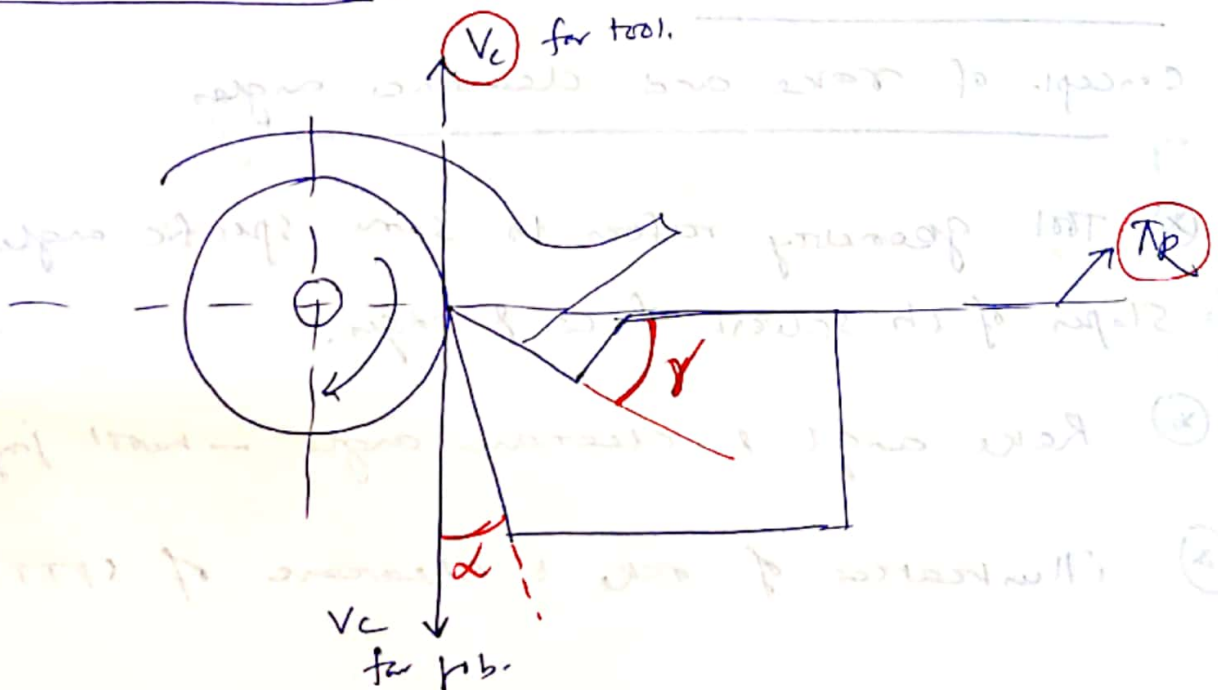


Ref. plane  $\div$  A plane  $\perp$  to the velocity vector

Rake angle  $\div$  Inclination of the rake surface of the tool from the ref. plane.

Clearance angle - Angle or inclination of the clearance surface of the tool from the m/cut surface.  
OR Angle of flank surface from finished surface

In case of turning



Purpose of rake angle:  
(for)

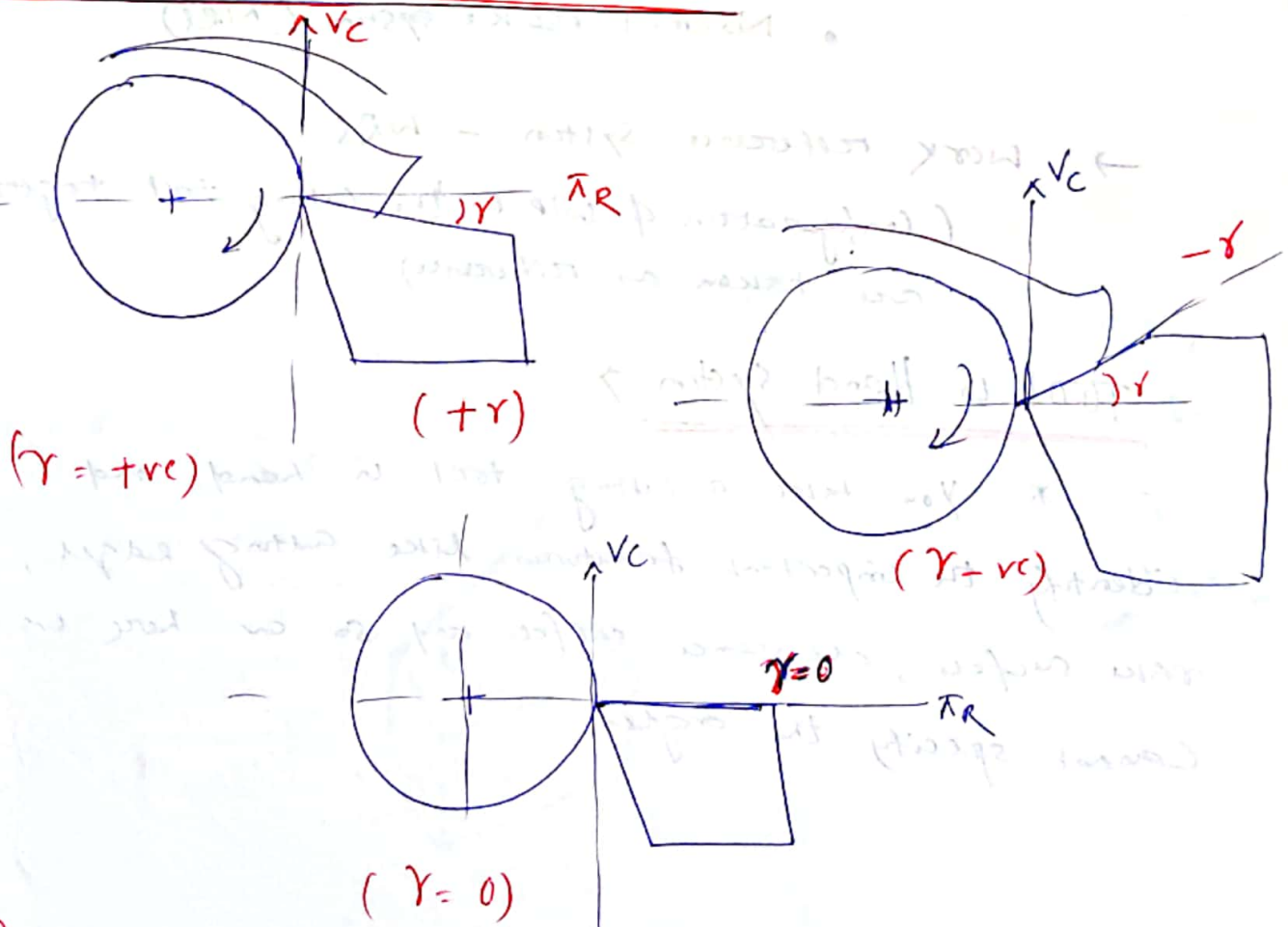
is provided in cutting tool to facilitate m/cy easier.  
m/cy becomes, less effort, less cutting force, less energy requirement.  
Less, temp, required.

Purpose of clearance angle

to avoid rubbing bet<sup>n</sup> the flank surface of the tool and the m/ced surface.

tool damaged, surface of w/p damaged if clearance angle is not there.

Sign of rake & clearance angle



(\*) But clearance angle has to be +ve (always)  
it cannot be (-ve)

OM

(ii) Reference system of description of tool geometry

Tool geometry can be described in different systems of reference.

for diff. country for different purposes for different environments / situations

→ Tool-in-hand nomenclature

→ M/C reference system (ASA System).  
(M/C Configuration is taken as the reference)

→ Tool reference system (Configuration of cutting tool is taken as reference)

- Orthogonal reference system (ORS)
- Normal rake system (NRS)

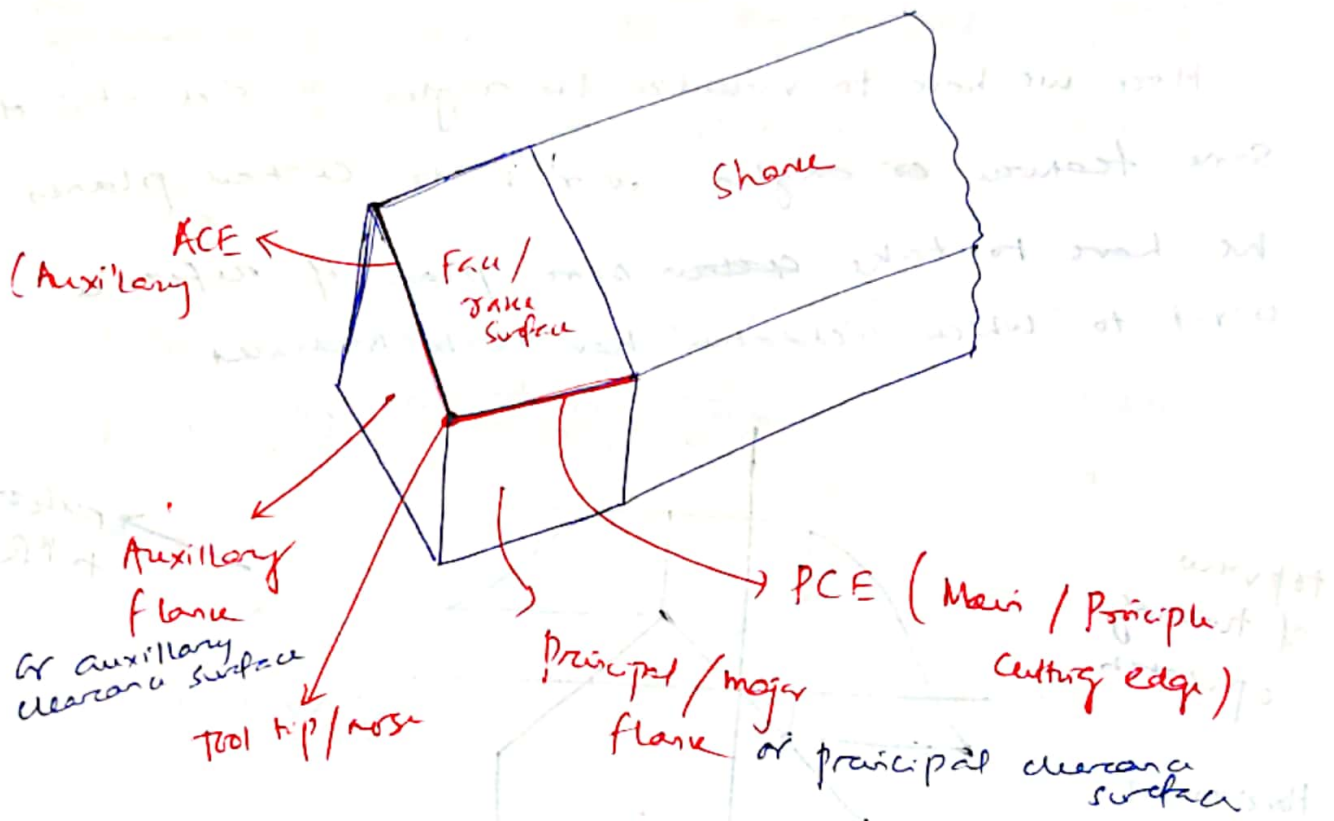
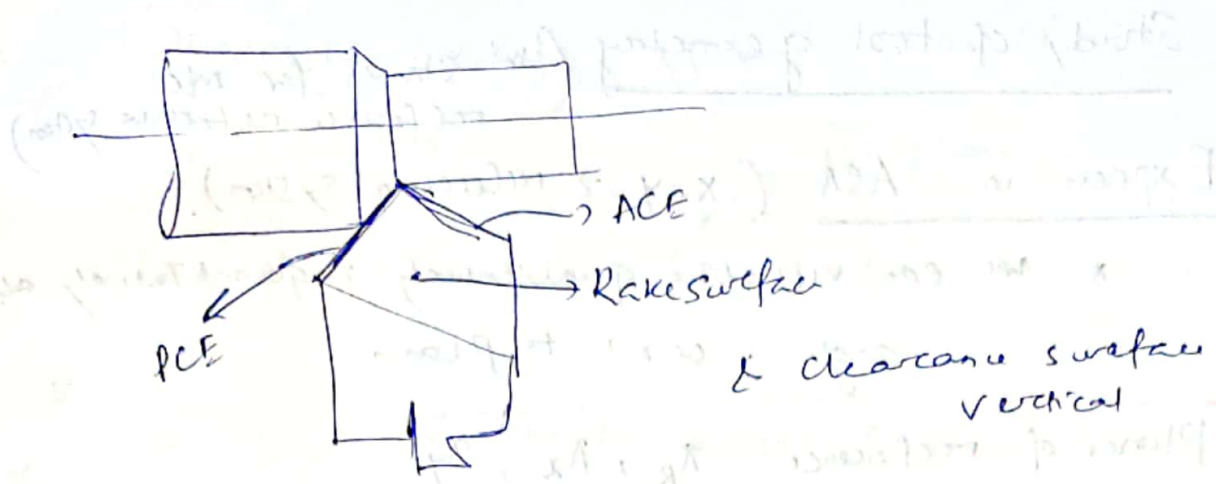
→ Work reference system - WRS

(Configuration of WIP & the cutting tool together are taken as reference)

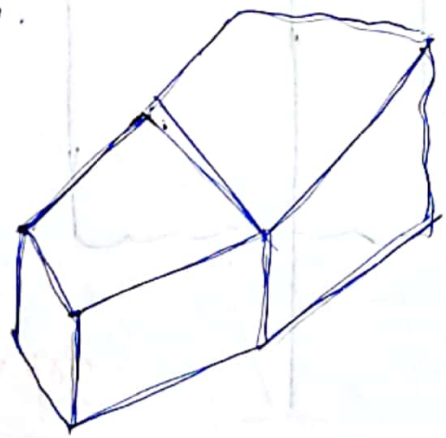
Tool in Hand System ⇒

\* You take a cutting tool in hand and identify the important features like cutting edges, rake surface, clearance surface and so on here we cannot specify the angles





\* Here we cannot quantitatively determine the value of the angles.





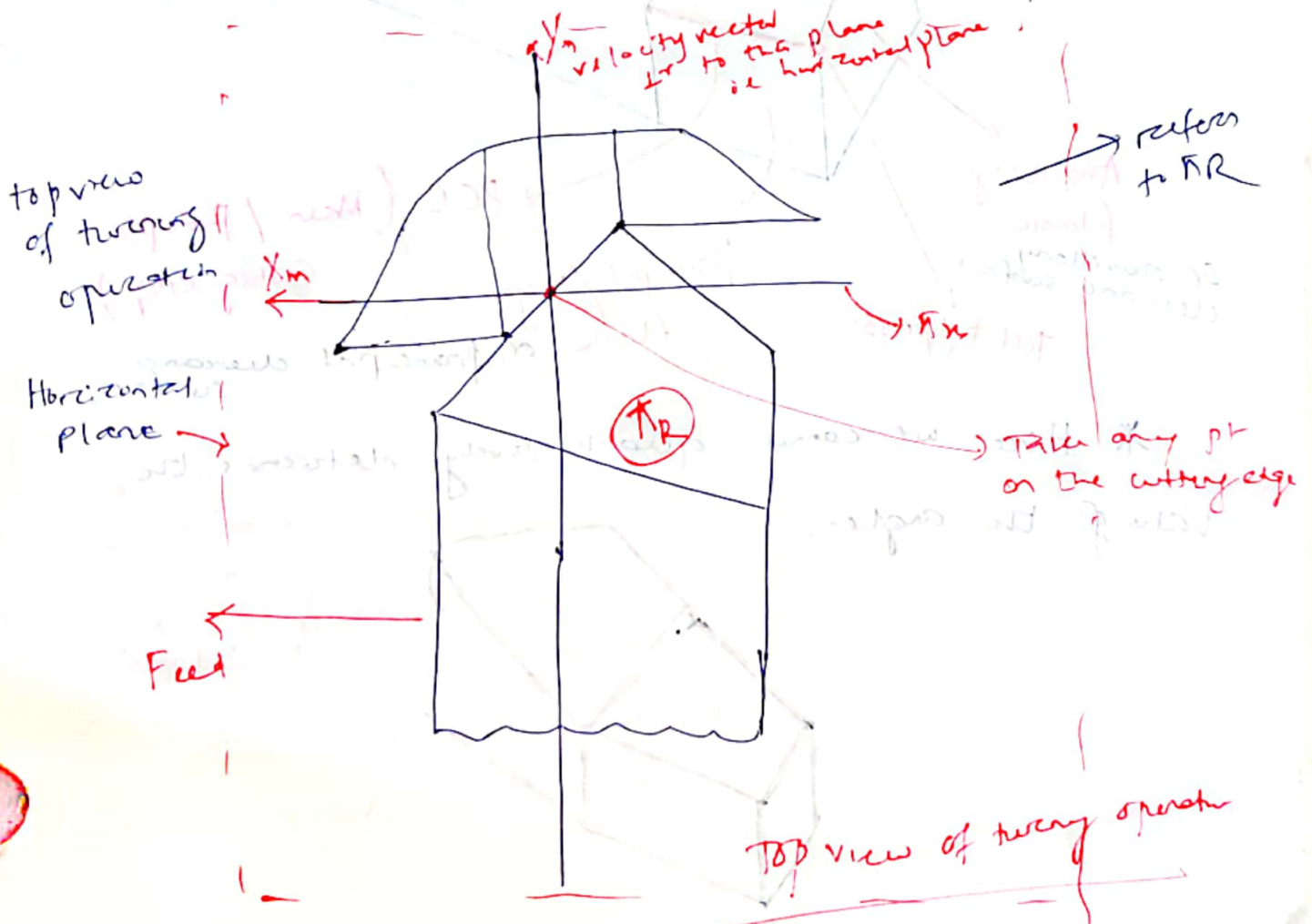
Study of tool geometry ('m' stands for mlt reference reference system)

Express in ASA (x-y-z reference system).

\* we can visualize qualitatively & quantitatively also.  
angle w.r.t to planes

Planes of reference:  $\pi_R, \pi_x, \pi_y$

Here we have to visualize the angles or inclination of some features or angles w.r.t to certain planes. we have to take ~~certain~~ some plane of reference w.r.t to which inclination have to be measured.



Ref. plane is to the velocity vector.

Three perpendicular planes taken to be as reference

Plane of reference  $\div \pi_R$ , 1st plane

One plane  $\parallel$  to the direction of feed & this plane  $\perp$  to  $\pi_R$ , vertical plane called M/C longitudinal plane expressed by  $\pi_x$

Another plane  $\perp$  to  $\pi_R$  as well as  $\pi_x$  taken in the direction  $\perp$  to the longitudinal feed or taken in the direction of cross-feed & this is also one vertical plane called M/C transverse plane expressed by  $\pi_y$ .

This can be visualized in 3-D

From 3-D visualization <sup>on</sup> we can see

M/C reference plane  $\pi_R$  contains  $X_m$  axis  
&  $Y_m$  axis

M/C longitudinal plane ( $\pi_x$ ) contains  $Z_m$  axis

&  $X_m$  axis

M/C transverse plane ( $\pi_y$ ) contains  $Y_m$  axis  
&  $Z_m$  axis

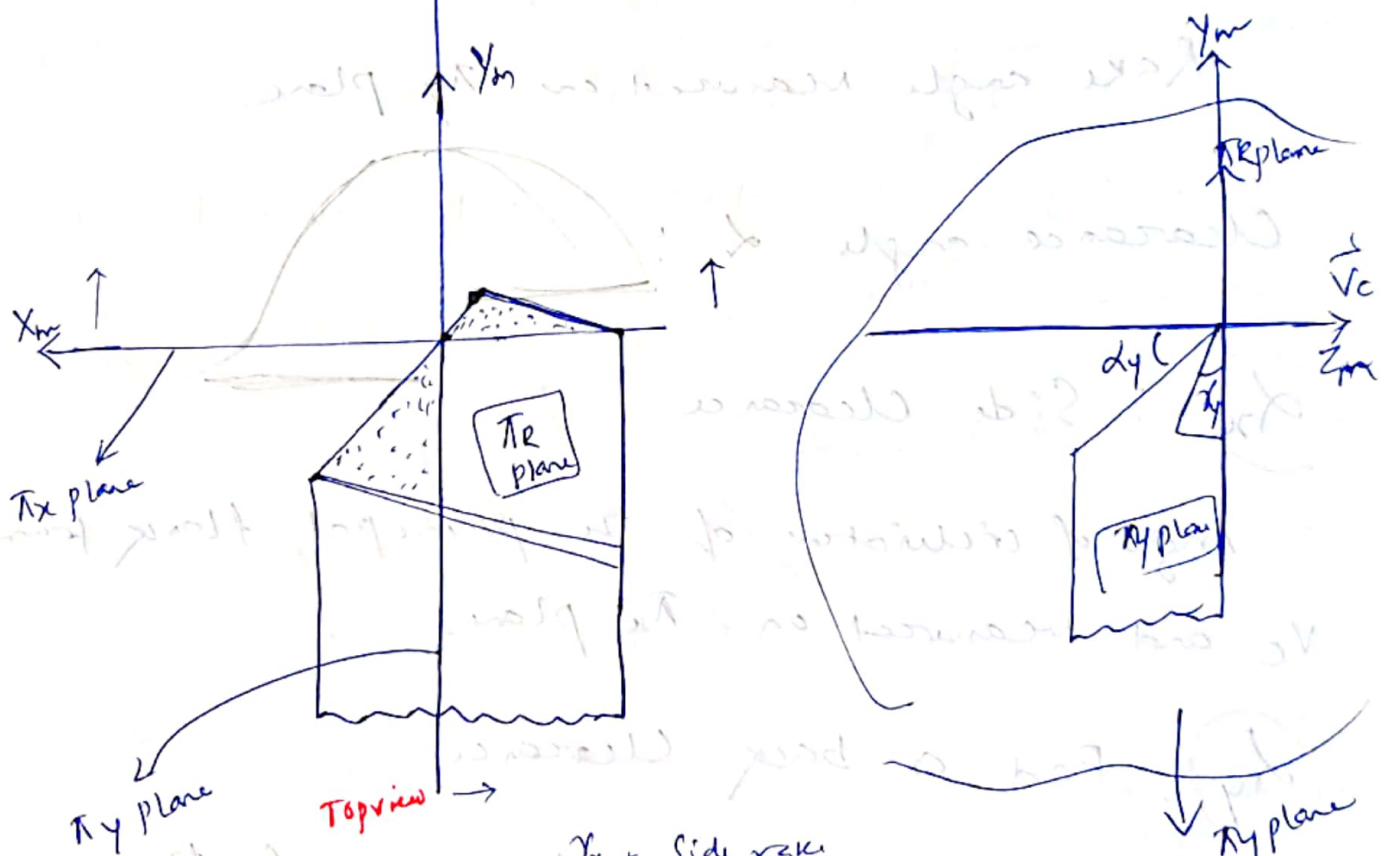
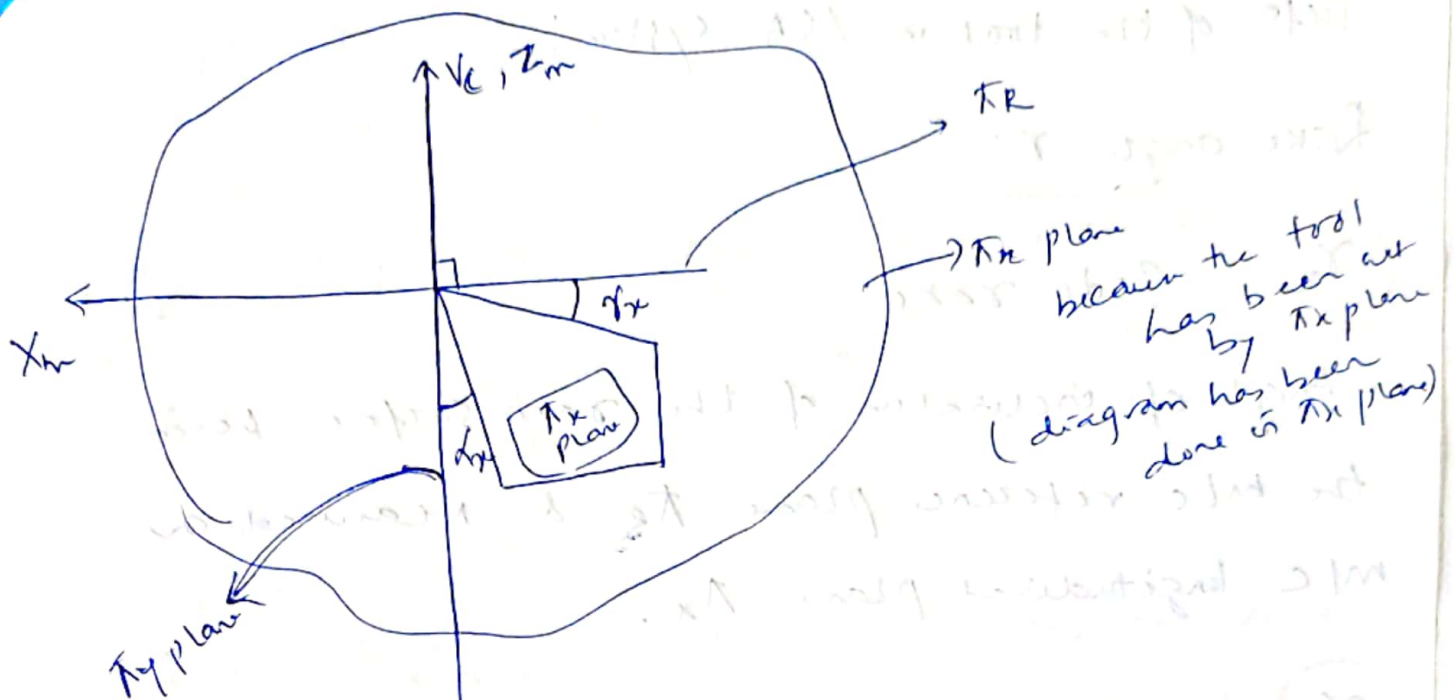
So the axes are :  $X_m, Y_m, Z_m$

taken in the direction  
of longitudinal feed or  
main axis of the  
M/C tool

transverse axis  
of the M/C tool

velocity  
vector  $v_c$   
(vertical  
line)

# Angles in MLC Reference system



$\gamma_r$  = Side rake  
 $\gamma_x$  = Side clearance  
 $\gamma_y$  = Back rake  
 $\gamma_z$  = Back clearance



## Tool geometry in ASA system

Defn of the tool in ASA system:

Rake angle  $\gamma$ :

$\gamma_x$  : Side rake

Angle of inclination of the rake surface from the MLC reference plane  $\pi_r$  & measured on MLC longitudinal plane  $\pi_x$ .

$\gamma_y$  : Back rake or end rake

Rake angle measured on  $\pi_y$  plane

Clearance angle  $\alpha$ :

$\alpha_x$  : Side Clearance

Angle of inclination of the principal flank from  $V_c$  and measured on  $\pi_x$  plane

$\alpha_y$  : End or back Clearance

Angle of inclination or clearance of the principal flank measured on  $\pi_y$  plane.

Cutting angles:

$\phi_s$  = approach angle: inclination of PCE from  $\pi_y$  and measured on  $\pi_R$ .

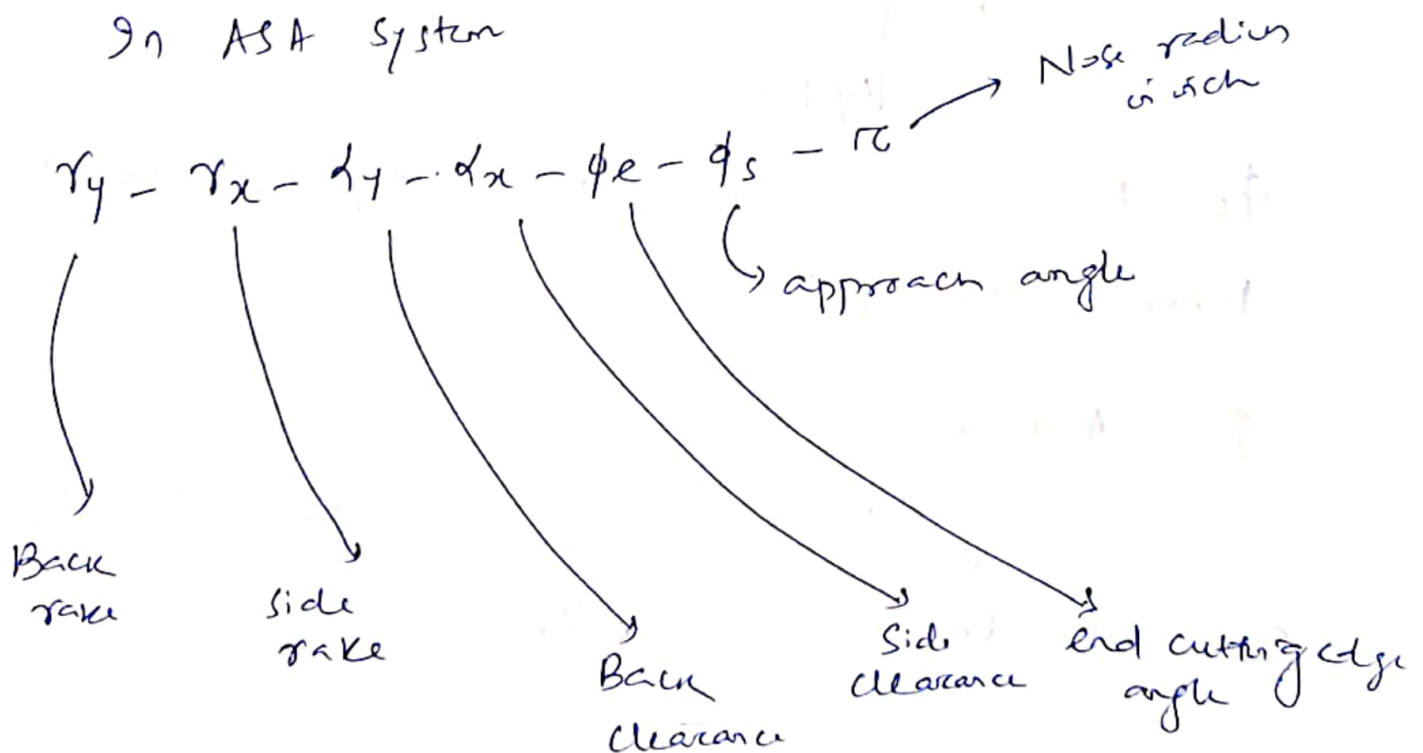
$\phi_e$  = end cutting edge angle: inclination of ACE from  $\pi_x$  and measured on  $\pi_R$ .

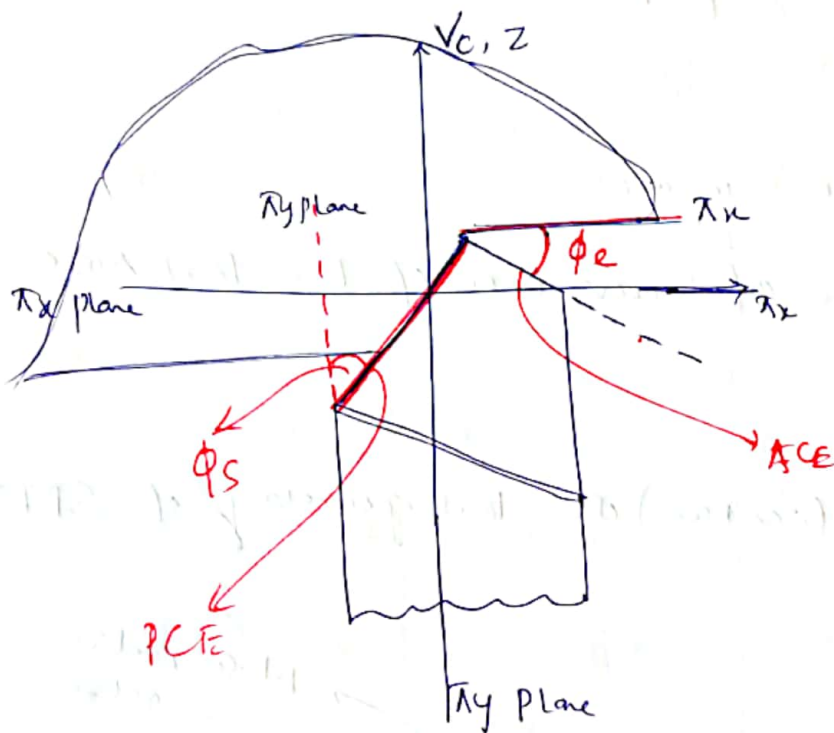
Nose radius ( $r$ ) in inch

$r$  = radius of curvature of the tool nose

Designation (specification) of tool geometry of SPTT

In ASA system





$\phi_s$  = Angle of inclination of the main cutting edge from n/c transverse plane  $T_y$ .

$\phi_e$  = End cutting angle

angle of inclination of the auxiliary cutting edge from n/c longitudinal plane  $T_x$ .

# Study of tool geometry in Tool reference system

ORS System, Orthogonal rake system, ISO-old

Here the configuration of cutting <sup>tool</sup> edge is taken as the tool reference system.

Tool reference system is classified into 2 types,

Orthogonal rake system, ISO-old.

In tool orthogonal system the axes are different.

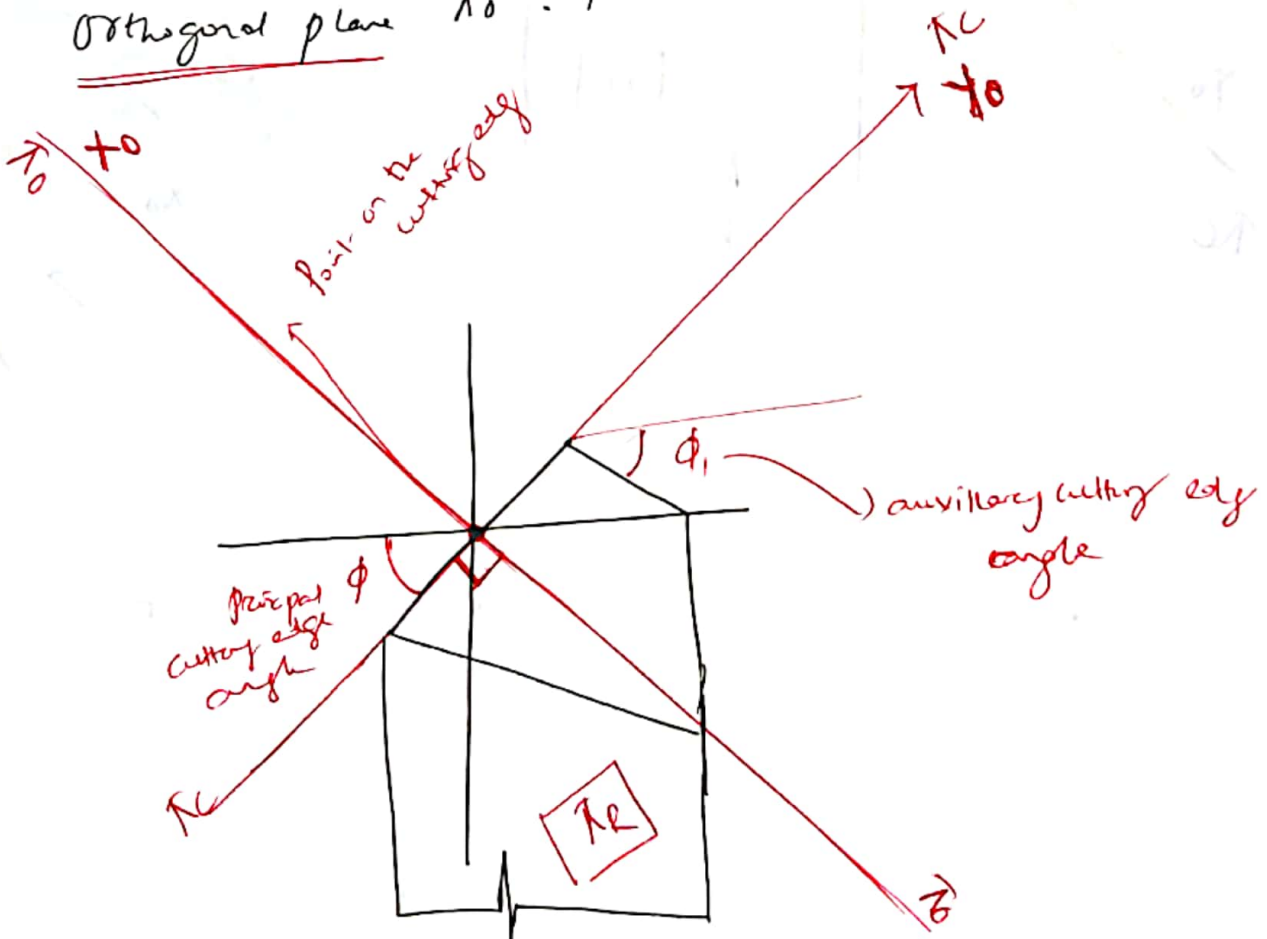
Plane of ref:  $\pi_R = \pi_C - \pi_0$  Axes:  $X_0, Y_0, Z_0$

All the planes should be  $\perp$  to each other. ( $Z_0$  velocity vector)

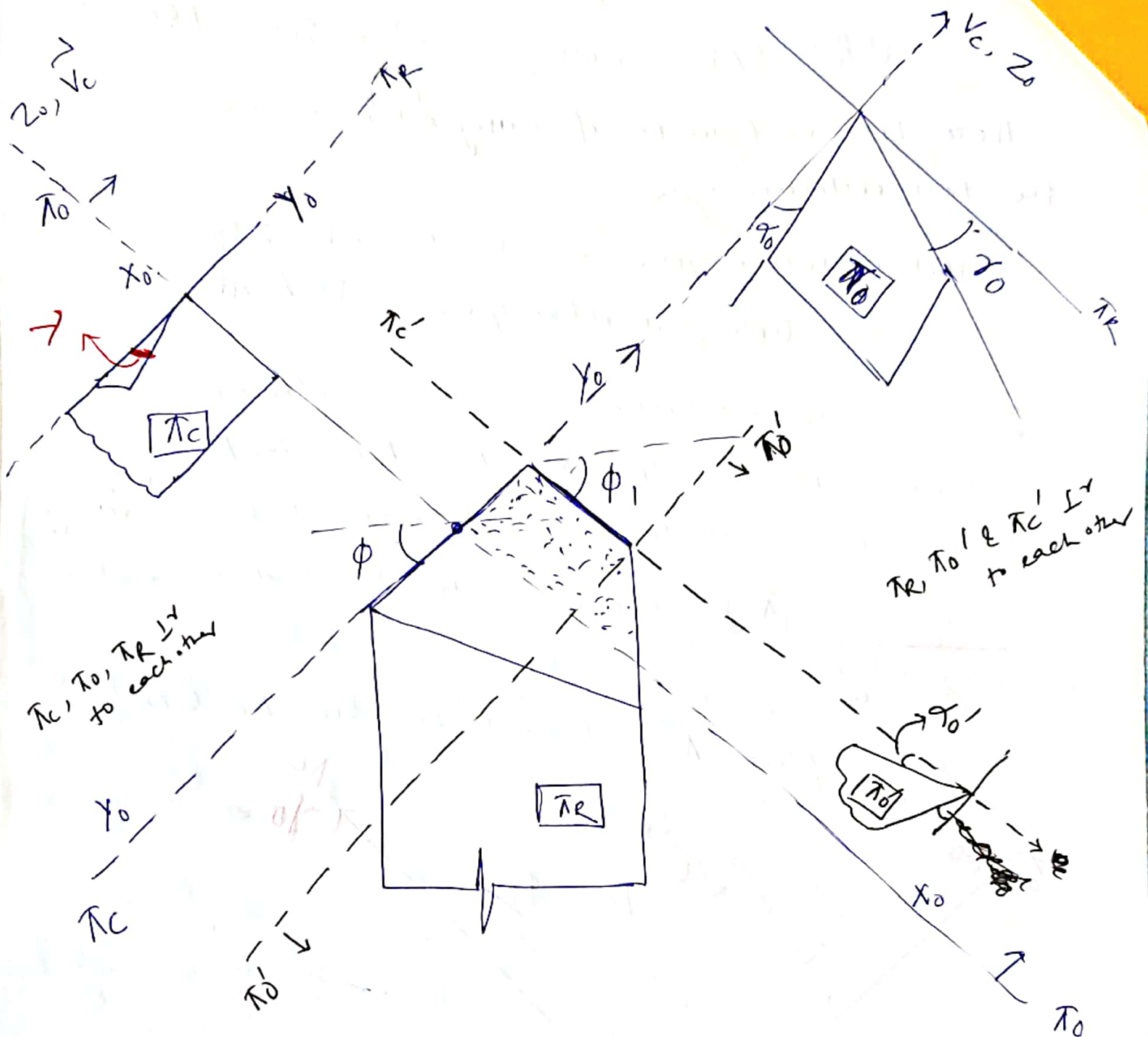
cutting plane  $(\pi_C)$ : Plane  $\perp$  to  $\pi_R$  and contains the

main cutting edge

Orthogonal plane  $\pi_0$ : Plane  $\perp$  to both  $\pi_R$  &  $\pi_C$







(\*)

Main cutting edge is the part of the rake surface

$\gamma_o$  : Orthogonal rake

Angle of inclination of the rake surface from  $\pi_R$ , and measured on the orthogonal plane  $\pi_o$ .

$\alpha_o$  : orthogonal clearance

$\lambda$ : Rake angle or angle of inclination of rake surface from  $\pi_R$  but as it contains the main cutting-edge it has its special name called  $\lambda$ , i.e. inclination angle

$\lambda$ : Angle of inclination

Angle of inclination

$\pi_c$ : cutting plane (principal or main cutting plane)  
 Plane  $\perp$  to  $\pi_R$  & contains the principal or main cutting edge

$\pi_c'$ : auxiliary cutting plane  
 Plane containing the auxiliary cutting edge

$\pi_o'$ : auxiliary orthogonal plane

$\pi_o$ : principal orthogonal plane

$d_o'$ : auxiliary orthogonal clearance

$\gamma_o'$ : auxiliary orthogonal rake.

Tool reference system

NRS system: Normal rake system

NRS - ISO New

Characteristics of NRS:

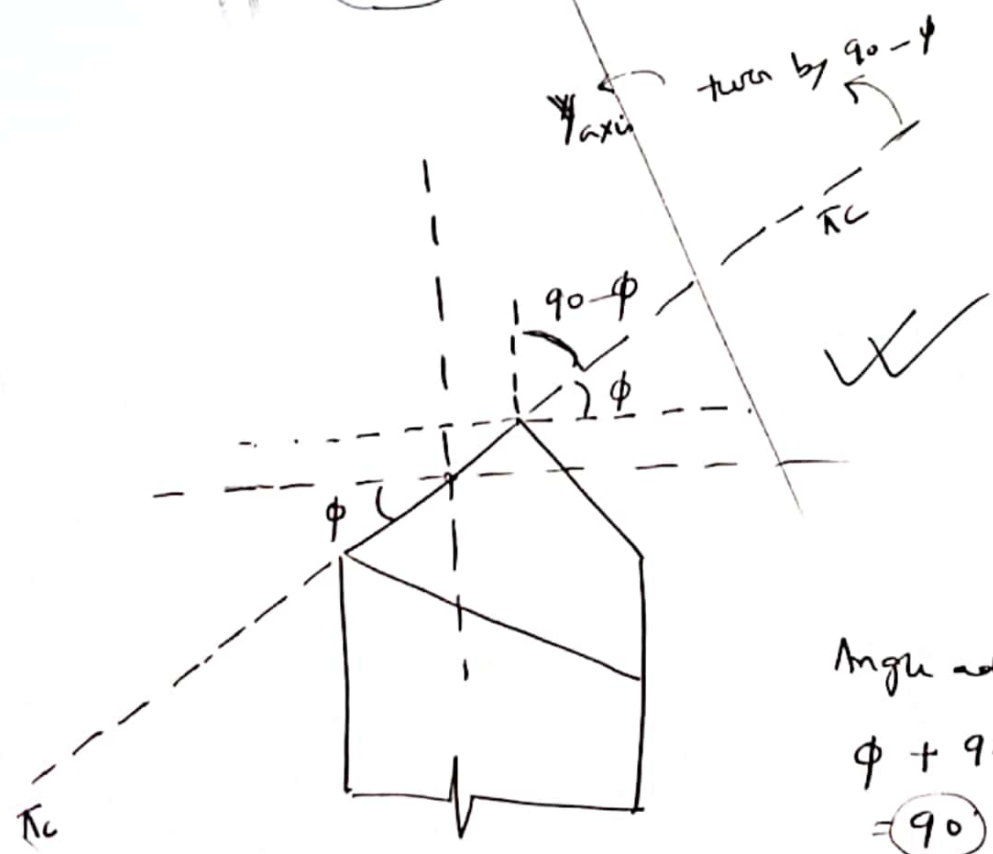
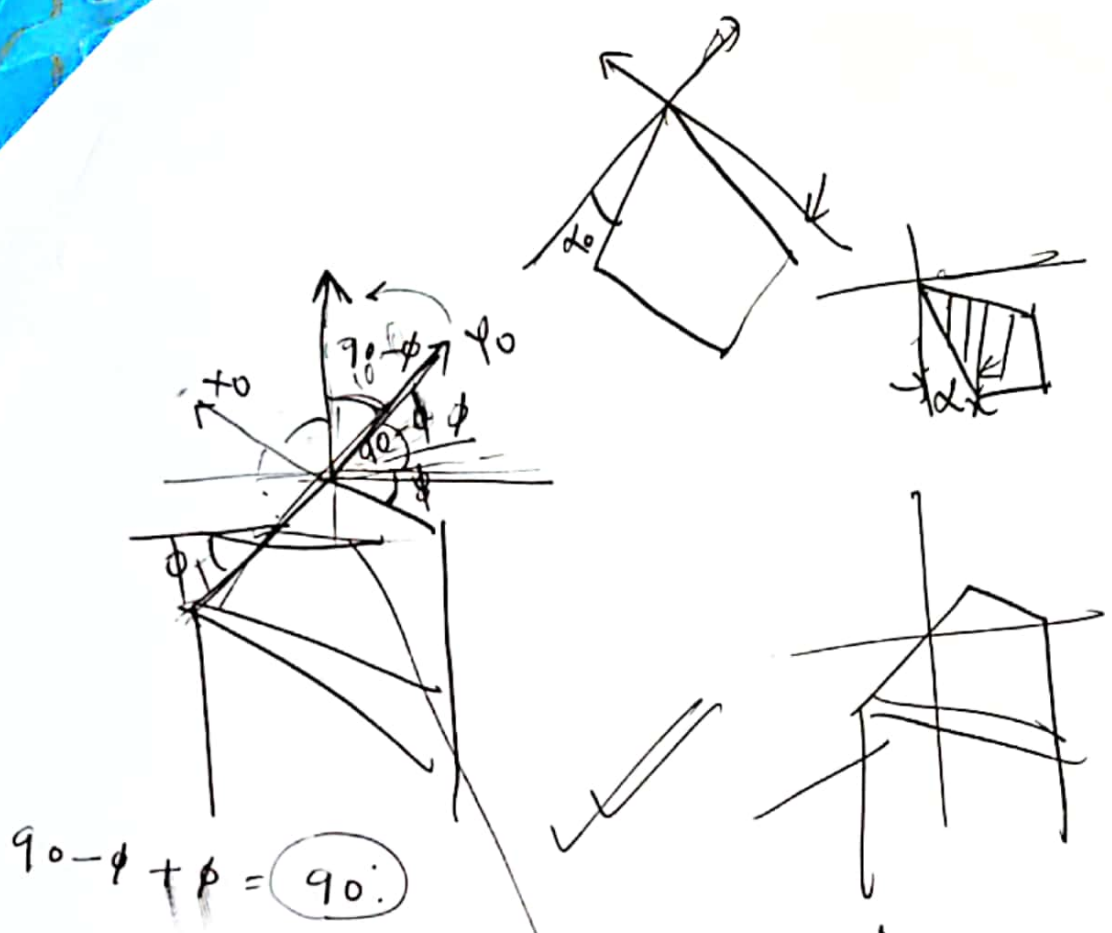
Advantages ~~of~~ over ASA and ORS

- ASA system is convenient, only for inspection.
- ORS is convenient for analysis & research but does not reveal the true geometry & needs additional corrections for grinding.
- NRS gives true geometry & need no correction.

In ORS  $\gamma$  &  $\alpha$  are measured in  $\pi_0$  but in NRS, in  $\pi_n$  (Plane  $\perp$  to cutting edge only)

When  $\lambda = 0$  : ORS & NRS become same





Angle add  
 $\phi + 90 - \phi$   
 $= 90$   
 then  $\alpha_x = \alpha_0$

$\lambda$  = inclination angle; angle between  $\pi_c$  from the direction of assumed longitudinal feed  $[\pi_x]$  and measured on  $\pi_c$

- Clearance angles [Fig. 3.7]

$\alpha_o$  = orthogonal clearance of the principal flank: angle of inclination of the principal flank from  $\pi_c$  and measured on  $\pi_o$

$\alpha_o'$  = auxiliary orthogonal clearance: angle of inclination of the auxiliary flank from auxiliary cutting plane,  $\pi_c'$  and measured on auxiliary orthogonal plane,  $\pi_o'$  as indicated in Fig. 3.8.

- Cutting angles [Fig. 3.7]

$\phi$  = principal cutting edge angle: angle between  $\pi_c$  and the direction of assumed longitudinal feed or  $\pi_x$  and measured on  $\pi_R$

$\phi_1$  = auxiliary cutting angle: angle between  $\pi_c'$  and  $\pi_x$  and measured on  $\pi_R$

- Nose radius,  $r$  (mm)

$r$  = radius of curvature of tool tip

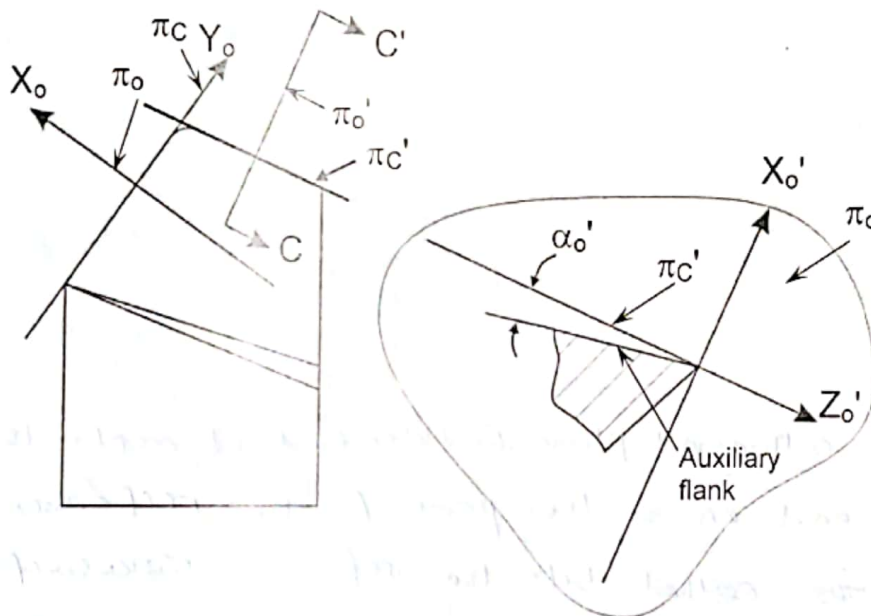


Fig. 3.8 Auxiliary orthogonal clearance angle

- Normal Rake System – NRS

This system is also known as ISO – new.

ASA system has limited advantage and use like convenience of inspection. But ORS is advantageously used for analysis and research in machining and tool performance. But ORS does not reveal the true picture of the tool geometry when the cutting edges are inclined from the reference plane, i.e.,  $\lambda \neq 0$ . Besides, sharpening or resharpening, if necessary, of the tool by grinding in ORS requires some additional calculations for correction of angles.

ASA system is conv. only for inspection.

ORS system is conv. for analysis and research, but does not reveal the true geometry & needs add. corrections for regrinding.

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These two limitations of ORS are overcome by using NRS for description and use of tool geometry.

The basic difference between ORS and NRS is the fact that in ORS, rake and clearance angles are visualized in the orthogonal plane,  $\pi_o$ , whereas in NRS those angles are visualized in another plane called Normal plane,  $\pi_N$ . The orthogonal plane,  $\pi_o$  is simply normal to  $\pi_R$  and  $\pi_C$  irrespective of the inclination of the cutting edges, i.e.,  $\lambda$ , but  $\pi_N$  (and  $\pi_N'$  for auxiliary cutting edge) is always normal to the cutting edge. The differences between ORS and NRS have been depicted in Fig. 3.9.

The planes of reference and the coordinates used in NRS are:

$$\pi_{RN} = \pi_C = \pi_N \text{ and } X_n = Y_n = Z_n$$

where,

$\pi_{RN}$  = normal reference plane

$\pi_N$  = Normal plane: plane normal to the cutting edge

and

$X_n = X_o$

$Y_n$  = cutting edge

$Z_n$  = normal to  $X_n$  and  $Y_n$

It is to be noted that when  $\lambda = 0$ , NRS and ORS become same, i.e.  $\pi_o \equiv \pi_N$ ,  $Y_N \equiv Y_o$  and  $Z_N \equiv Z_o$ .

Definition (in NRS) of

- Rake angles

$\gamma_n$  = normal rake: angle of inclination of the rake surface from  $\pi_R$  and measured on normal plane,  $\pi_N$

$\alpha_n$  = normal clearance: angle of inclination of the principal flank from  $\pi_C$  and measured on  $\pi_N$

$\alpha_n'$  = auxiliary clearance angle: normal clearance of the auxiliary flank (measured on  $\pi_N'$  - plane normal to the auxiliary cutting edge).

The cutting angles,  $\phi$  and  $\phi_1$  and nose radius,  $r$  (mm) are same in ORS and NRS.

\* When the orthogonal plane is extended it meets the rake surface OB and OA is the part of the reference plane. & this angle is called between the rake surface & the reference plane is called the rake angle & since it is orthogonal plane it is called orthogonal rake  $\gamma_o$ . This orthogonal plane is  $\perp$  to  $\pi_o$  but not  $\perp$  to the cutting edge because cutting edge is inclined. & this inclination is denoted by  $\lambda$ . So orthogonal plane is not  $\perp$  to the cutting edge because of this inclination angle  $\lambda$ .

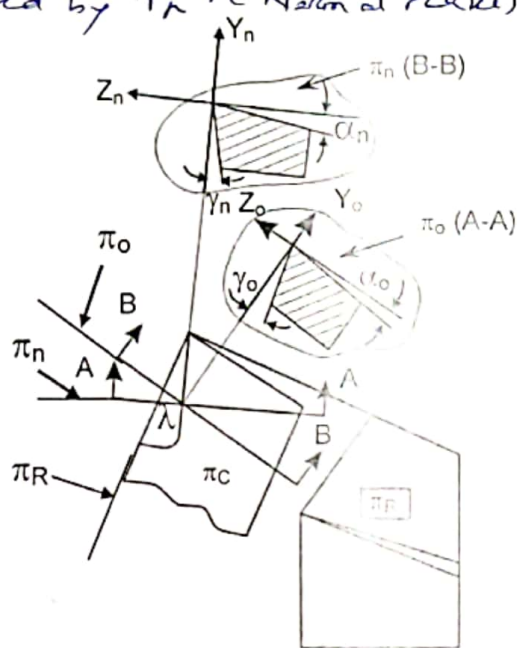
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Now we have to consider a plane which is  $\perp$  to the cutting edge. So the orthogonal plane has to be tilted by an angle  $\lambda$  so that it will be  $\perp$  to the cutting edge. & that

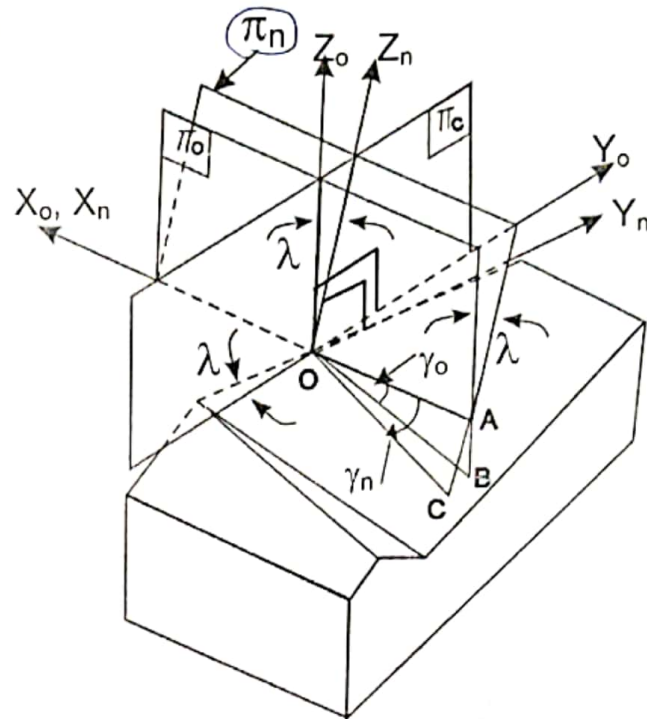


plane is called Normal plane ( $\pi_n$ ) & that axis cutting edge axis is called  $Y_n$  axis. So in NRS  $X_n, Y_n$  &  $Z_n$  are the axes.

Normal plane  $\pi_n$  which are extended, then it meets the rake surface & then the rake angle but this rake angle is different from  $\gamma_o$ , as it is measured on Normal plane so this is denoted by  $\gamma_n$  (Normal rake).



(a)



(b)

Similarly in the same plane  $\pi_n$ , the clearance angle has to be measured.

Fig. 3.9 Differences of NRS from ORS w.r.t. cutting tool geometry.

#### (b) Designation of tool geometry

The geometry of a single point tool is designated or specified by a series of values of the salient angles and nose radius arranged in a definite sequence as follows:

Designation (signature) of tool geometry in

- ✓ ASA System -  $\gamma_y, \gamma_x, \alpha_y, \alpha_x, \phi_e, \phi_s, r$  (inch)
- ✓ ORS System -  $\lambda, \gamma_o, \alpha_o, \alpha_o', \phi_1, \phi, r$  (mm)
- ✓ NRS System -  $\lambda, \gamma_n, \alpha_n, \alpha_n', \phi_1, \phi, r$  (mm)

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In NRS,  $\pi_R$  is not  $\perp$  to  $\pi_n$  for the cutting edge tool.  $\pi_c$ .



### Exercise – 3

#### Quiz Test:

Select the correct answer from the given four options :

1. Back rake of a turning tool is measured on its
  - (a) machine longitudinal plane
  - ☒ (b) machine transverse plane
  - (c) orthogonal plane
  - (d) normal plane
2. Normal rake and orthogonal rake of a turning tool will be same when its
  - (a)  $\phi = 0$
  - (b)  $\phi_1 = 0$
  - ☒ (c)  $\lambda = 0$
  - (d)  $\phi_1 = 90^\circ$
3. Normal plane of a turning tool is always perpendicular to its
  - (a)  $\pi_X$  plane
  - (b)  $\pi_Y$  plane
  - ☒ (c)  $\pi_C$  plane
  - (d) none of them
4. Principal cutting edge angle of any turning tool is measured on its
  - ☒ (a)  $\pi_R$
  - (b)  $\pi_Y$
  - (c)  $\pi_X$
  - (d)  $\pi_O$
5. A cutting tool can never have its
  - (a) rake angle – positive
  - (b) rake angle – negative
  - (c) clearance angle – positive
  - ☒ (d) clearance angle – negative
6. Orthogonal clearance and side clearance of a turning tool will be same if its perpendicular cutting edge angle is
  - (a)  $\phi = 30^\circ$
  - (b)  $\phi = 45^\circ$
  - (c)  $\phi = 60^\circ$
  - ☒ (d)  $\phi = 90^\circ$

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7. Inclination angle of a turning tool is measured on its  
(a) reference plane  
✓ (b) cutting plane  
(c) orthogonal plane  
(d) normal plane
8. Normal rake and side rake of a turning tool will be same if its  
(a)  $\phi = 0^\circ$  and  $\lambda = 0^\circ$   
✓ (b)  $\phi = 90^\circ$  and  $\lambda = 0^\circ$   
(c)  $\phi = 90^\circ$  and  $\lambda = 90^\circ$   
(d)  $\phi = 0^\circ$  and  $\lambda = 90^\circ$

Answer of the objective questions

- 1 - (b)
- 2 - (c)
- 3 - (c)
- 4 - (a)
- 5 - (d)
- 6 - (d)
- 7 - (b)
- 8 - (b)

Interrelation among the tool angles expressed in different systems

Objectives:

- (1) State the purposes of conversion of tool angles from one system to another.
- (2) Name the different methods of conversion of tool angles.
- (3) Convert tool angles by graphical method
  - Rake angle: ASA  $\rightleftharpoons$  ORS
  - Clearance angle: ASA  $\rightleftharpoons$  ORS
- (4) Convert tool angles from ORS to NRS.
- (5) Ascertain tool geometry in some specific cond<sup>n</sup>.

Purposes of conversion of tool angles:

- Understand geometry if expressed in a different system.
- Decide benefits of different systems.
- Communicate geometry to others who follow different tool designation system.

## Different methods of conversion of tool angles

- ① Analytical method — Simple but tedious.  
(geometrical method)
- ② Graphical method — Simple, quick & popular.  
(Masterline principle)
- ③ Transformation Matrix method — Suitable for complex tool —  
ex: Horen tool, Hobb cutters, gear shaping cutters, Drill geometry, geometry.
- ④ Vector method — very simple & quick but needs concept  
of vectors.  
~~associated with~~



## Conversion of ASA to ORS - old system

$$(1) \quad \tan \gamma_0 = \tan \gamma_y \cdot \cos \phi + \tan \gamma_x \cdot \sin \phi$$

$$(2) \quad \tan \lambda = \tan \gamma_y \cdot \sin \phi - \tan \gamma_x \cdot \cos \phi$$

The above formulae are used to convert ASA system to ORS system, in terms of rake angle.

$$(3) \quad \cot \alpha_0 = \cot \alpha_y \cdot \cos \phi + \cot \alpha_x \cdot \sin \phi$$

$$(4) \quad \tan \lambda = \cot \alpha_y \cdot \sin \phi - \cot \alpha_x \cdot \cos \phi$$

These formulae are used to convert ASA - ORS in terms of clearance angle.

$$\gamma_0, \lambda = f(\gamma_x, \gamma_y, \phi)$$

→ Principle cutting

$$\phi = 90^\circ - \phi_s$$

$\phi_s$  : Approach angle

## Conversion of ORS to ASA system

$$(5) \quad \tan \gamma_y = \tan \gamma_0 \cdot \cos \phi + \tan \lambda \cdot \sin \phi$$

$$(6) \quad \tan \gamma_x = \tan \gamma_0 \cdot \sin \phi - \tan \lambda \cdot \cos \phi$$

$$(7) \quad \cot \alpha_y = \cot \alpha_0 \cdot \cos \phi + \tan \lambda \cdot \sin \phi$$

$$(8) \quad \cot \alpha_x = \cot \alpha_0 \cdot \sin \phi - \tan \lambda \cdot \cos \phi$$

## Formula for NRE system

(9)

$$\tan \gamma_n = \tan \gamma_0 \cdot \cos \lambda$$

(10)

$$\cot \alpha_n = \cot \alpha_0 \cdot \cos \lambda$$

(11)

$$\tan \gamma_n = \sqrt{\tan^2 \gamma_x + \tan^2 \gamma_y}$$

(12)

$$\tan \gamma_n = \sqrt{\tan^2 \gamma_0 + \tan^2 \lambda}$$

(13)

$$\begin{aligned} \cot \alpha_n &= \sqrt{\cot^2 \alpha_x + \cot^2 \alpha_y} \\ &= \sqrt{\cot^2 \alpha_0 + \tan^2 \lambda} \end{aligned}$$

→ when  $\phi = 90^\circ$

$$\pi_x = \pi_0$$

$$\gamma_x = \gamma_0$$

→ when  $\lambda = 0$

$$\pi_0 = \pi_n$$

$$\gamma_n = \gamma_0$$

$$\alpha_n = \alpha_0$$

→ when  $\lambda = 0$ , &  $\phi = 90^\circ$

$$\gamma_x = \gamma_0 = \gamma_n$$

$$\alpha_x = \alpha_0 = \alpha_n$$



Determine the values of orthogonal rake angle ( $\gamma_o$ ), the inclination angle ( $\lambda$ ) of the main cutting edge & the nose of the turning tool specified in the ASA system as  $10^\circ, -10^\circ, 8^\circ, 6^\circ, 15^\circ, 30^\circ, 0$  (in cw).

$$\tan \gamma_o = \tan \gamma_x \cdot \cos \phi + \tan \gamma_y \cdot \sin \phi$$

Given that  $\gamma_x = 10^\circ$

$\phi_s = 30^\circ$

$\gamma_y = -10^\circ$

$\phi = 90 - \phi_s$

$= 90 - 30 = 60^\circ$

$$\tan \gamma_o = \tan 10^\circ \cdot \sin 60^\circ + \tan(-10^\circ) \cdot \cos 60^\circ$$

$$\Rightarrow \boxed{\gamma_o = 3.68^\circ}$$

To determine inclination ( $\lambda$ ):

$$\tan \lambda = \tan \gamma_y \cdot \sin \phi - \tan \gamma_x \cdot \cos \phi$$

$$\tan \lambda = \tan(-10^\circ) \cdot \sin(60^\circ) - \tan 10^\circ \cdot \cos(60^\circ)$$

$$\Rightarrow \boxed{\lambda = -13.5^\circ}$$

$$\tan \gamma_n = \sqrt{\tan^2 \gamma_x + \tan^2 \gamma_y}$$

$$\Rightarrow \gamma_n =$$

Q2

Determine the values of the side rake ( $\gamma_x$ ), back rake and the normal rake ( $\gamma_n$ ) angles of the single point turning tool whose geometry is specified or designated as

$$0^\circ, 10', 8', 6', 15', 60', 0 \text{ (mm)}$$

$$\lambda = \gamma_0 = \alpha_0 = \alpha_0' = \phi_1 = \phi = r \text{ (mm)}$$

$$\tan \gamma_y = \tan \gamma_0 \cdot \cos \phi + \tan \lambda \cdot \sin \phi$$

$$= \tan 10' \cdot \cos 60' + \tan 0 \cdot \sin 60'$$

$$\Rightarrow \gamma_y =$$

$$\tan \gamma_x = \tan \gamma_0 \cdot \sin \phi - \tan \lambda \cdot \cos \phi$$

$$= \tan 10' \cdot \sin 60' - \tan 0 \cdot \cos 60'$$

$$\Rightarrow \gamma_x =$$

$$\tan \gamma_n = \sqrt{\tan^2 \gamma_0 + \tan^2 \lambda}$$

$$=$$

Under what geometrical cond<sup>n</sup> of a turning tool w.  
the values of its side rake ( $\gamma_x$ ), orthogonal rake ( $\gamma_o$ )  
& normal rake ( $\gamma_n$ ) angles be the same?

We know that.

$$\boxed{\tan \gamma_n = \tan \gamma_o \cdot \cos \lambda} \quad \text{--- (A)}$$

Where  $\lambda$  is the inclination angle.

$$\boxed{\text{Therefore when } \lambda = 0, \\ \gamma_o = \gamma_n}$$

$$\text{Again } \boxed{\tan \gamma_x = \tan \gamma_o \cdot \sin \phi - \tan \lambda \cdot \cos \phi} \quad \text{--- (B)}$$

$$\text{As } \lambda = 0$$

$$\Rightarrow \tan \gamma_x = \tan \gamma_o \cdot \sin \phi$$

$$\boxed{\text{Hence } \gamma_x \text{ \& } \gamma_o \text{ will be same} \\ \text{if } \phi = 90^\circ}$$

$$\therefore \boxed{\gamma_x = \gamma_o = \gamma_n \text{ will occur if } \lambda = 0 \text{ \& } \phi = 90^\circ}$$

oment

Q1

Calculate the inclination angle  $\lambda$  & orthogonal rake  $\gamma_0$

when  $\gamma_x = 10^\circ$ ,  $\gamma_y = 8^\circ$ ,  $\phi_s = 15^\circ$

(2) Now reverse the process i.e

$$\text{If } \gamma_0 = 11.67^\circ \text{ \& } \lambda = 5.74^\circ$$

then calculate the side rake angle  $\gamma_x$  & Back rake angle  $(\gamma_y)$ .

(3) Calculate the inclination angle  $\lambda$  & orthogonal rake angle  $\gamma_0$

when  $\gamma_x = -05^\circ$ ,  $\gamma_y = -7^\circ$  &  $\phi_s = 15^\circ$  are given.

(4) Tool signature 7-10-7-7-15-60-0 (ORS)

Find  $\gamma_n$ ,  $\lambda_n$  & convert into NRS.

(5) Tool Signature is ASA as 10-8-7-7-15-30-0

Convert it into NRS.

(6) Convert first the ASA to ORS & then to NRS.



# MECHANICS OF MACHINING

## MECHANISM OF CHIP FORMATION :

### Instructional objectives :

To enable the students, at the end of this lecture,

(i) Describe with illustration the mechanism of chip formation in machining

- o Ductile materials &

- o Brittle materials

(ii) Describe geometric characteristics of ductile chips

- o chip reduction coefficient & cutting ratio

- o Shear angle & cutting strain.

(iii) Identify the causes, characteristics & effects of built-up-edge (BUE) formation.

(iv) Classify chips & identify the cond'y. under which such kind of chip form.

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The form of chips is an important index of machining as it indicates.

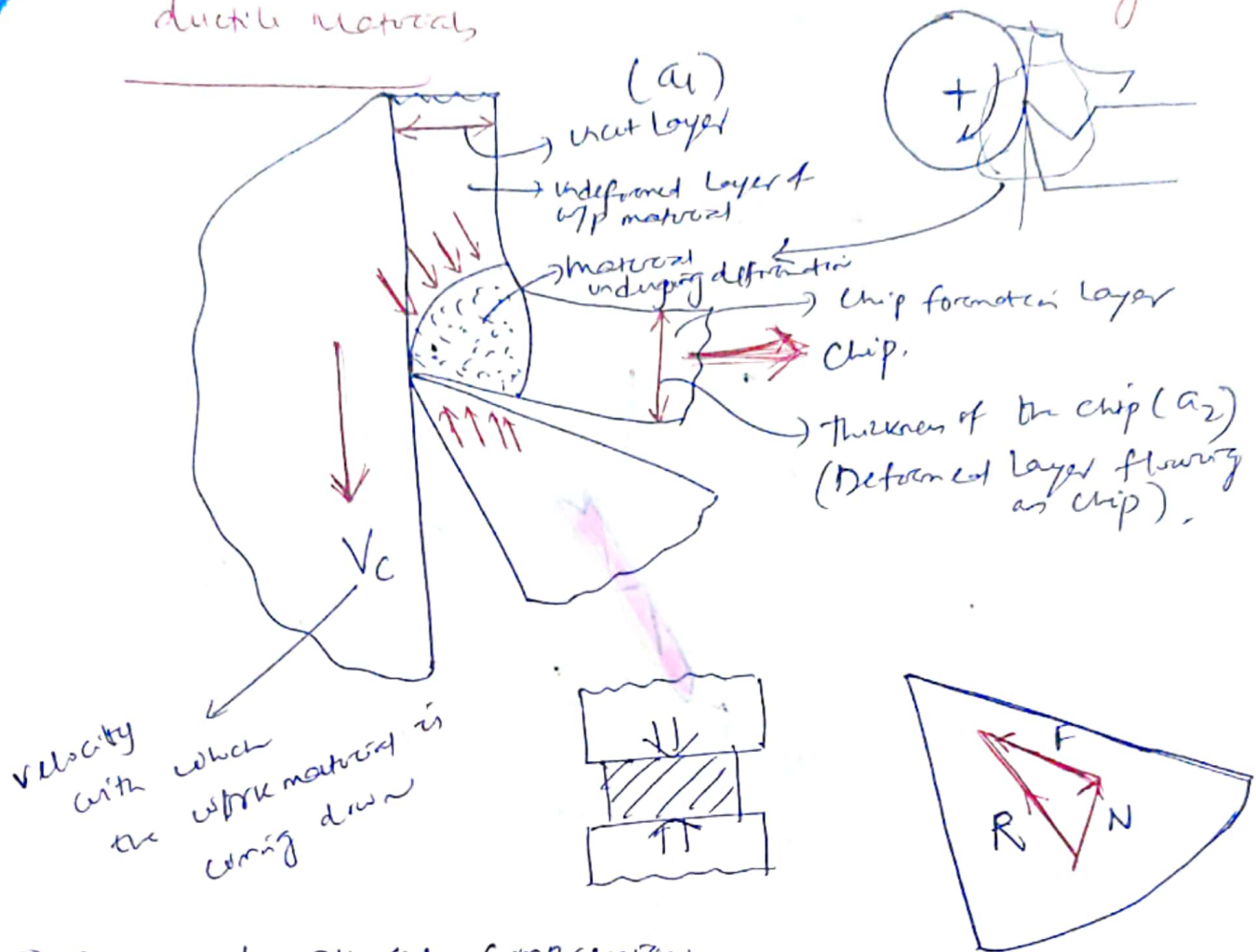
- Nature & behavior of the work material under machining.
- Specific energy requirement to machine.
- Nature & degree of interaction at the work-tool interfaces.

form of the chips that are produced during machining is an important index of machining. Index of machining means from the chip configuration & colour one can understand qualitatively whether the machining is favorable/not favorable.

The Form & Colour of Chips depend upon :-

- (1) Work material : Brittle, ductile
- (2) Cutting tool : material & geometry  $\rightarrow$  chemical affinity (chips better),  
stickiness (chips poorer)
- (3) Level of the process parameters.
- (4) Application of cutting fluid (cooling & lubrication  
both affect the form & color of the chips).

# Mechanism of chip formation in machining ductile materials

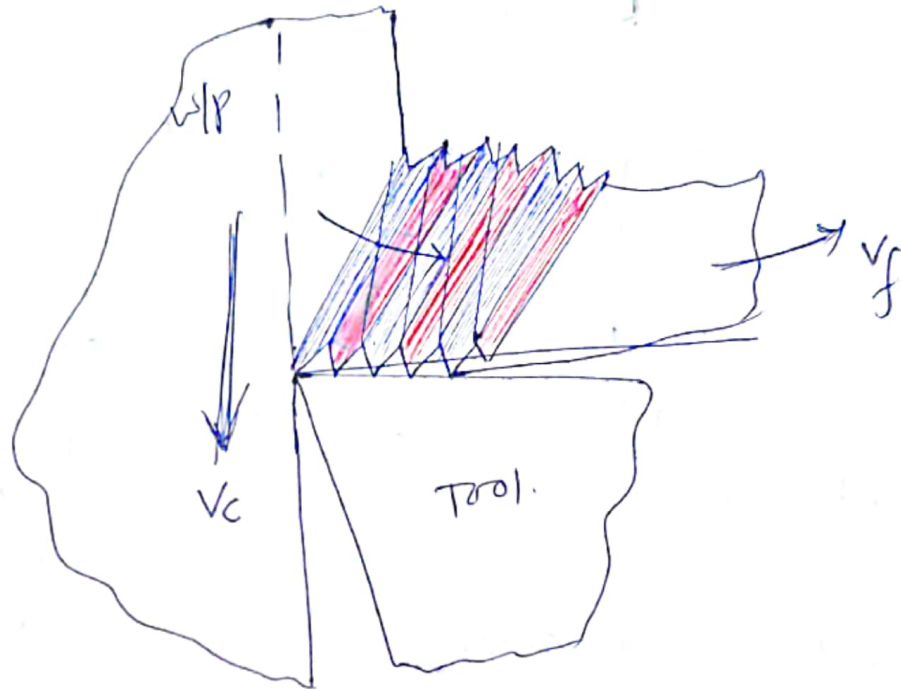
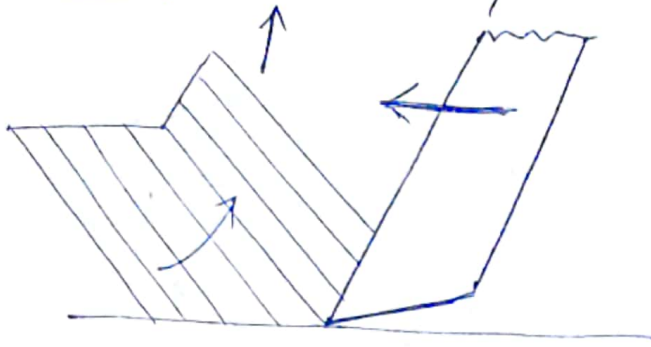


⊛ Suspected to all side compression

+ Shear stress simultaneously develops in different direction & in different magnitude.

\* whenever & wherever the value of the shear stress reaches or exceeds the shear strength of that work material in the deformation zone, then failure will start i.e. separation takes place of the chip material from the parent material.

## Lamellar chip formation <sup>OM</sup> by shear



Prispannen model of card analogy to explain chip formation in machining ductile materials:

- (a) Shifting of postcards by partial sliding.
- (b) Chip formation in lamella by shear.



## Experimental Study of Chip formation

Purpose: Chip formation mechanism is the deformation process which is the permanent failure.

It is the mechanical failure i.e. ~~by stress~~ it caused by stress i.e. either by compression stress, shear stress, tensile stress / combination.

What is the real cause / which one is <sup>more</sup> predominant? acting on the formation of chips. It is very complex phenomenon & difficult to understand. For that reason some experiments has to be conducted to understand all these things. What kind of stress is responsible for chip formation / Real mechanism for ductile material.

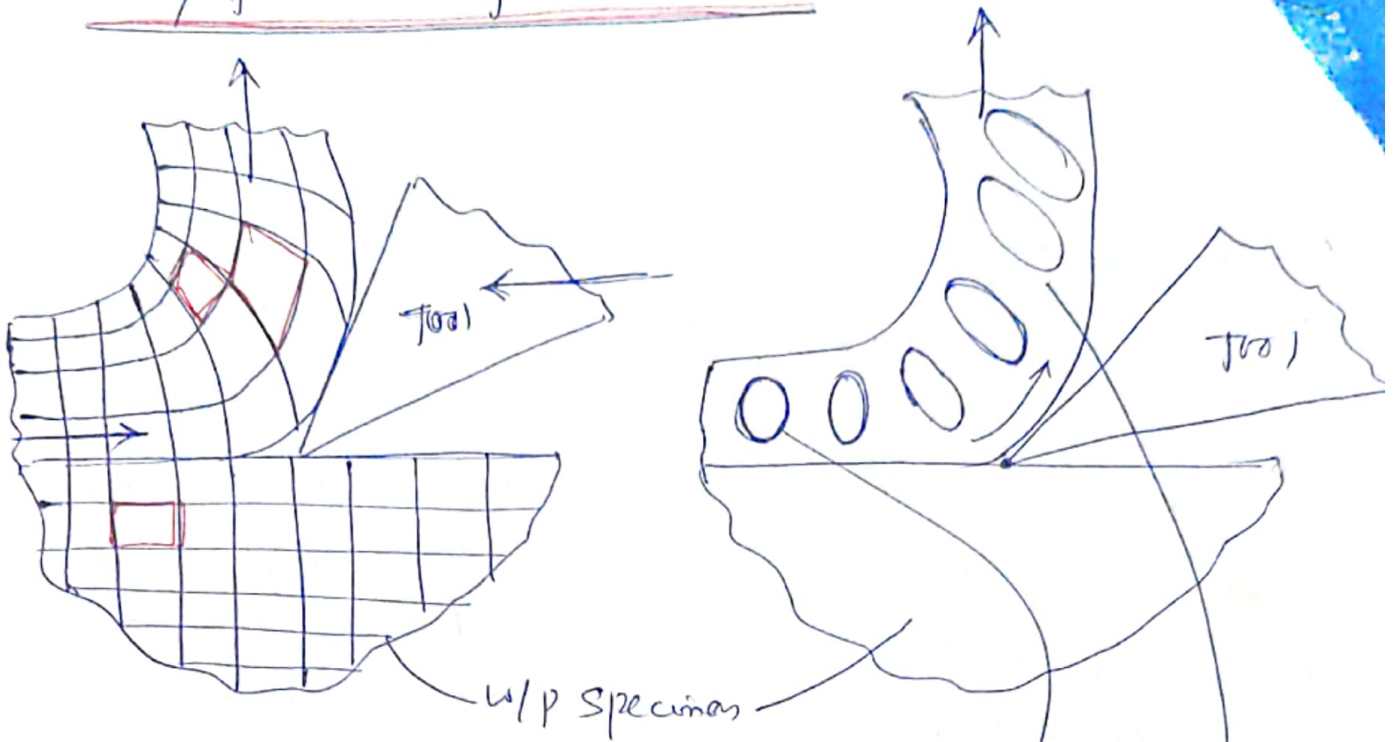
~~Some~~ Some feasible & popular experimental methods for this purpose are as follows:

- (1) Observing the chip formation (directly)
  - (a) On the side surface of the w.p.
  - (b) on the inner surface of a compound w.p.

- (2) Exp Microscopic study of chips frozen by
  - (a) Drop tool or
  - (b) Quick stop apparatus.

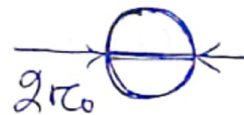
- (3) Study of running chips photographed by high speed camera fitted with low magnification microscope

# (1) Study of circular grid formation

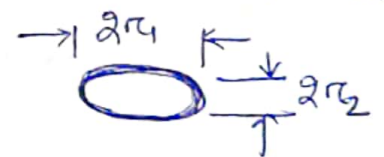


(Pattern of grid deformation during chip formation)

- (a) Rectangular grids
- (b) Circular grids



Circular grid after deformation

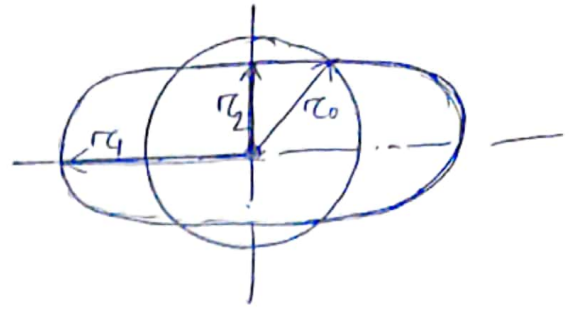


The type of stress actually responsible for transformation of a grid into an elliptical form, can be experimentally identified by using Levy-Lode's theorem:

$$\frac{l_1 + l_2 - 2l_3}{l_1 - l_2} = \frac{\tan(30 - \phi)}{\tan 30}$$

where  $l_1 = \ln \left( \frac{r_1}{r_0} \right)$

$$l_2 = \ln \left( \frac{r_2}{r_0} \right)$$



$$\& \quad l_1 + l_2 + l_3 = 0$$

where  $l_1, l_2$  &  $l_3$  are the strains in the three principal axes.

$\phi$  = angle of index

$r_0$  = Original radius of circles. (radius of un-deformed circular grid)

$r_1$  = Major axis of the deformed elliptical grid,

$r_2$  = Minor axis of the deformed elliptical grid.

If values for  $l_1, l_2, l_3$  are substituted for any deformation,  $\phi$  can be determined.

It has been found that

$$\phi = 0, \text{ for pure tension}$$

$$\phi = 60', \text{ for pure compression}$$

$$\phi = 30', \text{ for pure shear}$$

By studying circular grid deformation in machining ductile metals, it has been found by KUFARE that the actual value of the angle of index  $\phi$  is very close to  $30^\circ$ . Thus, it is inferred that chip formation in machining ductile metals is caused mainly by shearing process.

## Geometrical characteristics of continuous chips

The geometry of the chips being formed at the cutting zone follows a particular pattern especially in machining ductile materials.

The major sections of the engineering materials being machined are ductile in nature.

The pattern and degree of deformation during chip formation are quantitatively assessed & expressed by some factors, the values of which indicate the forces & energy reqd for a particular machining work.

Chip reduction Coefficient ( $\zeta$ )

Cutting ratio ( $r_c$ ).

$a_1$  = Thickness of uncut layer or chip

$a_2$  = Thickness of chip or cut layer

$t$  = Depth of cut i.e.

amount of penetration of the cutting tool from unfinished to finished surface.

$b_1$  = Width of cut

$$\zeta = \text{Chip reduction coefficient} = \frac{a_2}{a_1}$$

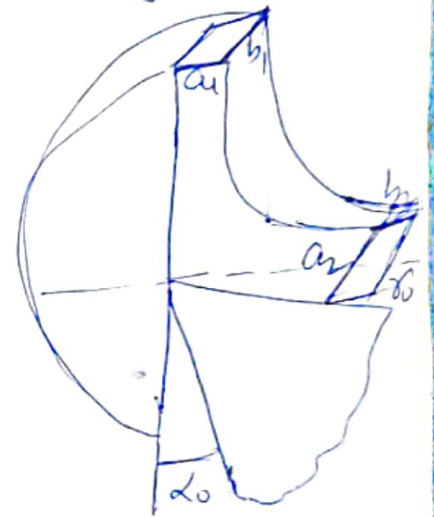
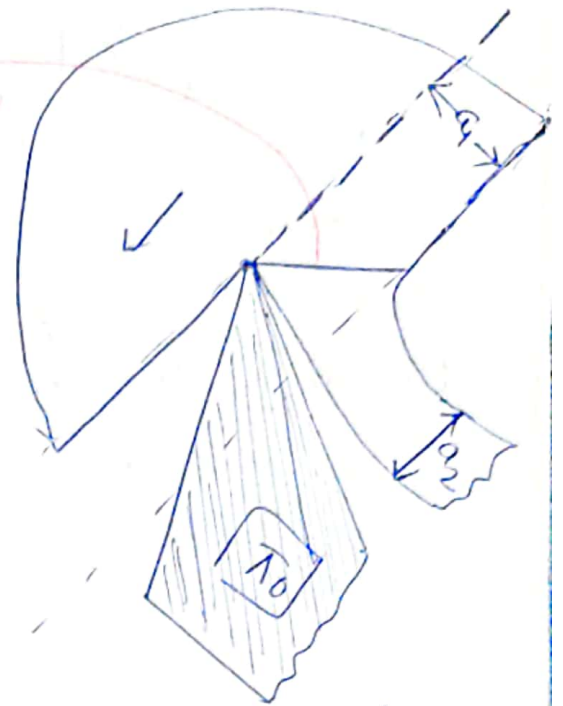
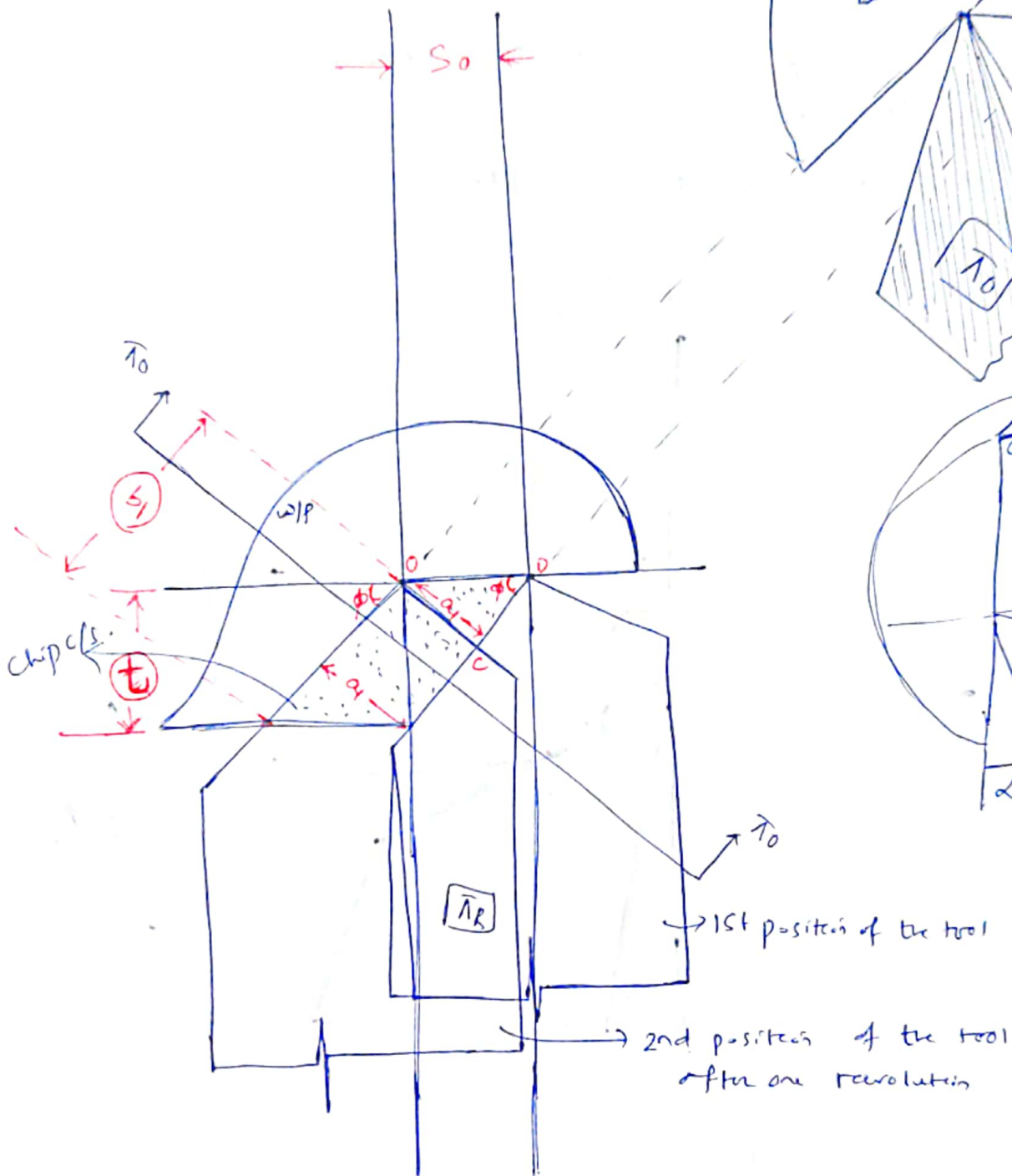
= Ratio of thickness of the chip after & before

cut  
= Degree of thinning of the chip.

It is an index of the degree of deformation involved in chip formation process.



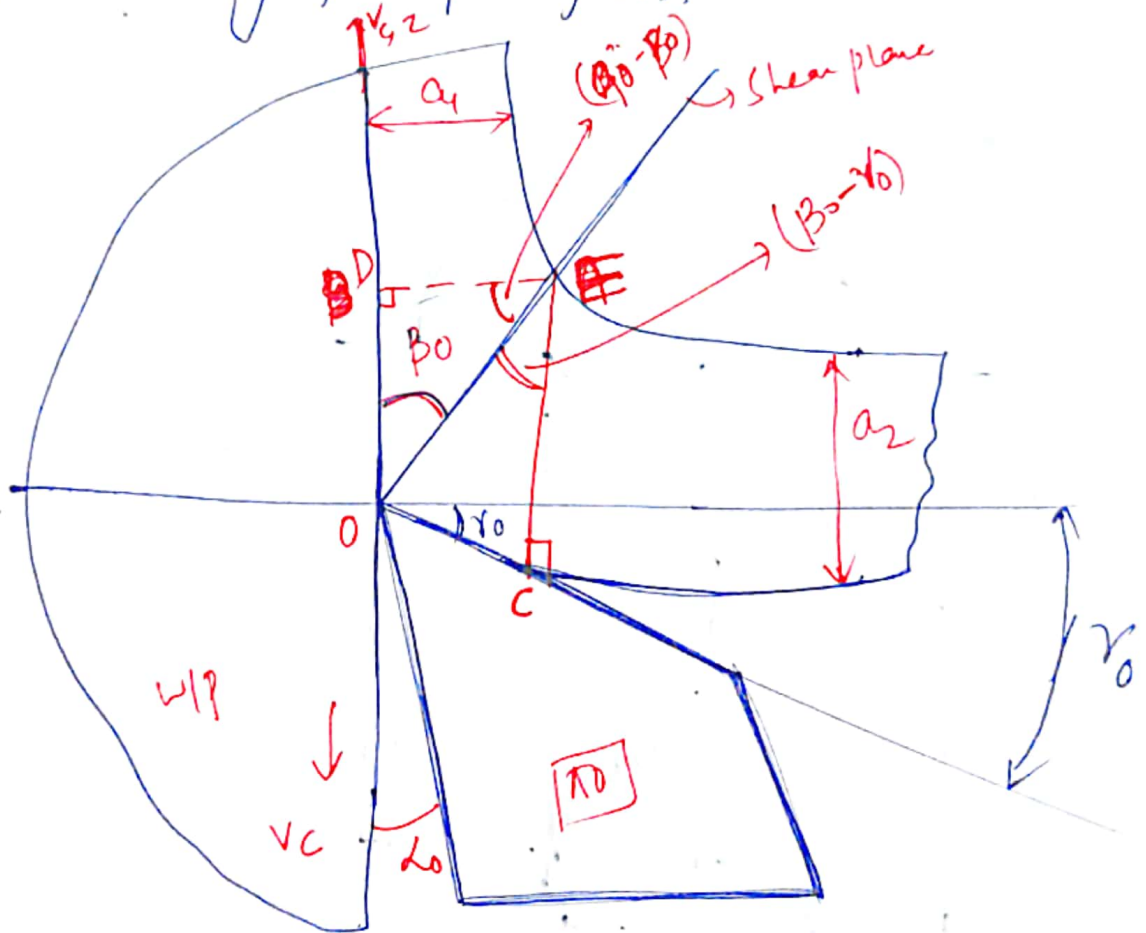
Plane of shear or lamellar sliding



$$\zeta = \frac{a_2}{a_1} > 1.00$$

The reason :

- (\*) because the work material is ahead of the cutting tool is always in compression, the chip becomes thicker & the length shrinks.
- (\*) Frictional resistance to chip flow.
- (\*) Lamellar sliding of chip segments.



$\beta_0 = \text{Shear angle}$

$$\zeta = \frac{a_2}{a_1}$$

$$DE = a_1 = OE \sin \beta_0$$

$$a_2 = CE = OE \cos(\beta_0 - \gamma_0)$$

$$\zeta = \frac{OE \cos(\beta_0 - \gamma_0)}{OE \sin \beta_0}$$

$$\Rightarrow \boxed{\zeta = \frac{\cos(\beta_0 - \gamma_0)}{\sin \beta_0}}$$

$$\Rightarrow \zeta = \frac{\cos \beta_0 \cdot \cos \gamma_0 + \sin \beta_0 \cdot \sin \gamma_0}{\sin \beta_0}$$

$$\Rightarrow \boxed{\tan \beta_0 = \frac{\cos \gamma_0}{\zeta - \sin \gamma_0}}$$

$$\Rightarrow \boxed{\beta_0 = f(\gamma_0, \zeta)}$$

\* The inverse of  $\zeta$  is denoted by  $\eta_c$  & is known as cutting ratio.

$$\boxed{\eta_c = \frac{1}{\zeta}}$$

Relation bet<sup>n</sup> shear angle and rake angle.

$\beta_0$  is a f<sup>n</sup> of rake angle & chip reduction coefficient.

## EFFECT OF CUTTING VARIABLES ON CHIP REDUCTION COEFFICIENT

The chip reduction co-efficient is an important parameter through which the degree of deformation and hence the forces required for cutting can be gauged.

The basic characteristics of the chip formation process include:

- (a) Direction of primary shear.
- (b) Magnitude of chip deformation.
- (c) Chip shrinkage.
- (d) The specific chip formation work.

① The chip reduction co-efficient is affected by:

- (i) Cutting variables i.e. cutting speed ( $V_c$ ).
- (ii) The feed ( $S_o$ )
- (iii) The depth of cut ( $b$ )
- (iv) The rake angle ( $\gamma$ )

② The cutting environment and auxiliary variable i.e. frictional resistance at the rake surface, the cutting fluid, the material of the tool, the surface of the work material, the auxiliary shape of the tool etc.

Mechanism of chip formation is usually investigated by two sets of experimentations  $\Rightarrow$

- (A) Studies at low cutting speeds.
- (B) Studies at high cutting speeds.

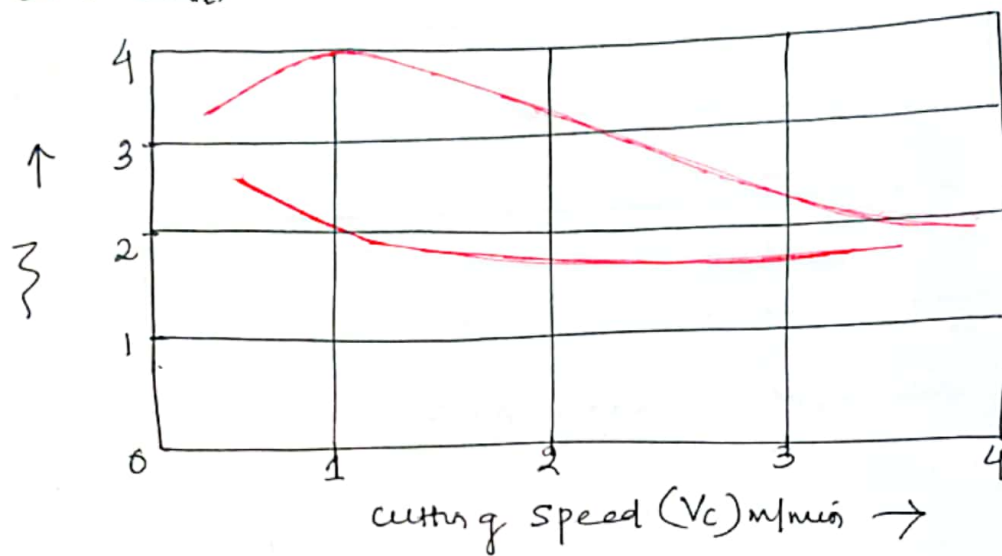


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### (A) STUDIES AT LOW CUTTING SPEEDS $\Rightarrow$

#### (i) Effect of cutting speed on $\zeta$ $\Rightarrow$

The influence of cutting speed at low range on the chip reduction co-efficient at two different uncut depths is shown.



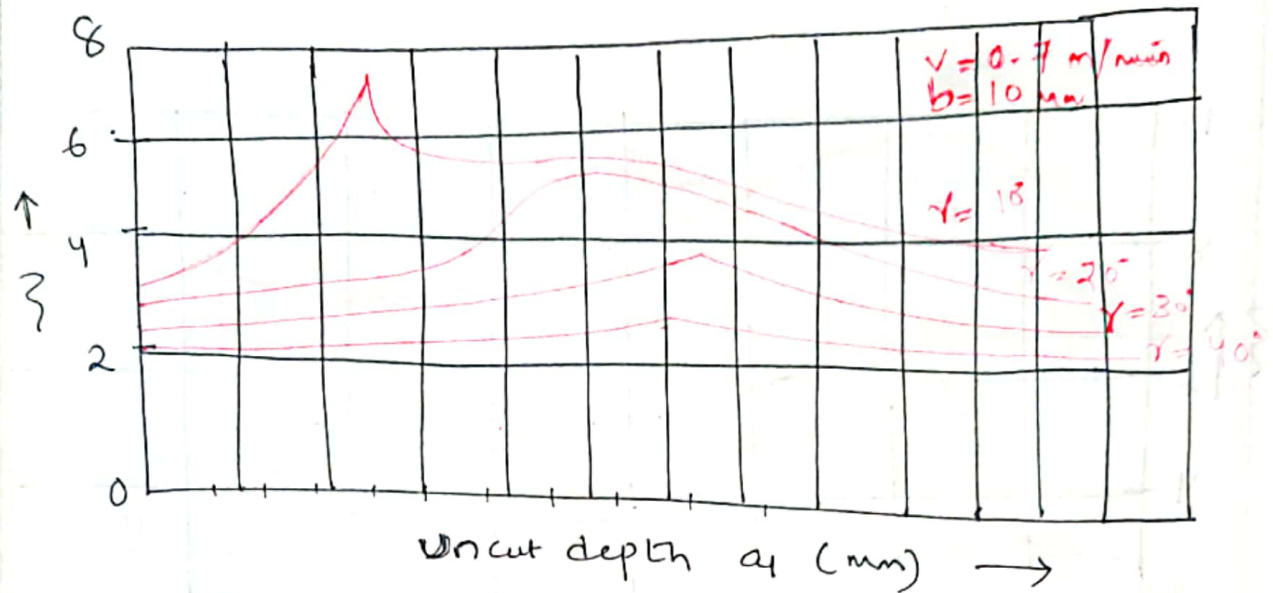
At larger value of  $a_1$ , the chip reduction co-efficient first rises and after reaching a maxm, it decreases with increase of cutting speed.

With smaller depth of cut of layer ( $a_1 = 0.058$ ) the built-up edge starts forming at the lower cutting speed and a continuous drop in  $\zeta$  results.

#### (ii) Effect of uncut - depth ( $a_1$ ) $\Rightarrow$

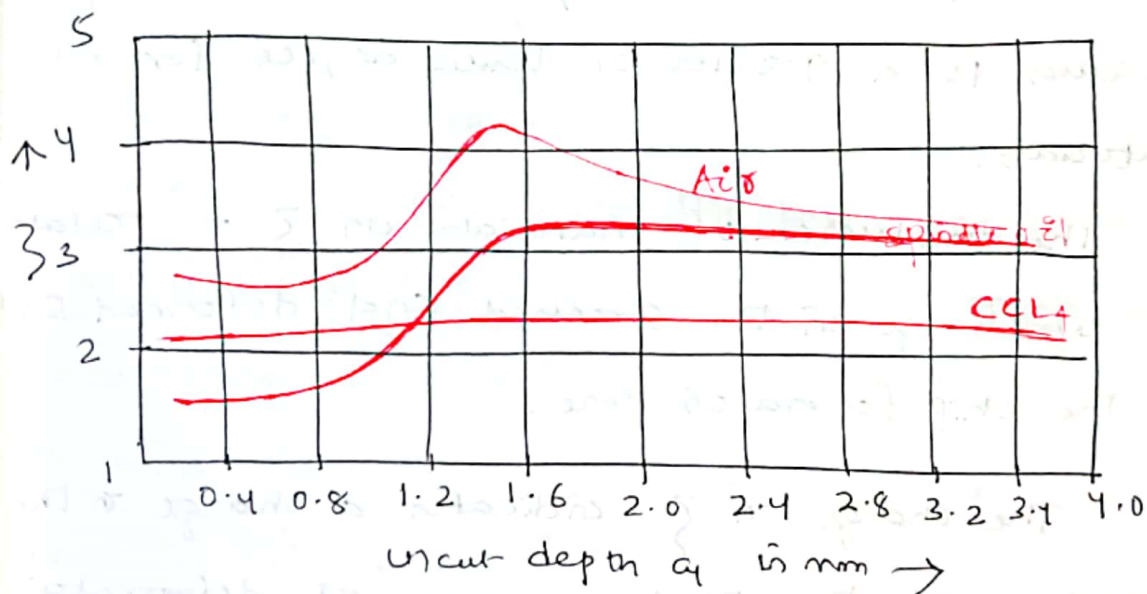
At larger rake angle  $\gamma_0 \gg 40^\circ$ , the chip reduction-co-efficient curves run horizontally remaining almost constant for all values of  $a_1$ .

At lower values of rake angles, there is a sharply discernible maxm after which  $\zeta$  drops with increase of  $a_1$ .



The critical values of  $a_1$  beyond which  $Z$  starts dropping shifts to the right with the increase of  $\gamma_0$ .

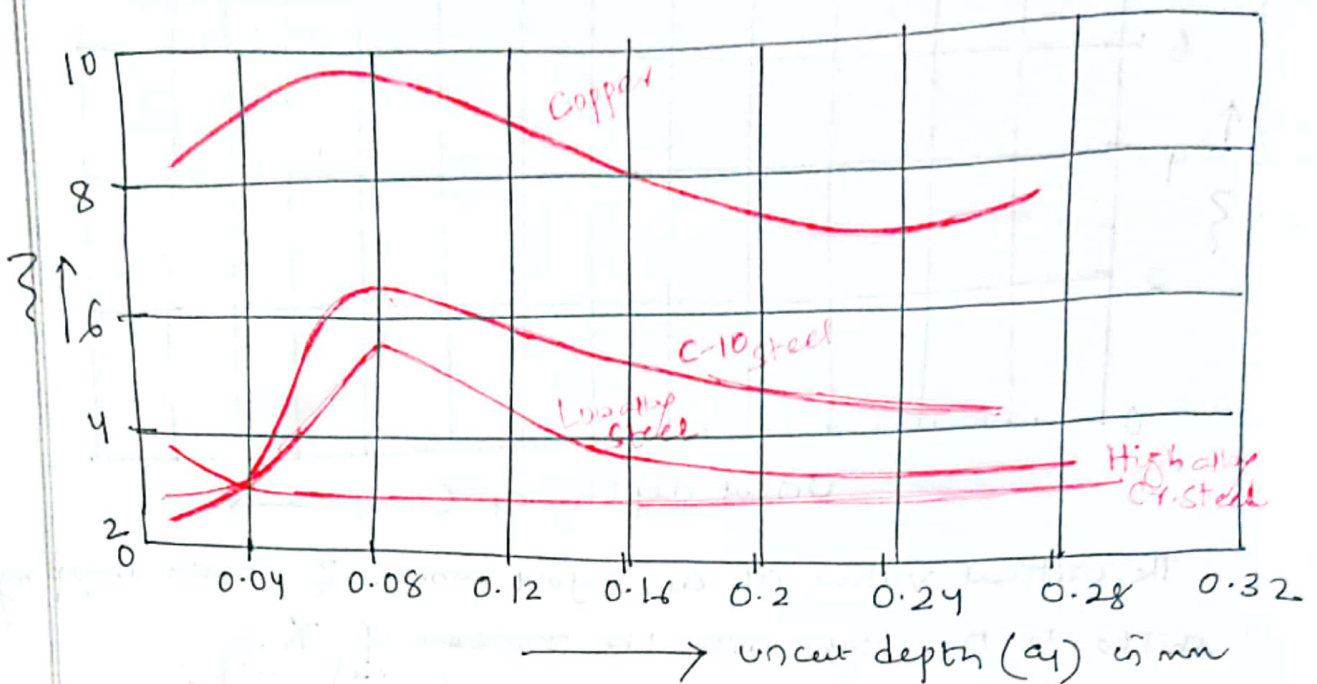
(ii) Effect of cutting fluids  $\Rightarrow$



With air or spindle oil as cutting fluid, the  $Z$  vs  $a_1$  curves goes through a change over from rising to dropping nature. The spindle oil reduces the BUE formation but doesn't totally eliminate. But with  $\text{CCl}_4$ , the formation of BUE is substantially eliminated and the characteristics doesn't reveal any hump.

Therefore, chemically active fluids have greatest influence on the cutting process.

(iv) Effect of cutting materials  $\Rightarrow$



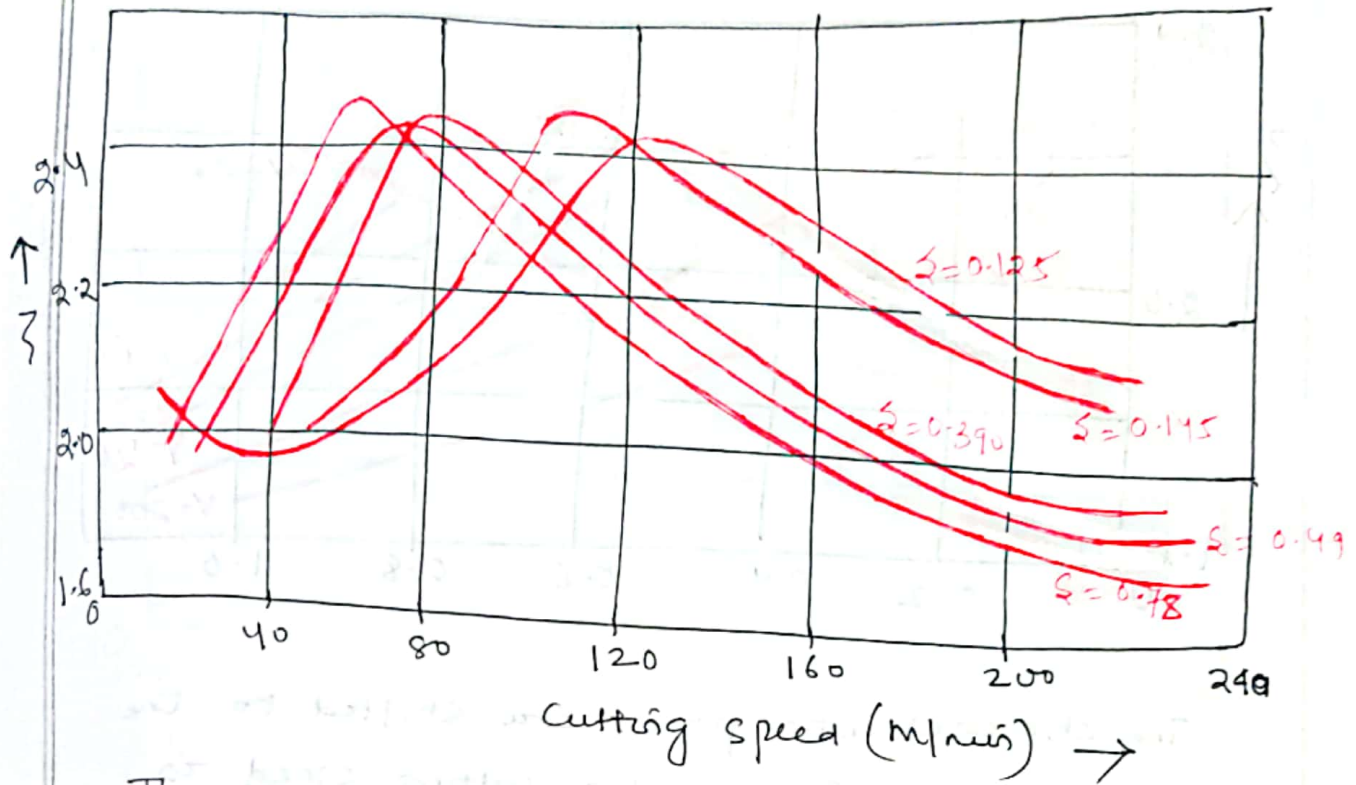
As the depth of uncut layer is reduced,  $\gamma$  increases to a greater or lesser degree for all materials.

The influence of materials on  $\gamma$  is related to the change in the stressed and deformed state in the chip formation zone.

The change in  $\gamma$  indicates a change in the direction of the principal axes of deformation in the shear zone and can be judged from the change of shear angle.



## (V) Effect of cutting speed $\rightarrow$ (High cutting speed)

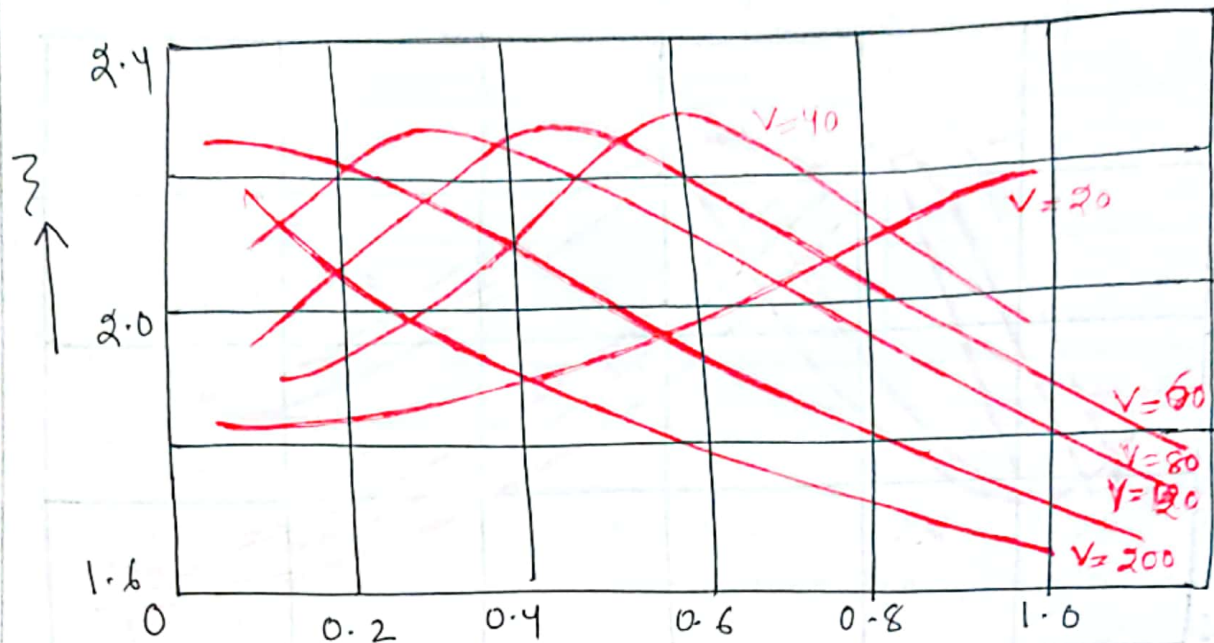


The cutting speed has no direct influence on the chip formation process in the low speed range. The indirect influence of the cutting speed on the chip formation process takes place through a change in mean value of frictional co-efficient and Built up edge formation.

It is seen that the maxima and minima of the curves are displaced towards the region of lower cutting speed, when the feed is increased.



## (v) Effect of feed $\Rightarrow$

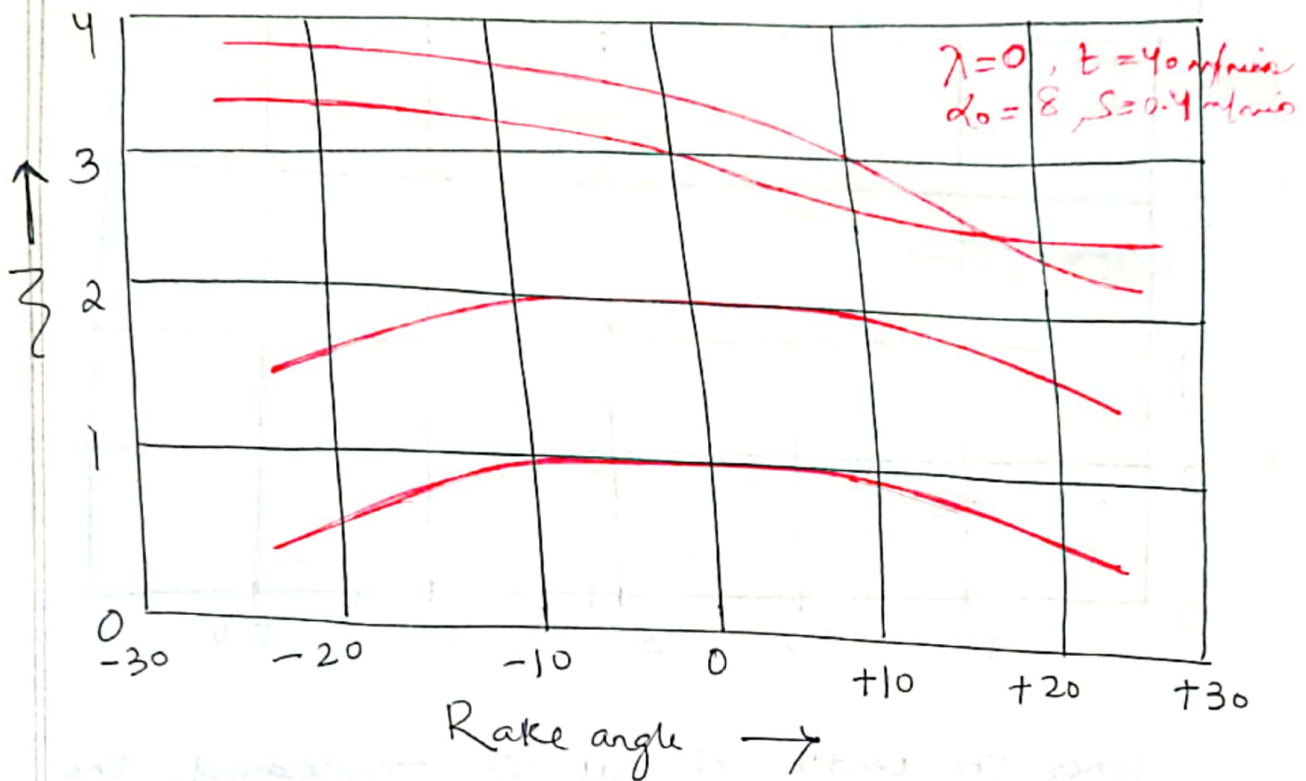


The characteristics points are shifted to the region of smaller feed, when cutting speed is increased.

An increase in feed results in a reduction of chip reduction coefficient ( $Z$ ) which increases to a particular value and finally decreases again.

The influence of feed on  $Z$  may be attributed to the variation of specific friction force along the rake surface to the variation of interface surface.

### (vii) Effect of rake angle ( $\gamma_0$ ) $\rightarrow$



The influence of rake angle on  $Z$  may be considered from 2 aspects :

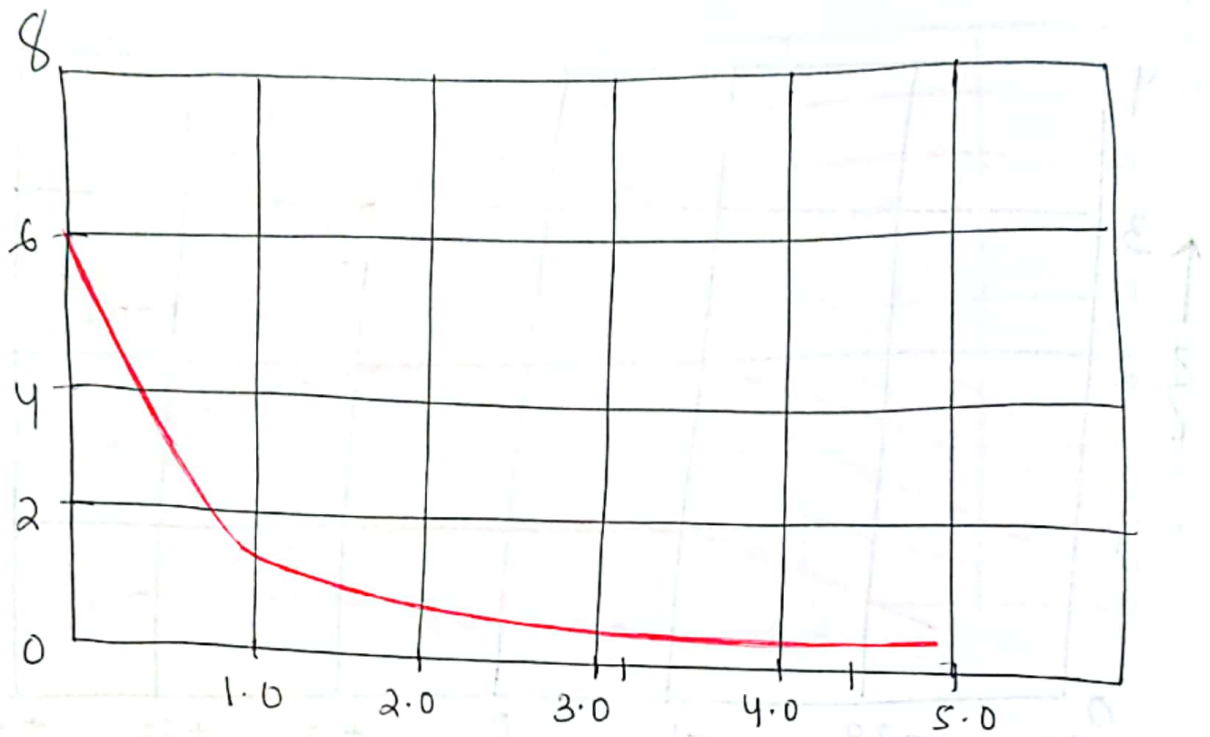
a direct influence due to change in the magnitude and direction of resultant cutting force and

an indirect influence due to the variation of the interface temperature on the specific friction force.

At low cutting speed decrease in  $Z$  is large when rake angle is increased.

But at high speeds decrease in  $Z$  may become negligible and often opposite in nature.

### (viii) Effect of width of cut $\Rightarrow$



When the width of cut is increased, the chip reduction coefficient } is sharply decreased at first and on reaching a particular value remains constant.

If the width of cut is nearly equal to uncut chip thickness (feed), the influence of width of cut comes significant.

- (B) The factors that may affect the value of chip reduction coefficient at high cutting speed includes:-
- (i) Increasing in rounding of cutting edge affects an increase in  $\lambda$  accompanied by side flow.
  - (ii) Application of cutting fluids exerts a considerable influence on chip reduction coefficient at low cutting speeds.
  - (iii) The material of cutting tool affect  $\lambda$  due to change in frictional aspects at the chip tool interface.
  - (iv) The micro structure of the work material also has a considerable influence on the value of  $\lambda$ .
- ex + Hardened steel results in decrease in  $\lambda$ .

### Conclusion →

The characteristics of  $\lambda$  can be analysed by the study of forces and temperature at the interfacial surfaces as a fn of cutting variables.

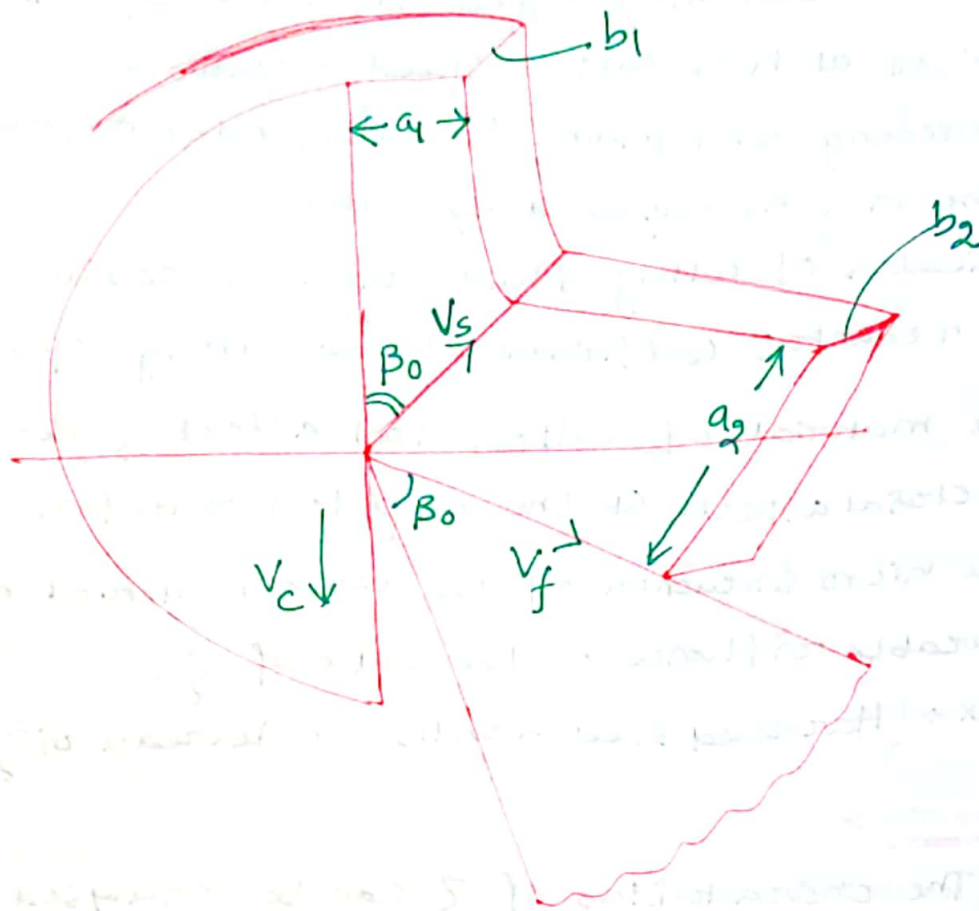
These factors affect  $\lambda$  by influencing the frictional aspects at the interface.

The determination of chip reduction coefficient and its behavior with various variables is very important in the calculation of tool forces and explaining cutting tool performance.

— x —

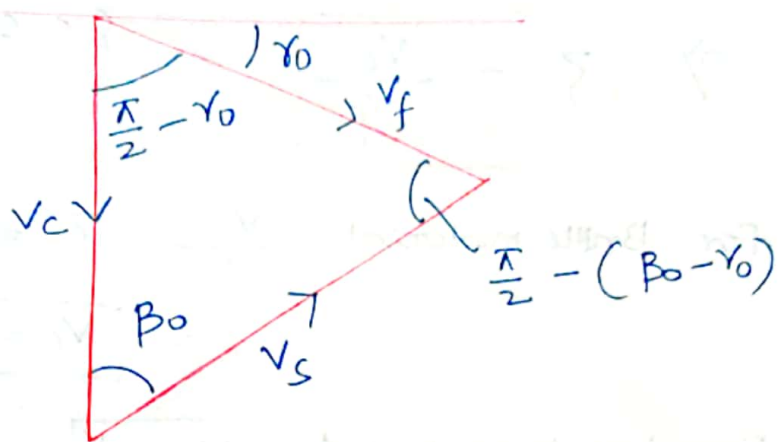


## VELOCITY RELATION IN MACHINING $\Rightarrow$



There are 3 velocities in cutting process.

- (i) Cutting velocity ( $V_c$ ) which is the velocity of the cutting tool relative to the work and directed parallel to the new surface.
- (ii) The chip velocity ( $V_f$ ) which is the speed of the chip relative to the tool and its vector is  $\parallel$  to the rake surface.
- (iii) The shear velocity ( $V_s$ ) which is the speed of the chip relative to the workpiece and directed along the shear plane.



- ie ① cutting velocity ( $V_c$ )  $\rightarrow$  velocity of tool w.r.t w/p.  
 ② chip velocity ( $V_f$ )  $\rightarrow$  velocity of chip w.r.t tool.  
 ③ shear velocity ( $V_s$ )  $\rightarrow$  velocity of chip w.r.t w/p.

Analytically, using property of  $\Delta$  we get =

$$\frac{V_f}{\sin \beta} = \frac{V_c}{\sin \left[ \frac{\pi}{2} - (\beta - \gamma_0) \right]} = \frac{V_s}{\sin \left[ \frac{\pi}{2} - \gamma_0 \right]} \quad \text{--- (A)}$$

$$\text{Hence, } V_s = V_c \cdot \frac{\cos \gamma_0}{\cos (\beta - \gamma_0)} \quad \text{--- (1)}$$

$$\begin{aligned} \& \quad V_f &= V_c \cdot \frac{\sin \beta}{\sin \left[ \frac{\pi}{2} - (\beta - \gamma_0) \right]} \\ &= V_c \cdot \frac{\sin \beta}{\cos (\beta - \gamma_0)} \quad \text{--- (2)} \end{aligned}$$

$$\Rightarrow V_f = \frac{V_c}{\zeta} \quad \text{--- (3)}$$

$$\Rightarrow \boxed{\zeta = \frac{V_c}{V_f}} \quad \zeta = \text{chip reduction coefficient}$$

for Brittle material,  $\frac{V_c}{V_f} = \zeta = 1$

$$\Rightarrow \boxed{V_c = V_f}$$

for ductile material,  $\boxed{V_c > 1}$

$\Rightarrow$  Rate of heat flow is constant.

$$\text{i.e. } Q_1 = Q_2$$

i.e. vol. of cut chip = vol. of uncut chip

$$\Rightarrow \text{area of cut chip} \times V_c = \text{area of uncut chip} \times V_f$$

$$\Rightarrow (a_1 \times b_1) \times V_c = (a_2 \times b_2) \times V_f$$

$\Rightarrow$  Assuming  $b_1 = b_2$ , but in practice it is false.

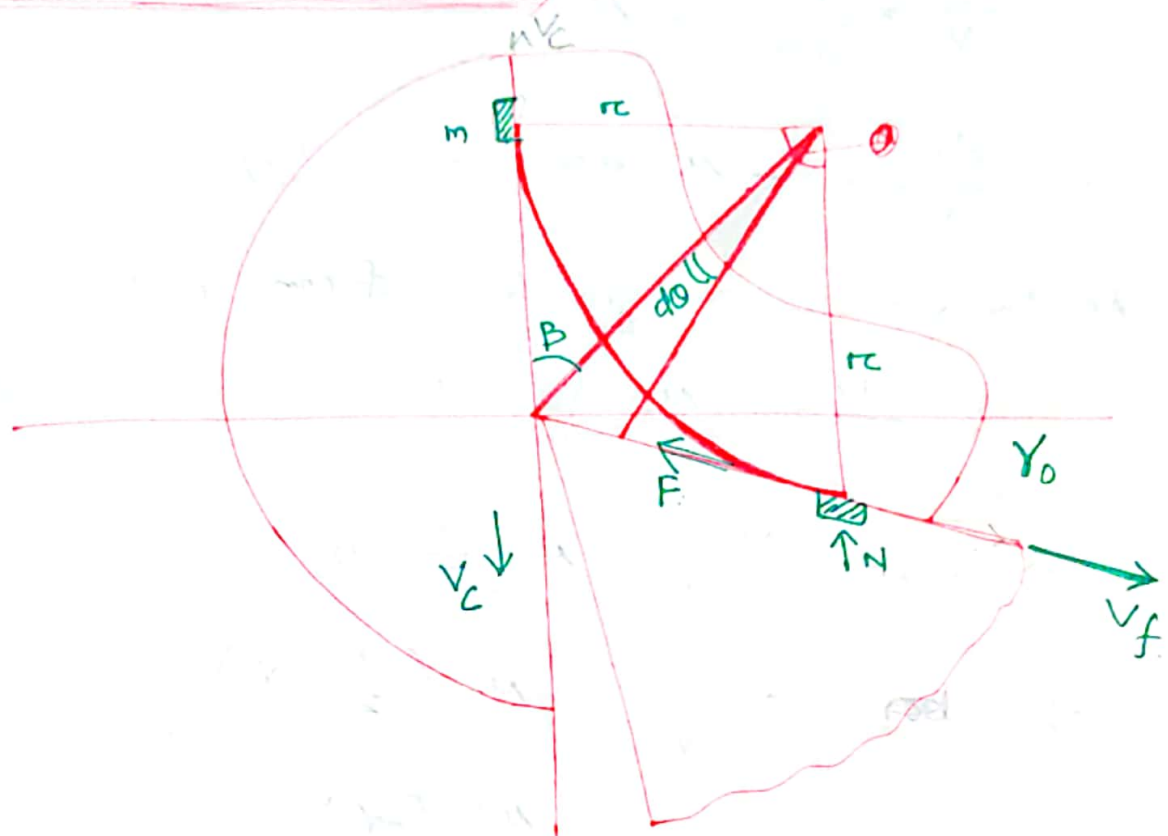
$$\Rightarrow \frac{a_2}{a_1} = \frac{V_c}{V_f} = \zeta$$

$$\Rightarrow \frac{V_s}{V_c} = \frac{\cos \gamma_0}{\cos (\beta - \gamma_0)}$$

$$\Rightarrow \boxed{\beta = f(\zeta, \gamma_0)}$$

The cutting velocity  $V_c$  and rake angle  $\gamma_0$  are known in questions. From the measurement of chip thickness, the shear angle  $\beta$  can be determined which permits numerical evaluation of the velocities  $V_f$  and  $V_s$  as a function of  $V_c$ ,  $\beta$  and rake angle ( $\gamma_0$ ).

# KRONENBERG'S FORMULA FOR $\xi$ AS A FUNCTION OF $V_0$ AND CO-EFFICIENT OF FRICTION $\mu$



Kronenberg's derived an interesting relation for  $\left(\frac{V_c}{V_f}\right) = \xi$  which is of considerable physical significance.

Considering the motion of any chip particle to which principles of momentum change are applied.

Consider a small mass 'm'.

$$m \frac{dv}{dt} = -F = -\mu N \quad \text{--- (1)}$$

$$\text{Again } m \omega^2 r = N$$

$$\Rightarrow m v \omega = N$$

$$\Rightarrow m v \cdot \frac{d\theta}{dt} = N \quad \text{--- (2)}$$



Dividing eqn ① by eqn ② we get

$$\frac{1}{V} \times \frac{dV}{d\theta} = -\mu$$

$$\Rightarrow \frac{dV}{V} = -\mu \cdot d\theta \quad \text{--- (3)}$$

As the velocity changes from  $V_c$  to  $V_f$ , hence

$$\int_{V_c}^{V_f} \frac{dV}{V} = -\mu \cdot \int_0^{\pi/2 - \gamma_0} d\theta$$

$$\Rightarrow -\ln\left(\frac{V_f}{V_c}\right) = \mu\left(\frac{\pi}{2} - \gamma_0\right)$$

$$\Rightarrow \ln \frac{V_c}{V_f} = \mu\left(\frac{\pi}{2} - \gamma_0\right)$$

$$\Rightarrow \frac{V_c}{V_f} = e^{\mu\left(\frac{\pi}{2} - \gamma_0\right)}$$

$$\Rightarrow \zeta = e^{\mu\left(\frac{\pi}{2} - \gamma_0\right)}$$

$$\therefore \zeta = f(\gamma_0)$$

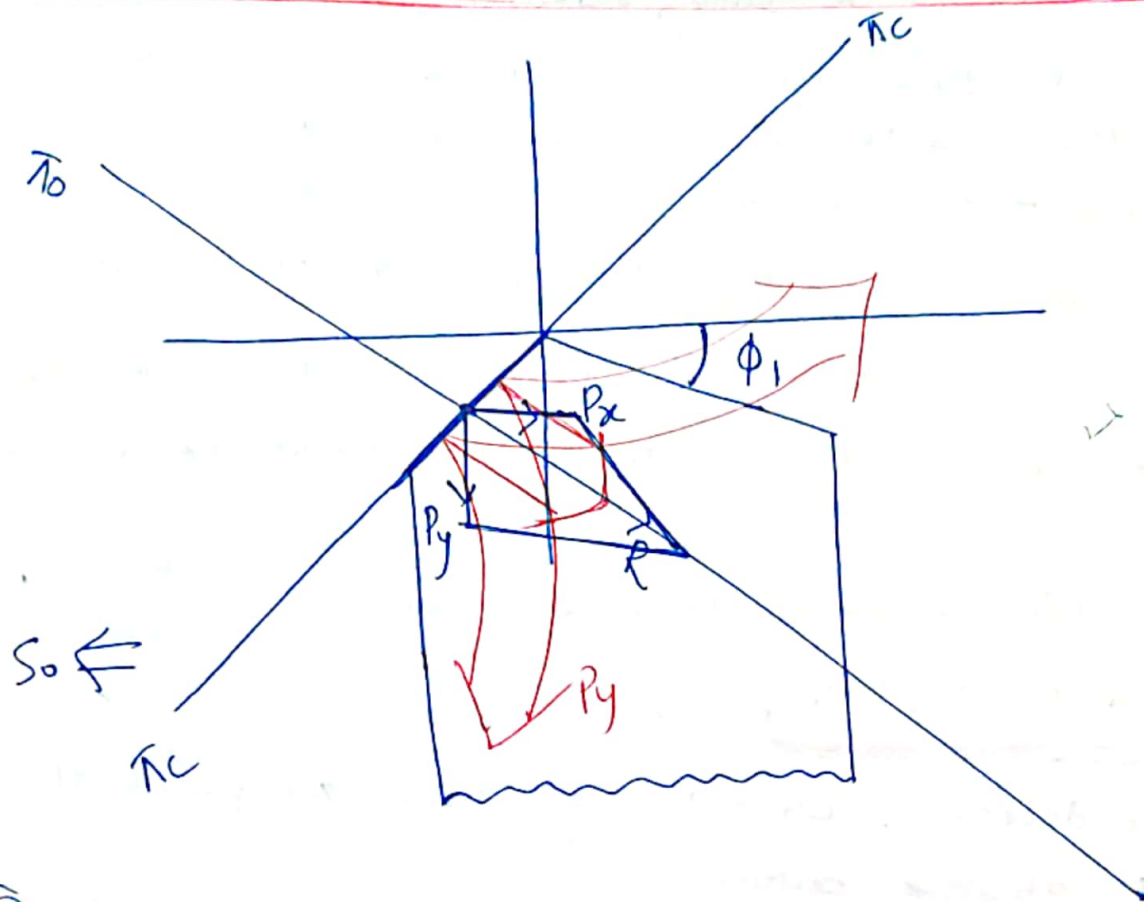
$$\zeta = f(\gamma_0, \mu)$$

This equation states that the chip reduction coefficient and chip flow velocity depends on the frictional aspects ~~that~~ at the interface as well as on the orthogonal rake angle.

If  $\gamma_0$  is increased, chip reduction coefficient decreases.

—x—

# ORTHOGONAL AND OBLIQUE CUTTING



- ① What do you mean by orthogonal and oblique cutting.
- ② What is the advantage & disadvantage?

## Orthogonal cutting

If the chip flow takes place along the  $T_0$  plane, it is called as orthogonal cutting.

### Main features:

- (i) The cutting edge of the tool remains normal to the direction of tool feed or work feed.
- (ii) The direction of chip flow velocity is normal to the cutting edge of the tool.

(iii) The angle of inclination of the cutting edge of the tool with normal to the cutting velocity  $V_c$  is zero.

(iv) The chip flow angle i.e. angle bet<sup>n</sup> the direct<sup>n</sup> of chip flow and the normal to the cutting edge of the tool, measured in the plane of tool face is zero.

(v) The cutting edge is longer than the width of the cut.

(\*) The last cond<sup>n</sup> may not be fulfilled in some cases.

It is then called Semiorthogonal or Restricted orthogonal cutting.

(iv) Tool life is less due to small area.

## OBLIQUE CUTTING

If the direct<sup>n</sup> of chip deviates from  $\pi_0$  plane it's called oblique cutting.

main features:

(i) The cutting edge of tool always remains inclined at an acute angle to the direct<sup>n</sup> of tool feed or work feed.

(ii) The direct<sup>n</sup> of the chip flow velocity is at an angle  $\beta$ , with the normal to the cutting edge of the tool. The angle is known as chip flow angle.

(iii) The cutting edge of the tool is inclined at an angle with the normal to the direct<sup>n</sup> of work feed or tool feed i.e. the velocity  $V_c$ .

(iv) Three mutually  $\perp$  components of cutting forces act at the cutting edge of the tool.



(v) The cutting edge may or may not be longer than the width of cut.

(vi) It acts on larger area & thus tool life is more.

✓ If the resultant forces on the chip deviates from the plane then the chip deviates and thus oblique cutting takes place.

Reason for deviation:

- (i) Restricted cutting effect (iii) Tool nose rounding
- (ii) Due to  $\lambda$ . (inclination angle  $\lambda \neq 0$ )

If the cutting angle is less than  $15^\circ$ , rubbing occurs betw the w/p & tool and thus thrust is caused which deviates the chip.

$$\tan \psi = \frac{\sin(\phi + \phi_1)}{\frac{2t}{S - \sin \phi} + \cos(\phi + \phi_1)}$$

Stabblers rule :- (oblique due to inclination angle  $\lambda$ )

$$\phi_1 = \pm \lambda \Rightarrow \tau = \psi \pm \lambda$$

↪ Deviation of chip from the plane

When  $\lambda$  is +ve, the deviation is away from the machining surface & vice versa.

### Effective rake angle

Effective rake angle is the angle of inclination of rake surface from the reference plane measured on the plane which is  $\perp$  to  $\tau_r$  & taken in the direction of <sup>actual</sup> chip flow. This plane may be called as effective plane.



In orthogonal cutting

$$\gamma_{\text{normal}} (\gamma_n) = \gamma_o = \gamma_e \text{ (Effective rake)}$$

In oblique cutting

$$\gamma_e \neq \gamma_n$$

$$\sin \gamma_e = \cos \lambda \cdot \cos \phi_c \cdot \sin \gamma_n + \sin \lambda \cdot \sin \phi_c$$

When  $\phi_c = \lambda$  (not in a restricted cutting)

$$\sin \gamma_e = \cos^2 \lambda \cdot \sin \gamma_n + \sin^2 \lambda$$

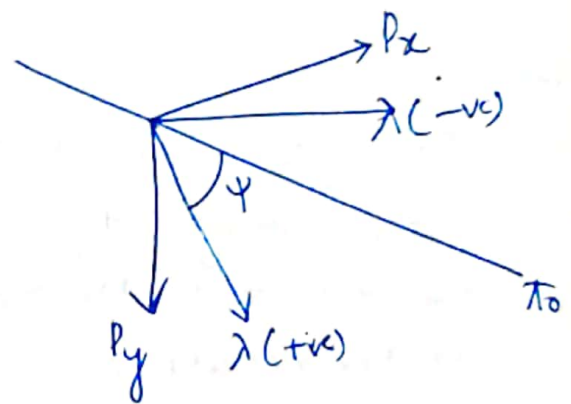
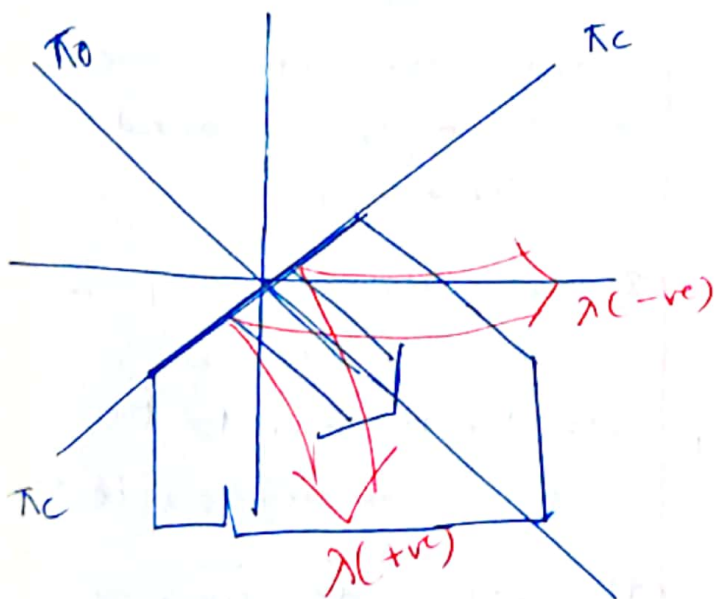
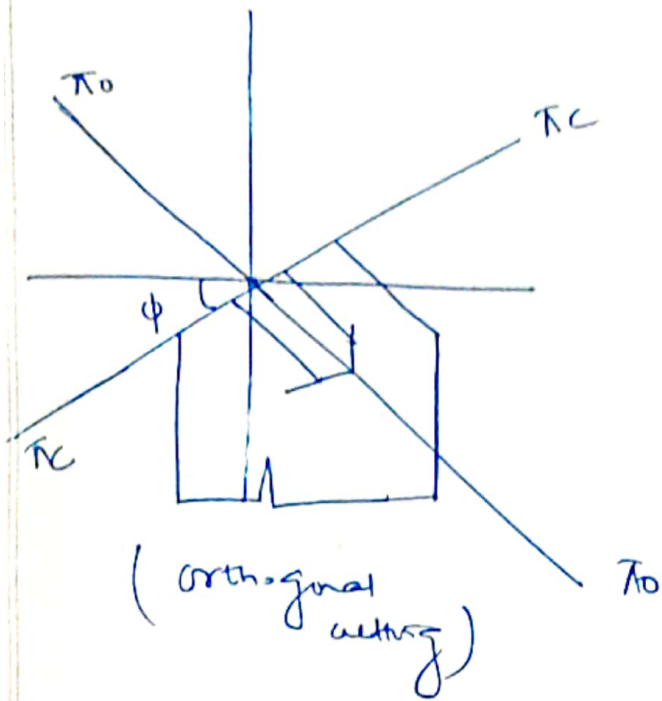
↪ effective rake angle

## ORTHOGONAL CUTTING & OBLIQUE CUTTING

Orthogonal cutting takes place when the direction of chip velocity is along to plane. In orthogonal cutting, the cutting face of the tool is at  $90^\circ$  to the line of action of tool.

Oblique cutting takes place when direction of chip velocity is deviated from to plane. In oblique cutting, the cutting face of the tool is less than  $90^\circ$  to the line of action of tool.

This deviation is due to the thrust force  $P_y$  acting in the direction  $\perp$  to the produced surface. The thrust force is acting both w/p machined surface and auxiliary cutting force.



This thrust force exist if  $\phi$  is effected. Also due to circulation chip gets deviated. If chip is deviated from  $\pi_0$  plane towards  $-ve$  side, then  $-ve$  rake angle pushes it towards  $\pi_0$  plane, thus maintaining the chip to flow along  $\pi_0$  plane.

So most advantageous is the -ve inclination angle ( $\lambda$  -ve). Because + $\lambda$  value of  $\lambda$  deviated the chip from the plane and thus increases the  $P_y$  force decrease  $P_x$  force.

### Orthogonal cutting

### oblique cutting

- |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>(1) cutting edge of the tool remains normal to the direction of tool feed or work feed.</p> <p>(2) Direction of chip flow velocity is normal to the cutting edge of tool.</p> <p>(3) The angle of inclination <math>\lambda</math> of the cutting edge of the tool with the normal to the velocity <math>V_c</math> is zero.</p> <p>(4) cutting edge is longer than the width of cut.</p> <p>(5) Contact area is less for same depth of cut.</p> <p>(6) Stresses produced is more.</p> | <p>(1) Remains inclined at an acute angle to the direction of tool feed or work feed.</p> <p>(2) The direction of chip flow velocity is at <math>\beta</math> with the normal to the cutting edge of the tool. The angle is called chip flow angle.</p> <p>(3) The cutting edge of the tool is inclined at an angle <math>\lambda</math> with the normal to the direction of work feed i.e. <math>V_c</math>.</p> <p>(4) cutting edge may or may not longer than the width of cut.</p> <p>(5) Contact area is more for same depth of cut.</p> <p>(6) Stresses produced is less.</p> |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

(7) Heat dissipation is less.

(8) cutting force & power req<sup>d</sup> is more.

(9) Turning, parting, grooving operations can be done but facing operation cannot be done.

(10) Tool life is less.

(7) Heat dissipation is more

(8) cutting force & power req<sup>d</sup> is less.

(9) Turning, facing, chamfering can be done but grooving & parting cannot be done.

(10) Tool life is more.



# Mechanics of Metal Cutting

## Forces of deformation at the cutting edge

The deformation of cutting forces necessary for deforming the work material at the shear zone is essential for several important requirements.

- (a) To estimate the power requirement of a m/c tool.
- (b) To estimate the streaming action that must be resisted by m/c tool components, bearings, rigs and fixtures.
- (c) To survey the cutting characteristics of work and cutting tool materials by investigating cutting tool forces.

The removal of excess materials in the form of chips by which finished surface is produced involves tool geometry, internal & frictional processes at the interfaces and several other physical & chemical factors. However, the analysis is made by means of experimental observations or by using simplified models of the actual cutting process.

The magnitude and direction of the resultant cutting force " $R$ " acting at the wedge is unknown, it is necessary to establish a convenient reference co-ordinate system so as to determine the magnitude and direction of  $R$  with respect to the chosen co-ordinate. Three different techniques may be applied

to evaluate 'R' in magnitude and direction.

- (i) To find R in a two dimensional force system directly from a chosen axis, such as the "Black or white brush" of photo elastic fringe pattern.
- (ii) To find R in a 2-dimensional force system with respect to two chosen reference coordinates.
- (iii) To find R in a three 3-dimensional force system with respect to three-chosen reference co-ordinates.

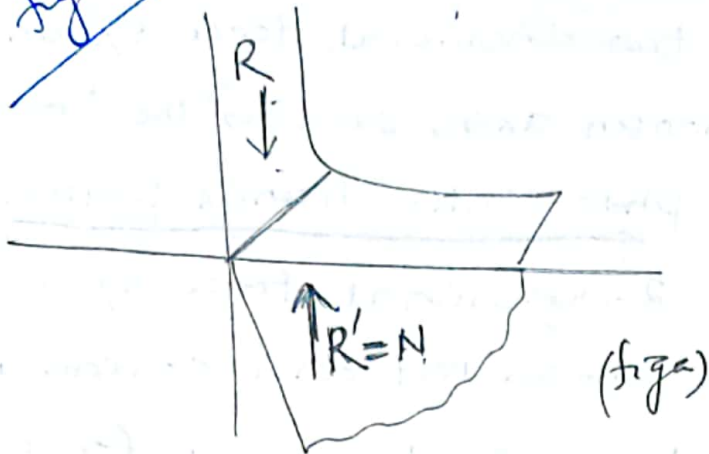
In some special cases, a three dimensional system can be reduced to a two dimensional system for analysis. This is done by adopting a form of machining in which orthogonal cutting, resulting in plain strain, is employed.

Consider the basic case of a wedge shown in figure below, where the work material is being deformed between the force system  $R - R'$ .

For the convenience of analysis, the schemes are built one after another as shown.

In fig (a),  $\gamma_0 = 0$  and the deformation takes place in absence of friction (i.e.  $\mu = 0$ ) at the interface.

fig a

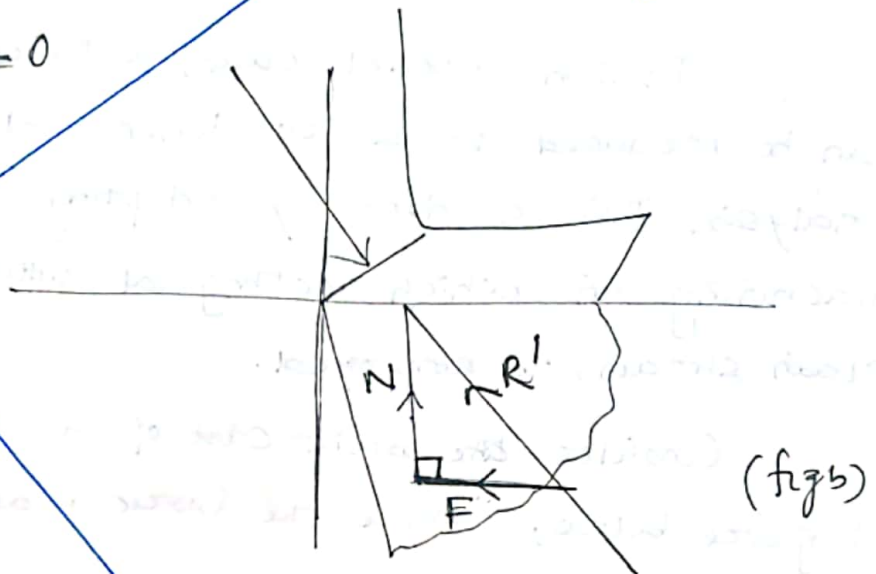


$$\lambda \neq 0$$

$$\gamma_0 = 0$$

$$\mu = 0$$

fig b

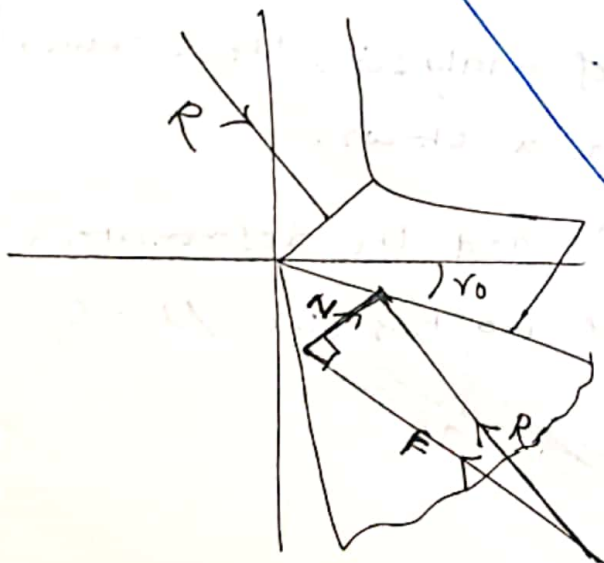


$$\lambda \neq 0$$

$$\gamma_0 = 0$$

$$\mu \neq 0$$

fig c



$$\gamma_0 \neq 0$$

$$\lambda \neq 0$$

$$\mu \neq 0$$

(fig c)

On  
In fig(b), due to frictional effects the force req<sup>d</sup> to deform and to overcome friction increases.

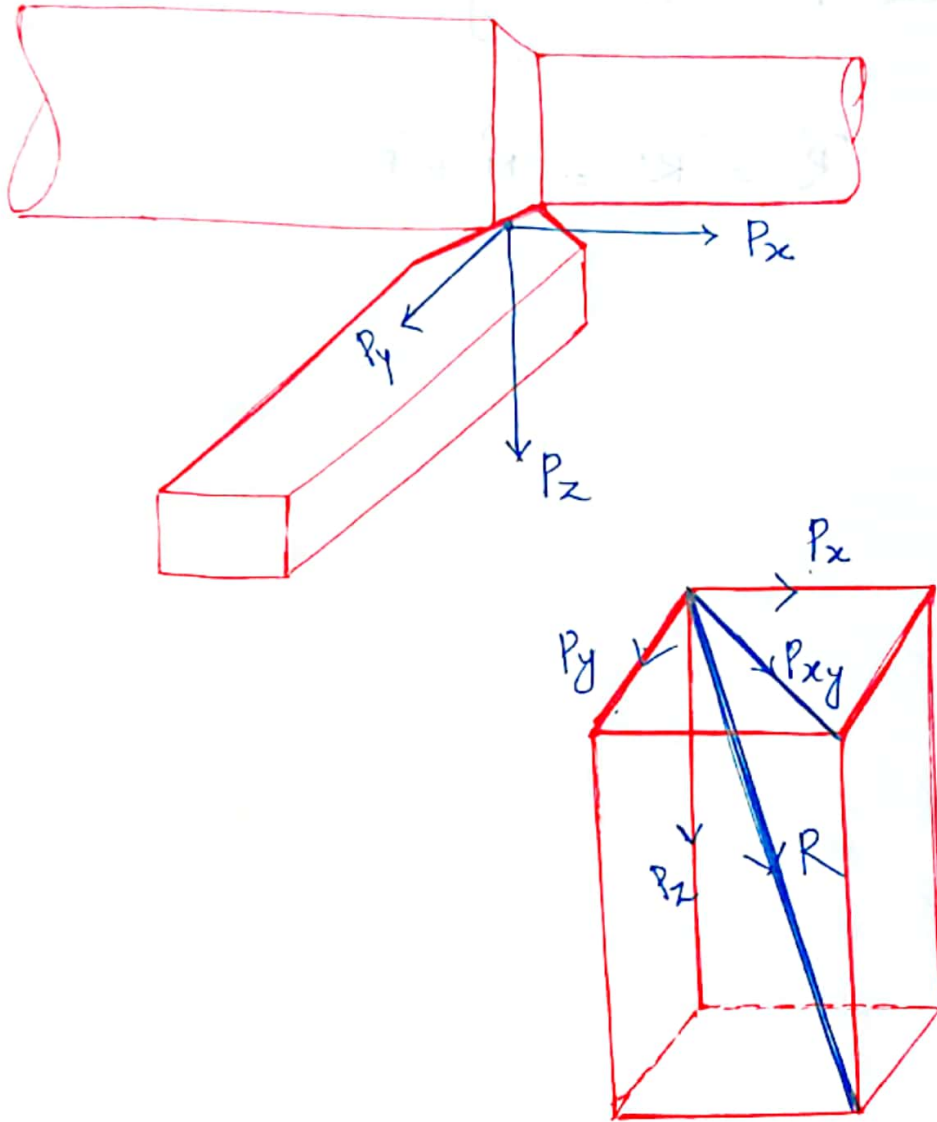
Assuming that same force  $N$  is req<sup>d</sup> to deform, the force necessary to ~~at~~ be applied is now

$$\vec{R} = \vec{R}' = \vec{N} + \vec{F}$$





## FORCES DURING TURNING



(Cutting force in conventional turning process)

The force system in general case of conventional turning process is shown in figure.

The resultant cutting force  $R$  is expressible by its components.

$P_x \Rightarrow$  It is known as the **feed force** in the direction of the tool travel.

$P_y \Rightarrow$  It is called as **thrust force** in the direction perpendicular ( $\perp$ ) to the produced surface.

$P_z \Rightarrow$  It is the **cutting force** or the **main force** acting in the direction of the cutting velocity vector.

These directions have been chosen for their suitability of being determined by properly designed tool force-dynamometers.

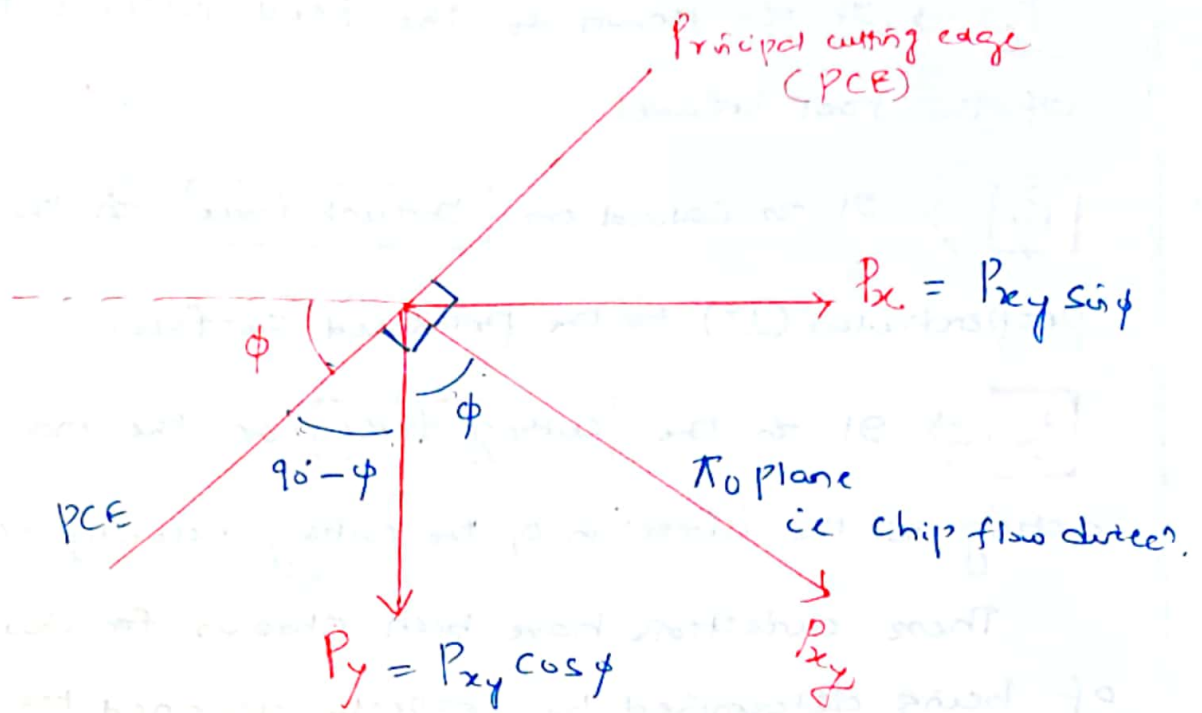
After determining the individual components  $P_x$ ,  $P_y$  &  $P_z$ , the resultant  $R$  can be evaluated.

$$R = \sqrt{P_x^2 + P_y^2 + P_z^2} \quad \text{--- (1)}$$

This 3-dimensional force system can be reduced to a 2-dimensional force system if in the orthogonal plane  $\pi_0$  the forces are considered, in such a way that, when

$$R = \sqrt{P_z^2 + P_{xy}^2} \quad \text{--- (2)}$$

$$\& \quad P_{xy} = \sqrt{P_x^2 + P_y^2} \quad \text{--- (3)}$$



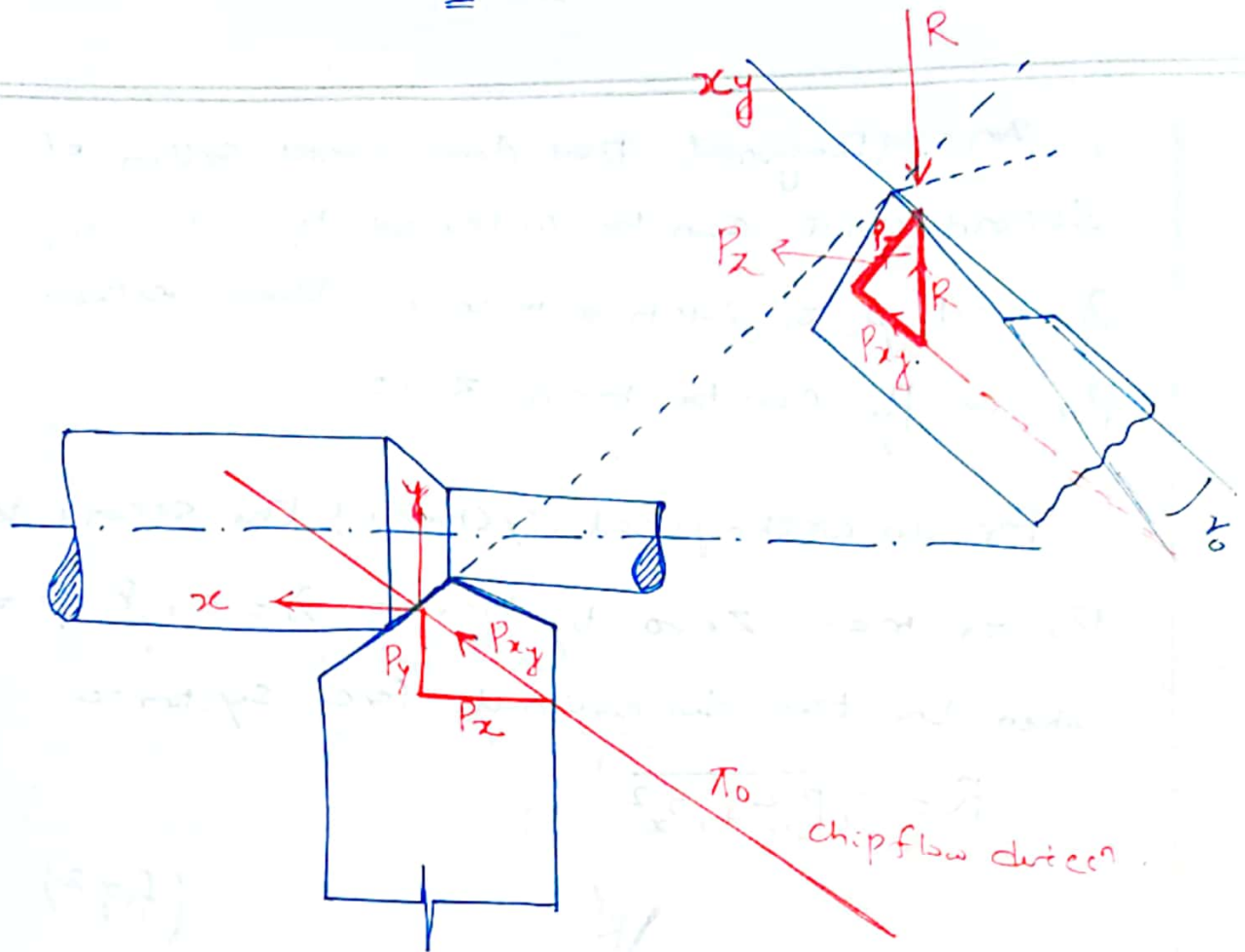
⇒ This is possible when the force  $P_{xy}$  is contained in plane  $\pi_0$ , which is possible under conditions of the Free orthogonal cutting.

This corresponds to the Orthogonal System of the first kind, for which the conditions are  $\Rightarrow$

(i)  $0 < \phi < 90^\circ$

(ii)  $\lambda = 0$ .

(iii) chip flow direction lies in the plane  $\pi_0$ .



( Reduced two dimensional force system to  
for orthogonal system of the first kind with  
 $\lambda = 0$ , &  $0 < \phi < 90^\circ$  )

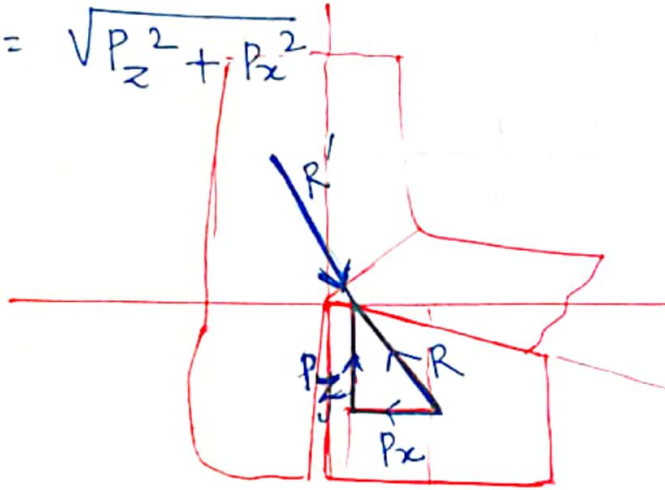
Figure above shows the cutting forces for  
the case of ORTHOGONAL SYSTEM OF FIRST KIND.



➤ An orthogonal two dimensional system of second kind can be obtained by choosing  $\lambda$  and  $\phi$  in such a manner that either  $P_x$  or  $P_y$  can be made zero.

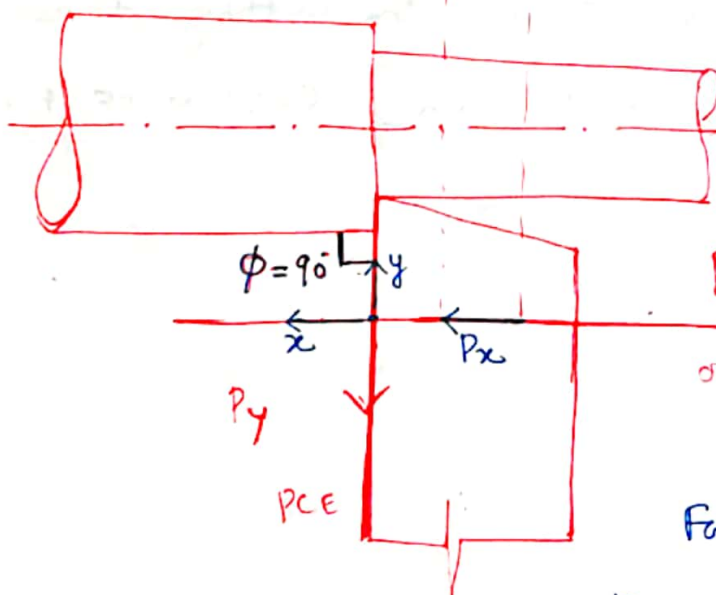
➤ For an orthogonal system of the second kind,  $P_y$  is made zero by having  $\lambda = 0$ , &  $\phi = 90^\circ$ , when the two dimensional force system is

$$R = \sqrt{P_z^2 + P_x^2}$$



(fig 2)

$$\begin{aligned} \lambda &= 0, \phi = 90^\circ \\ P_x &= P_{xy} \\ P_y &= 0 \\ R &= \sqrt{P_z^2 + P_x^2} \\ \text{chipflow in } \pi\text{-plane} \\ \text{ie } P_{xy} &= P_x \end{aligned}$$



$$P_x = P_{xy}$$

$\pi$  plane  
over which chipflow

For  $\phi = 90^\circ$

$$P_x = P_{xy} \sin \phi$$

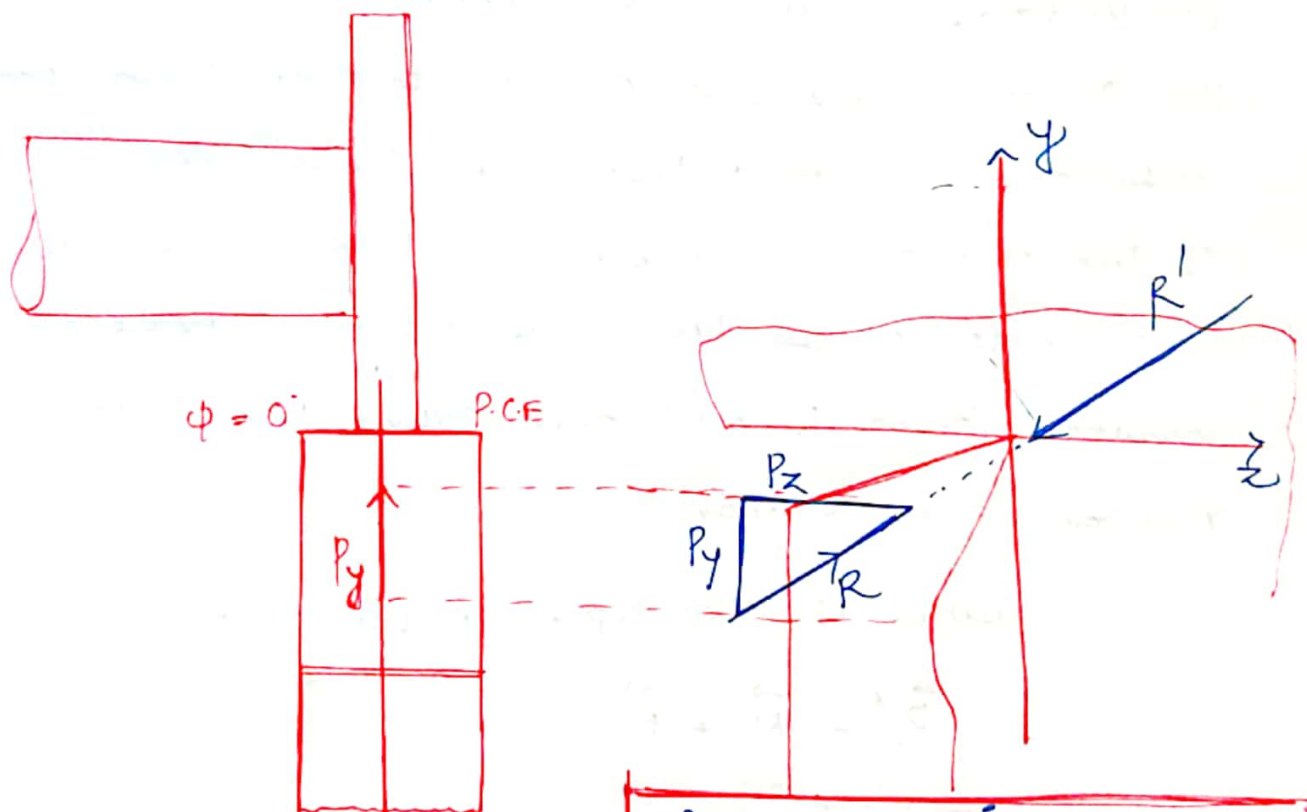
$$\Rightarrow P_x = P_{xy}$$

$$\begin{aligned} \& P_y = P_{xy} \cos \phi \\ \Rightarrow P_y &= 0 \end{aligned}$$

Figure (2) shows the orthogonal system of second kind where  $P_y$  is made zero.

It shows the disposition of the cutting force in plane orthogonal turning with  $\lambda = 0$  and  $\phi = 90^\circ$ .

➡ Another alternative way of having an orthogonal system of the second kind is to have  $P_x = 0$  during radial turning or facing operation when  $R = \sqrt{P_z^2 + P_y^2}$



$P_y = P_{xy} \cos \phi$   
 $\Rightarrow P_{xy} = P_y$

No plane over which chip flow

$\lambda = 0, \phi = 0^\circ$   
 $P_x = 0, P_y = P_{xy}$   
 $R = \sqrt{P_z^2 + P_y^2}$   
 chip flow over  $\pi_0$  plane i.e.  $P_{xy} = P_y$

# MERCHANT'S CIRCLE DIAGRAM

For the purpose of analysis, a simplified scheme of chip formation is developed with the following assumptions:

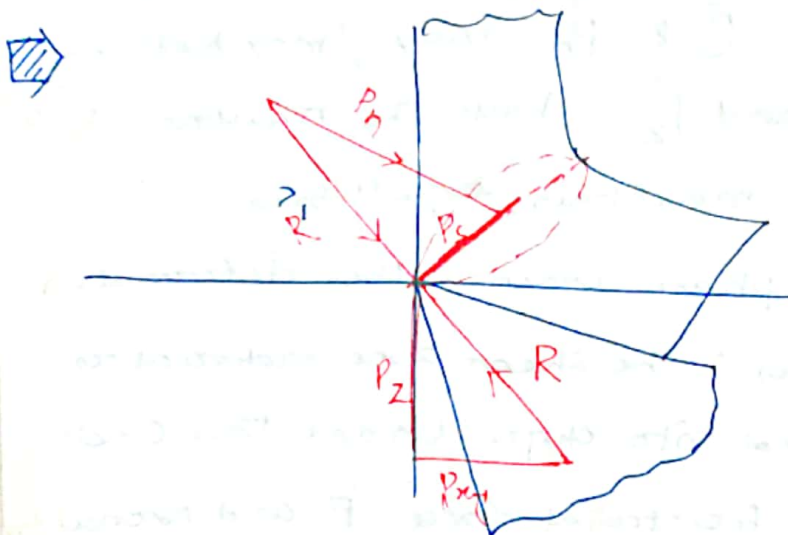
- (1) No contact at the flank i.e. the tool is perfectly sharp.
- (2) No side flow of the chips i.e.  $b_1 = b_2$ .
- (3) It is a orthogonal cutting i.e. chip will flow over to plane.
- (4) A continuous chip is produced with no built up edge.
- (5) Free cutting condition.
- (6) No variation in process parameter.
- (7) uniform <sup>cutting</sup> velocity i.e.  $V_c = \text{Constant}$ .
- (8) It is assumed that chip is in equilibrium state also it is assumed that single shear plane.
- (9) The chip is considered to be held in equilibrium by the action of the two equal & opposite ~~resultant~~ - resultant forces  $R$  &  $R'$  and assume that these resultant are collinear.

Equation of chip segment under  $R-R'$  force system

$$\vec{R}' = \vec{N} + \vec{F}$$

$$\& \quad \vec{R} = \vec{P}_s + \vec{P}_n = \vec{P}_z + \vec{P}_x$$





while considering the metal cutting process, the edge of the tool applies a compressive effort, & there is a critical point where the material is divided into two portions: one is the machined surface of the part & the other is the chip.

Plastic deformation takes place in a certain region encompassed between the undeformed material on one side and the cutting tool on the other side.

The distinctive zone between the chip & chip, where the deformation gradually increases towards the cutting edge is called shear zone / primary deformation zone.

From the above figure,  $R$  is the resultant of  $P_z$  and  $P_{xy}$  force.

So for equilibrium an equal & opposite reaction  $\vec{R}'$  is there, & when we resolve it in shear zone we get two force  $\vec{P}_s$  &  $\vec{P}_n$



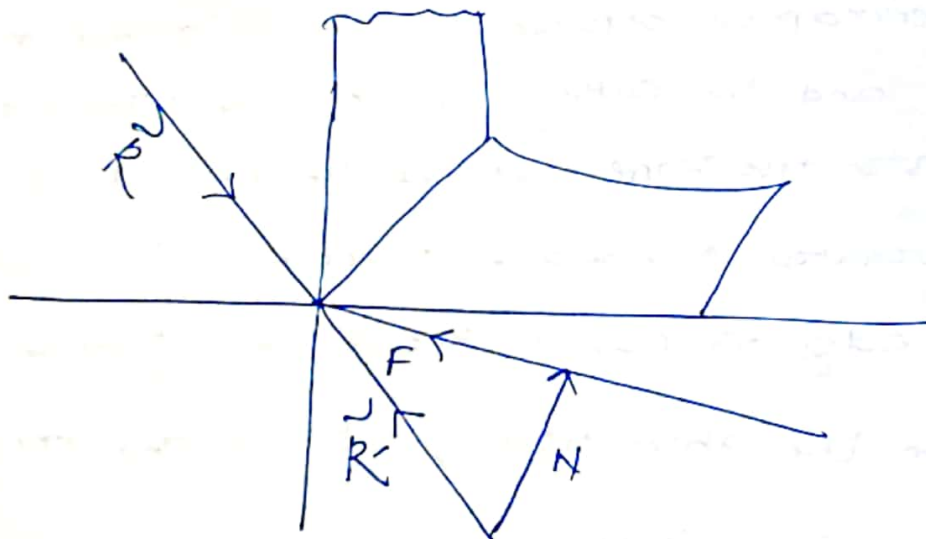
The magnitude of  $\vec{P}_s$  &  $\vec{P}_n$  may / may not be equal to  $\vec{P}_{xy}$  and  $\vec{P}_z$  but the resultant  $\vec{R}$  &  $\vec{R}'$  must be equal to maintain equilibrium.

On the second phase when the deformation gradually increases; the shear zone material is gradually converted into chips. Under this condition again there is frictional force  $\vec{F}$  and normal force  $\vec{N}$  exist whose resultant  $\vec{R}''$  act at the interface.

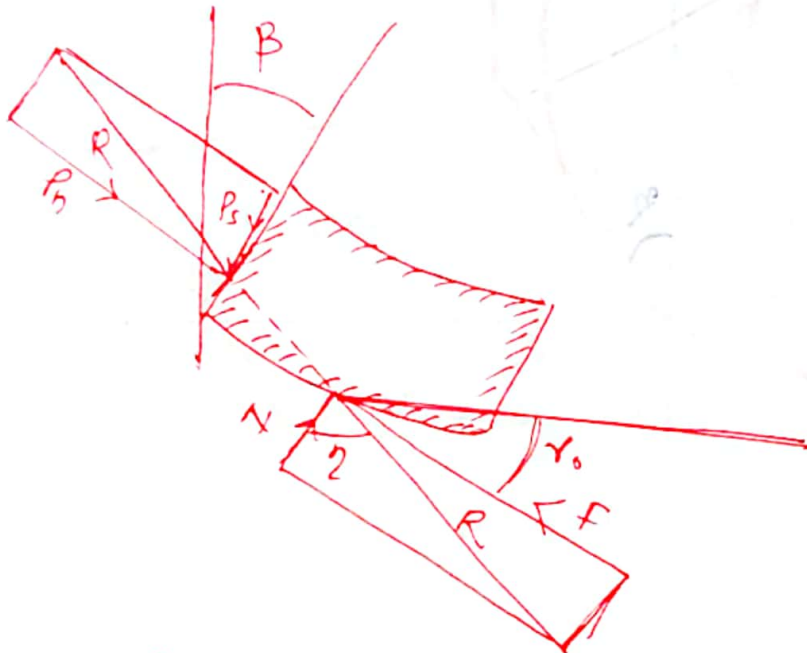
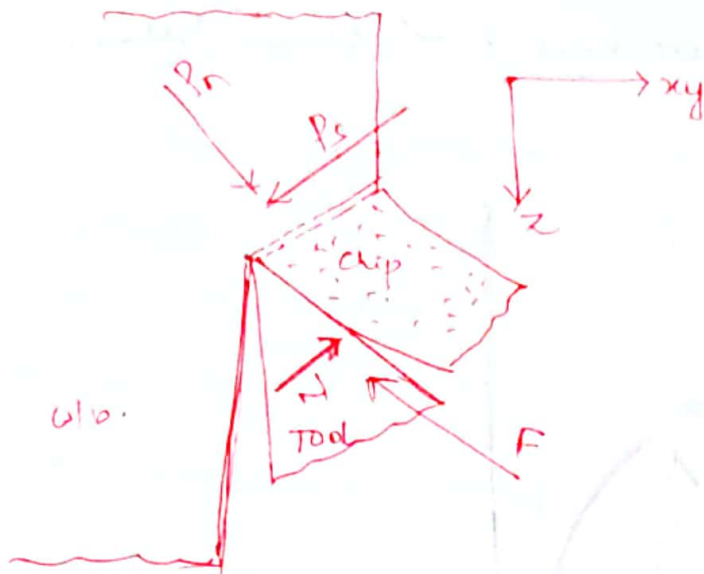
Again for the condition of equilibrium  $\vec{R}''$  must be equal to the  $\vec{R}'$  and  $\vec{R}$ .

$$\therefore \vec{R}' = \vec{N} + \vec{F}$$

$$\& \vec{R} = \vec{P}_s + \vec{P}_n = \vec{P}_z + \vec{P}_x$$

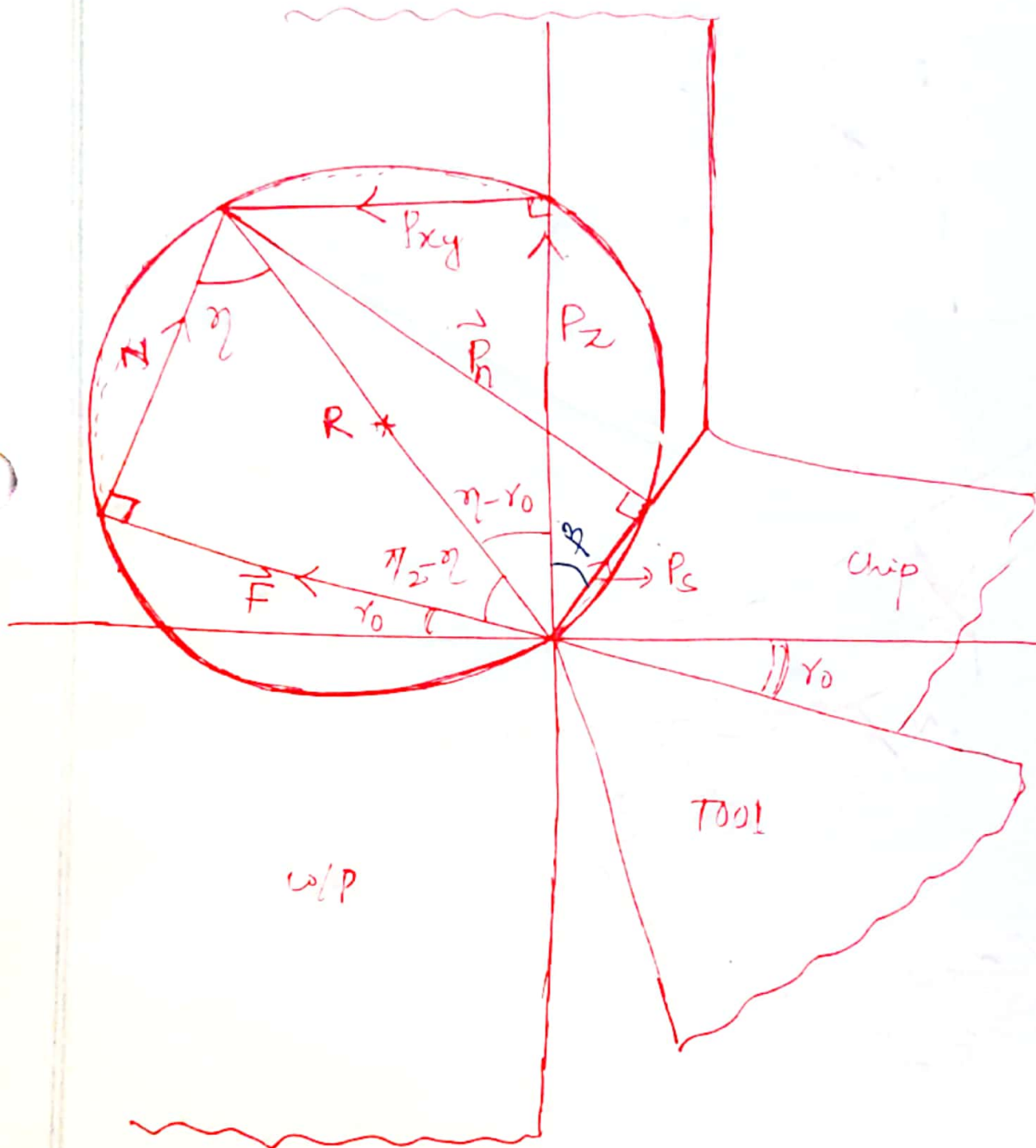


These



(Equilibrium of the chip segment under  $\vec{R} - \vec{R}'$  force system)

These above equations suggest a circle representation of forces done by Merchant & is shown below.



(Merchant circle diagram for cutting forces)

For a given tool shape, with  $\lambda=0$  and  $\gamma_0, \phi$  known, the tangential component of cutting force  $P_x$ , and thrust force  $P_y$  can be experimentally determined along  $P_z$  with  $\beta$ .

From the values of  $\gamma_0, \beta, P_z$ , &  $P_{xy}$ , other component forces  $F, N, P_s$  and  $P_n$  can be easily estimated.

For orthogonal cutting of the second kind,

$$P_{xy} = P_x \quad \text{when } \phi = 90^\circ \text{ \& } \lambda = 0$$

$$\& P_{xy} = P_y \quad \text{when } \phi = 0^\circ \text{ \& } \lambda = 0$$

But for the orthogonal cutting of the first kind i.e. when  $0 < \phi < 90^\circ$  with  $\lambda = 0$ ,

$$P_x = P_{xy} \sin \phi$$

$$P_y = P_{xy} \cos \phi.$$

## FRICTIONAL FORCE SYSTEM AT THE CHIP-TOOL INTERFACE

Assuming that the force system is due to concentrated equivalence of any distribution, a relation between  $P_z, P_{xy}$  and  $\gamma_0$  on one hand and  $F, N, \mu$  on the other hand can be estimated.





$$\mu = \frac{F}{N} = \tan \eta = \frac{P_{xy} \cos \gamma_0 + P_z \sin \gamma_0}{P_z \cos \gamma_0 - P_{xy} \sin \gamma_0}$$

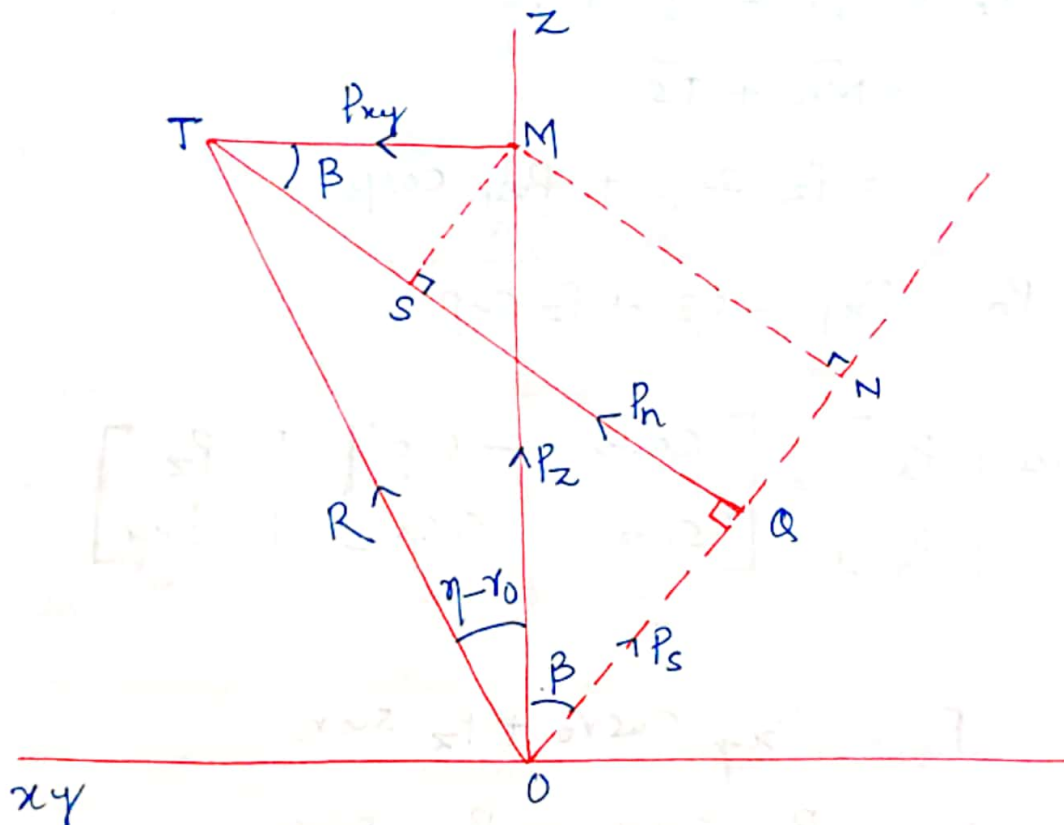
$$\Rightarrow \mu = \tan \eta = \frac{P_{xy} + P_z \tan \gamma_0}{P_z - P_{xy} \tan \gamma_0}$$

When  $\mu$  = Kinetic co-efficient of friction.

$\eta$  = mean angle of friction at the rake surface.

FRICTIONAL FORCE SYSTEM AT THE SHEAR PLANE

$$P_s, P_n = f(P_z, P_{xy})$$



Let  $P_s$  be the force acting tangentially at the - hypothetical single shear plane causing slip and deformation while  $P_n$  is the compressive load on the shear plane.

From the geometry,

$$P_s = \vec{OQ} = \vec{ON} - \vec{QN} = \vec{ON} - \vec{SM}$$

$$\Rightarrow P_s = P_z \cos \beta - P_{xy} \sin \beta$$

and similarly,

$$P_n = \vec{QT} = \vec{QS} + \vec{TS}$$

$$= \vec{MN} + \vec{TS}$$

$$= P_z \sin \beta + P_{xy} \cos \beta$$

$$\therefore P_n = P_{xy} \cos \beta + P_z \sin \beta$$

$$\text{Hence } \begin{bmatrix} P_s \\ P_n \end{bmatrix} = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} P_z \\ P_{xy} \end{bmatrix}$$



$$F = P_{xy} \cos \gamma_0 + P_z \sin \gamma_0$$

$$N = P_z \cos \gamma_0 - P_{xy} \sin \gamma_0$$

$$P_n = P_{xy} \cos \beta + P_z \sin \beta$$

$$P_s = P_z \cos \beta - P_{xy} \sin \beta$$

## STRESS IN CONVENTIONAL SHEAR PLANE (OR THEORETICAL ESTIMATION OF $P_z$ )

We know that

$$P_z = R \cos(\eta - \gamma_0) \quad \text{--- (1)}$$

$$P_s = R \cos(\beta + \eta - \gamma_0) \quad \text{--- (2)}$$

(From Merchant circle diagram)

Dividing eqn (1) by eqn (2)

$$\frac{P_z}{P_s} = \frac{\cos(\eta - \gamma_0)}{\cos(\beta + \eta - \gamma_0)}$$

$$\Rightarrow P_z = P_s \frac{\cos(\eta - \gamma_0)}{\cos(\beta + \eta - \gamma_0)}$$

But  $P_s = \tau_s A_s$

$P_s$  = Shear force

where  $\tau_s$  = Dynamic shear stress  $\text{kg/cm}^2$  on the conventional shear plane of area  $A_s \text{ mm}^2$

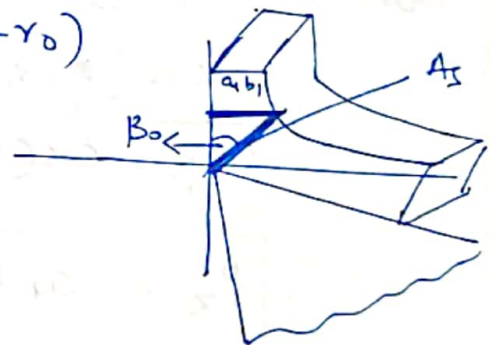
$$\text{Now } A_s = \frac{a_1 b_1}{\sin \beta} = \frac{S_0 t}{\sin \beta}$$

where  $S_0$  = feed

$t$  = Thickness of chip.

$A_s$  = area of shear plane

$a_1$  = chip thickness before cut.  $b_1$  = width of cut.





$$\therefore P_s = \frac{a_1 b_1}{\sin \beta} \times Z_s$$

$$= \frac{S_o t}{\sin \beta} \times Z_s.$$

$$\therefore P_z = \frac{a_1 b_1 Z_s \cos(\eta - \gamma_o)}{\sin \beta \cdot \cos(\beta + \eta - \gamma_o)} \quad \text{--- (A)}$$

$$\text{or } P_z = \frac{S_o t Z_s \cos(\eta - \gamma_o)}{\sin \beta \cdot \cos(\beta + \eta - \gamma_o)}$$

[only valid for orthogonal cutting]

But in eqn (A)

$$\frac{\cos(\eta - \gamma_o)}{\sin \beta \cdot \cos(\beta + \eta - \gamma_o)} = \text{form factor} = f$$

$$\therefore P_z = a_1 b_1 Z_s f = S_o t Z_s f$$

This form factor is always greater than one.

Similarly

$$P_{xy} = Z_s a_1 b_1 \frac{\sin(\eta - \gamma_o)}{\sin \beta \cdot \cos(\beta + \eta - \gamma_o)}$$

## ENERGY OF CUTTING PROCESS

(a) Cutting power

$$U_c = P_z V_c$$

(b) Feed power

$$U_f = P_x \cdot V_f$$

where  $V_f = \frac{S_o N}{1000} \text{ m/min}$

Total power

$$U_T = U_c + U_f$$

$$\text{Motor power} = \frac{U_T \cdot f_o \cdot f_s}{\eta_m \cdot \eta_e}$$

where  $f_o = \text{Overload}$   
(range from 1.25-1.2)

$f_s = \text{Shock load}$  (1.2-1.4)

$\eta_m = \text{mechanical efficiency}$  (0.85-0.9%)

$\eta_e = \text{Electrical efficiency}$  (0.9-0.95)

## SPECIFIC ENERGY

Specific energy may be defined as the energy consumed / utilised for removal of unit volume of material.

$$\frac{U_c}{MRR} = \frac{P_z \cdot V_c}{V_c \cdot t \cdot S_0} = \frac{P_z}{t S_0}$$

$$\Rightarrow \frac{U_c}{MRR} = \frac{P_z}{t S_0}$$

$$\Rightarrow \boxed{\frac{U_c}{MRR} = \frac{P_z}{A_0}}$$

~~Ans~~

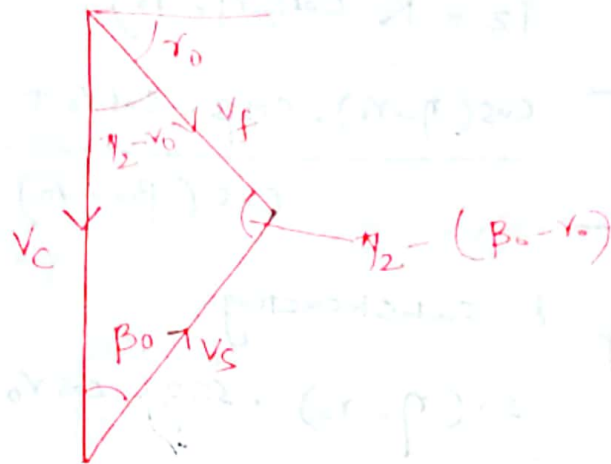
PROVE THAT THE ENERGY CONSUMED FOR CUTTING IS EQUAL TO SUM OF THE ENERGY CONSUMED FOR SHEAR AND FRICTION.

$$\text{i.e. } \boxed{U_c = U_s + U_f} \cdot \text{ PROVE THIS}$$

$$\text{or } \boxed{V_c F_c = F_s V_s + F_f \cdot V_f}$$

$$\text{We know that } U_c = \frac{P_z V_c}{A_0 V_c} = \frac{P_z}{A_0} \quad \text{--- (1)}$$

$$\text{But } V_c = V_s \cos \beta + V_f \sin \gamma_0$$



From the  $\Delta$  side

$$\frac{V_c}{\cos(\beta_0 - \gamma_0)} = \frac{V_f}{\sin \beta_0} = \frac{V_s}{\cos \gamma_0}$$

$$V_s = V_c \cdot \frac{\cos \gamma_0}{\cos(\beta_0 - \gamma_0)}$$

$$V_f = V_c \cdot \frac{\sin \beta_0}{\cos(\beta_0 - \gamma_0)}$$

$$U_c = \frac{P_z V_c}{A_0 V_c}$$

$$= \frac{P_z (V_s \cos \beta_0 + V_f \sin \gamma_0)}{A_0 V_c}$$

$$= \frac{P_z}{A_0} \cdot \frac{V_c}{V_c} \left[ \frac{\cos \gamma_0}{\cos(\beta_0 - \gamma_0)} \cdot \cos \beta_0 + \frac{\sin \beta_0}{\cos(\beta_0 - \gamma_0)} \sin \gamma_0 \right]$$

$$= \frac{P_z}{A_0} \left[ \frac{\cos \gamma_0 \cdot \cos \beta_0 + \sin \beta_0 \cdot \sin \gamma_0}{\cos(\beta_0 - \gamma_0)} \right]$$



$$\text{Again } P_z = R \cos(\eta - \gamma_0)$$

$$= \frac{R}{A_0} \left[ \frac{\cos(\eta - \gamma_0) \cdot \cos \beta \cdot \cos \gamma_0 + \cos(\eta - \gamma_0) \cdot \sin \beta \cdot \sin \gamma_0}{\cos(\beta - \gamma_0)} \right]$$

Adding & subtracting

$$\sin(\eta - \gamma_0) \cdot \sin \beta \cdot \cos \gamma_0$$

$$= \frac{R}{A_0} \left[ \frac{\cos(\eta - \gamma_0) \cdot \cos \beta \cdot \cos \gamma_0 - \sin(\eta - \gamma_0) \cdot \sin \beta \cdot \cos \gamma_0 + \sin(\eta - \gamma_0) \cdot \sin \beta \cdot \cos \gamma_0 + \cos(\eta - \gamma_0) \cdot \sin \beta \cdot \sin \gamma_0}{\cos(\beta - \gamma_0)} \right]$$

$$P_s = R \cos(\beta + \eta - \gamma_0)$$

$$= \frac{R}{A_0} \left[ \frac{\cos(\beta + \eta - \gamma_0) \cdot \cos \gamma_0 + \sin(\eta - \gamma_0) \cdot \sin \beta \cdot \cos \gamma_0 + \cos(\eta - \gamma_0) \cdot \sin \beta \cdot \sin \gamma_0}{\cos(\beta - \gamma_0)} \right]$$

$$= \frac{P_s}{A_0} \frac{\cos \gamma_0}{\cos(\beta - \gamma_0)} + \frac{R \sin \beta}{A_0 \cos(\beta - \gamma_0)} \left[ \frac{\sin(\eta - \gamma_0) \cdot \cos \gamma_0 + \cos(\eta - \gamma_0) \cdot \sin \gamma_0}{\cos(\beta - \gamma_0)} \right]$$

$$= \tau_s \cdot \varepsilon + \frac{1}{A_0} \left( P_z \cdot \cos \gamma_0 + P_z \sin \gamma_0 \right)$$

$$= \tau_s \cdot \varepsilon + \frac{F}{A}$$

$$= U_s + U_f$$

$$\Rightarrow \boxed{U_c = U_s + U_f}$$

$$\left[ \begin{array}{l} P_x = R \sin(\eta - \gamma_0) \\ P_z = R \cos(\eta - \gamma_0) \\ P_s = R \cos(\beta + \eta - \gamma_0) \end{array} \right]$$

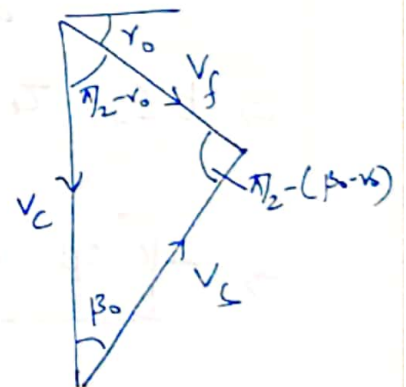
Alternative

$$P_z V_c = P_s V_s + F V_f$$

$$\begin{aligned} & P_s V_s + F V_f \\ &= (P_z \cos \beta_0 - P_{xy} \sin \beta_0) \cdot \frac{V_c \cos \gamma_0}{\cos(\beta_0 - \gamma_0)} \\ &\quad + (P_{xy} \cos \gamma_0 + P_z \sin \gamma_0) \cdot \frac{V_c \sin \beta_0}{\cos(\beta_0 - \gamma_0)} \\ &= P_z \cdot \frac{\cos \beta_0 \cdot V_c \cos \gamma_0}{\cos(\beta_0 - \gamma_0)} - P_{xy} \cdot \sin \beta_0 \cdot \frac{V_c \cos \gamma_0}{\cos(\beta_0 - \gamma_0)} \\ &\quad + P_{xy} \cdot \frac{\cos \gamma_0 \cdot V_c \sin \beta_0}{\cos(\beta_0 - \gamma_0)} + P_z \cdot \frac{\sin \gamma_0 \cdot V_c \sin \beta_0}{\cos(\beta_0 - \gamma_0)} \\ &= V_c \cdot \frac{1}{\cos(\beta_0 - \gamma_0)} \left[ P_z \cos \beta_0 \cdot \cos \gamma_0 + P_z \sin \gamma_0 \cdot \sin \beta_0 \right] \\ &= P_z \cdot V_c \cdot \frac{\cos(\beta_0 - \gamma_0)}{\cos(\beta_0 - \gamma_0)} \\ &= P_z \cdot V_c \end{aligned}$$

$$\therefore P_z V_c = P_s V_s + F V_f$$

$$\Rightarrow \boxed{U_c = U_s + U_f}$$



From the  $\Delta$

$$\frac{V_c}{\cos(\beta_0 - \gamma_0)} = \frac{V_f}{\sin \beta_0} = \frac{V_s}{\cos \gamma_0}$$

$$\Rightarrow V_s = V_c \frac{\cos \gamma_0}{\cos(\beta_0 - \gamma_0)}$$

$$\Rightarrow V_f = V_c \cdot \frac{\sin \beta_0}{\cos(\beta_0 - \gamma_0)}$$

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## CONDITION OF FORM FACTOR BY WHICH WE CAN FIND A MINIMUM CUTTING FORCE.

### ERNST AND MERCHANT'S ANGLE RELATIONSHIP

Ernst and merchant assumed that

- (a) The shear stress is a true property of the work-material.  $\tau_s = K$  (const.)
- (b) The total energy input to the system has been consumed in deformation.
- (c) The friction process at the chip tool interface is - invariant i.e.  $\eta = \text{constant}$

We know that  $P_z = \frac{t S_0 \tau_s \cos(\eta - \gamma_0)}{\sin \beta \cdot \cos(\beta + \eta - \gamma_0)} \quad \text{--- (1)}$

Applying principle of min energy

$$\text{i.e. } \frac{dP_z}{d\beta} = 0$$

$$\Rightarrow t S_0 \cdot \tau_s \cdot \cos(\eta - \gamma_0) \cdot \frac{d}{d\beta} \left[ \frac{1}{\sin \beta \cdot \cos(\beta + \eta - \gamma_0)} \right] = 0$$

$$\Rightarrow \frac{-[\cos \beta \cdot \cos(\beta + \eta - \gamma_0) - \sin \beta \cdot \sin(\beta + \eta - \gamma_0)]}{[\sin \beta \cdot \cos(\beta + \eta - \gamma_0)]^2} = 0$$

$$\Rightarrow \cos \beta \cdot \cos(\beta + \eta - \gamma_0) = \sin \beta \cdot \sin(\beta + \eta - \gamma_0)$$

$$\Rightarrow \cos \beta \cdot \cos(\beta + \eta - \gamma_0) - \sin \beta \cdot \sin(\beta + \eta - \gamma_0) = 0$$



$$\Rightarrow \cos(2\beta + \eta - \gamma_0) = 0$$

$$\Rightarrow \boxed{2\beta + \eta - \gamma_0 = \frac{\pi}{2}} \quad \text{--- (2)}$$

$\therefore P_z$  is min when  $2\beta + \eta - \gamma_0 = \frac{\pi}{2}$

Applying the condn in eqn (1) we get-

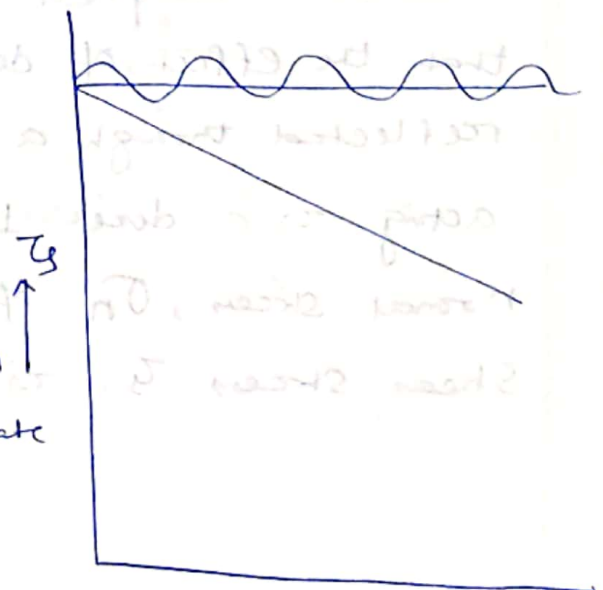
$$P_z = \frac{t S_0 \tau_g \cdot \cos(\frac{\pi}{2} - 2\beta)}{\sin \beta \cdot \cos(\frac{\pi}{2} - \beta)}$$

$$= \frac{t S_0 \cdot \tau_g \cdot \sin 2\beta}{\sin \beta \cdot \sin \beta}$$

$$\Rightarrow \boxed{P_z = 2t S_0 \tau_g \cot \beta_0}$$

If we verify the experimental result with the above theoretical result, a ~~the~~ wide variation will be there because here we take  $\tau_g$  as a constant but it is not true.

Experimental variations revealed that eqn (2) is an over estimate and the reasons are :-



Variation in cutting condition  $\rightarrow$

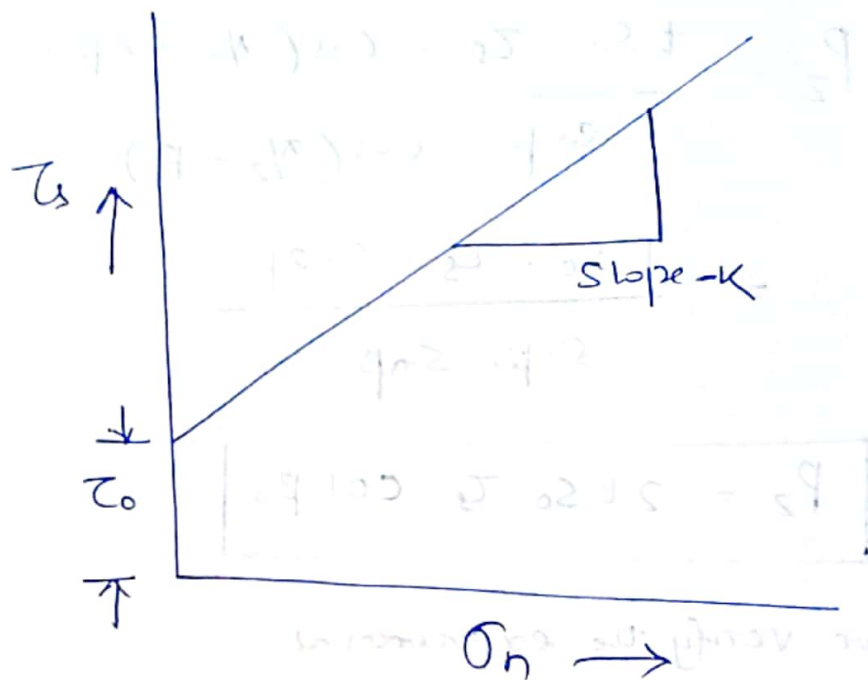


(a) Non-compatibility with the actual deformation process,

(b) Non-cognizance of the dependency of  $\beta$  on  $\eta$ .

Modified Merchant's Angle relationship or  
Merchant's Second Solution  $\Rightarrow$

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Mercant attempted an alternative sol<sup>n</sup> by assuming that the effect of deformation and friction are reflected through a change of normal force,  $P_n$  acting in a direction  $\perp$  to the plane of shear. The normal stress,  $\sigma_n$  of the shear plane affects the Shear stress  $\tau_s$ , in the direction of shear.

It was assumed that

$$\tau_s = f(\tau_0, \sigma_n)$$

$$\text{or } \tau_s = \tau_0 + k \sigma_n \quad \text{--- (1)}$$

From merchant circle diagram,

$$P_n = P_s \tan(\beta + \eta - \gamma_0)$$

Dividing this with area of the shear plane

$$\sigma_n = \tau_s \tan(\beta + \eta - \gamma_0) \quad \text{--- (2)}$$

By eqn (1) & (2) we get

$$\frac{\tau_s - \tau_0}{k} = \tau_s \tan(\beta + \eta - \gamma_0)$$

$$\Rightarrow \frac{\tau_s}{k} - \tau_s \tan(\beta + \eta - \gamma_0) = \frac{\tau_0}{k}$$

$$\Rightarrow \frac{\tau_s}{k} (1 - k \tan(\beta + \eta - \gamma_0)) = \frac{\tau_0}{k}$$

$$\Rightarrow \tau_s = \frac{\tau_0}{1 - k \tan(\beta + \eta - \gamma_0)}$$

Substituting the in eqn (A) we get

$$P_z = \frac{S_0 \cdot t \cdot \tau_0 \cdot \cos(\eta - \gamma_0)}{\sin \beta \cdot (1 - k \tan(\beta + \eta - \gamma_0)) \cdot \cos(\beta + \eta - \gamma_0)}$$

By applying principle of min energy,  $P_z$  is min

when 
$$\frac{dP_z}{d\beta} = 0$$

$$\Rightarrow t \cdot S_0 \cdot \tau_0 \cdot \cos(\eta - \gamma_0) \cdot \frac{d}{d\beta} \left[ \frac{1}{(1 - k \tan(\beta + \eta - \gamma_0)) \cdot \sin \beta \cdot \cos(\beta + \eta - \gamma_0)} \right] = 0$$

$$\Rightarrow -1 \cdot \frac{d}{d\beta} \left[ (1 - k \tan(\beta + \eta - \gamma_0)) \sin \beta \cdot \cos(\beta + \eta - \gamma_0) \right] = 0$$

$$\left[ (1 - k \tan(\beta + \eta - \gamma_0)) \cdot \sin \beta \cdot \cos(\beta + \eta - \gamma_0) \right]^2$$

$$\Rightarrow \frac{d}{d\beta} \left[ (1 - k \tan(\beta + \eta - \gamma_0)) \cdot \sin \beta \cdot \cos(\beta + \eta - \gamma_0) \right] = 0$$

$$\Rightarrow \frac{d}{d\beta} \left[ \sin \beta \cdot \cos(\beta + \eta - \gamma_0) - k \sin \beta \sin(\beta + \eta - \gamma_0) \right] = 0$$

$$\Rightarrow \cos \beta \cdot \cos(\beta + \eta - \gamma_0) - \sin \beta \cdot \sin(\beta + \eta - \gamma_0) - k \sin \beta \cdot \cos(\beta + \eta - \gamma_0) - k \cos \beta \cdot \sin(\beta + \eta - \gamma_0) = 0$$



$$\Rightarrow \cos \beta \cdot \cos (\beta + \eta - \gamma_0) - \sin \beta \cdot \sin (\beta + \eta - \gamma_0) \\ = K \left[ \cos \beta \cdot \sin (\beta + \eta - \gamma_0) + \sin \beta \cdot \cos (\beta + \eta - \gamma_0) \right]$$

$$\Rightarrow \cos (2\beta + \eta - \gamma_0) = K \sin (2\beta + \eta - \gamma_0)$$

$$\Rightarrow \boxed{\cot (2\beta + \eta - \gamma_0) = K}$$

$$\Rightarrow \boxed{2\beta + \eta - \gamma_0 = \cot^{-1} K = C} \quad \text{--- (3)}$$

where  $K$  is the slope of the curve.

$$\cot^{-1}(K) = C \quad (\text{constant})$$

&  $C$  is called the machining constant.

and experiment says that value of  $C$  will be from  $70^\circ$  to  $85^\circ$  i.e. it is  $< \pi/2$ .

By substituting the above value of eqn (3) to  $P_2$ , we get.

$$P_2 = \frac{t S_0 \tau_s \cos(\eta - \gamma_0)}{\sin \beta \cdot \cos(\beta + \eta - \gamma_0)} \\ = \frac{t S_0 \tau_s \cdot \cos(C - 2\beta)}{\sin \beta \cdot \cos(C - \beta)}$$

$$= \frac{t S_0 \tau_s \cdot \cos [(C - \beta) - \beta]}{\sin \beta \cdot \cos (C - \beta)}$$



$$= \frac{t S_0 \tau_s [\cos(C-\beta) \cdot \cot\beta + \sin(C-\beta) \cdot \sin\beta]}{\sin\beta \cdot \cos(C-\beta)}$$

$$\Rightarrow \boxed{P_z = t S_0 \tau_s [\cot\beta + \tan(C-\beta)]}$$

Again we know

$$\tan\beta = \frac{\cos\gamma_0}{3 - \sin\gamma_0}$$

$$\therefore \boxed{P_z = t S_0 \tau_s \left[ \frac{3 - \sin\gamma_0}{\cos\gamma_0} + \tan\left(C - \tan^{-1}\left(\frac{\cos\gamma_0}{3 - \sin\gamma_0}\right)\right) \right]}$$

According to Merchant,  $C$  is a property of work material unaffected by cutting angle but microstructure and grain size have effect on  $C$ . Cold working of material increases its mechanical constant.

## Lee and Shaffner's Angle Relation

An alternative angle relationship based on lower-bound concepts has been developed by Lee and Shaffer. Lower bound is any load for which a stress distribution can be found such that

- (1) Equilibrium condition can be satisfied everywhere.
- (2) The stress distribution is in equilibrium with the external loads.
- (3) The stresses are not greater than those which are in accordance with a yield or flow criterion.

### Assumptions :

- (1) The material in the stress field is ideally plastic & non-hardening.
- (2) The shear plane coincides with the direction of maximum shear stress.
- (3) The shear stress is maximum along the shear plane.
- (4) The work material ahead of the cutting tool in the zone of OAB is not undergoing hardening.
- (5) The whole system is in dynamic equilibrium.
- (6) The plane AB is at  $45^\circ$  to the shear plane OA. Along plane AB shear stress is zero.

The stress field in this region is composed of two families of slip lines which are  $11\pi$  & straight.

The condition of zero ~~stress~~ shear across the line AB determines that the slip lines are inclined at  $\pi/4$  to shear plane.

In Mohr's circle, b is the state of stress corresponding to the point b on the boundary between plastic & rigid zones. The stresses at b are zero because it has been assumed that no stress is transmitted across the boundary. a, d and c, f denotes the state of stress <sup>of the</sup> corresponding planes bounded by slip lines. Along the interface CB, the state of stress is denoted by the point e, such that

$$\tan \eta = \frac{\tau_c}{\sigma_c} = \mu$$

The plane of e is at an angle  $(\beta - \gamma_0)$  from the plane of d. From Mohr's ~~circle~~ circle it is seen that

$$\beta + \eta - \gamma_0 = \pi/4$$

This is the angle relationship derived by Lee and Shaffer.

Substituting this on cutting force ÷







$$\begin{aligned}
 P_z &= \frac{S_o t \tau_g \cos(\eta - \gamma_o)}{\sin \beta_o \cdot \cos(\beta + \eta - \gamma_o)} \\
 &= \frac{S_o t \tau_g \cos(\pi/4 - \beta_o)}{\sin \beta_o \cdot \cos \pi/4} \\
 &= \sqrt{2} \frac{S_o t \tau_g \cdot \cos(\pi/4 - \beta_o)}{\sin \beta_o} \\
 &= \sqrt{2} S_o t \cdot \tau_g \cdot \frac{\cos \pi/4 \cdot \cos \beta_o + \sin \pi/4 \cdot \sin \beta_o}{\sin \beta_o} \\
 &= \sqrt{2} \cdot \frac{1}{\sqrt{2}} S_o t \tau_g (\cot \beta_o + 1)
 \end{aligned}$$

$$\Rightarrow \boxed{P_z = S_o t \tau_g (1 + \cot \beta_o)}$$

We know that  $\tan \beta = \frac{\cos \gamma_o}{3 - \sin \gamma_o}$

$$\Rightarrow \cot \beta = \frac{3 - \sin \gamma_o}{\cos \gamma_o}$$

$$\Rightarrow \cot \beta \approx 3 - \tan \gamma_o$$

Since  $\gamma_o \approx \pm 5^\circ$

$$\therefore \boxed{P_z = S_o t \cdot \tau_g (3 - \tan \gamma_o + 1)}$$

It is applicable for ductile material.

From mechan O diagram we have

$$P_z = 2t s_0 \tau_s \cot \beta$$

$$= 2t s_0 \tau_s (3 - \tan \gamma_0)$$

Now

$$P_{xy} = P_z \tan(\eta - \gamma_0)$$

$$= \frac{t s_0 \tau_s \cos(\eta - \gamma_0)}{\sin \beta \cos(\beta + \eta - \gamma_0)} \times \tan(\eta - \gamma_0)$$

$$= \frac{t s_0 \tau_s \sin(\eta - \gamma_0)}{\sin \beta \cos(\beta + \eta - \gamma_0)}$$

$$= \frac{t s_0 \tau_s \sin(\pi/4 - \beta)}{\sin \beta \cos(\pi/4)}$$

$$(\because 2\beta + \eta - \gamma_0 = \pi/2)$$

$$= \frac{t s_0 \tau_s (\cot \beta - 1)}{\sin \beta \cos(\pi/4)}$$

$$= t s_0 \tau_s (\cot \beta - 1)$$

$$= t s_0 \tau_s (3 - \tan \gamma_0 - 1)$$

$$\therefore P_{xy} = t s_0 \tau_s (\cot \beta - 1) = t s_0 \tau_s (3 - \tan \gamma_0 - 1)$$

If  $\gamma_0 = -10^\circ$  &  $\eta = 35^\circ$  then  $\cot \beta = \infty$ , thus it indicates that this soln is untenable.

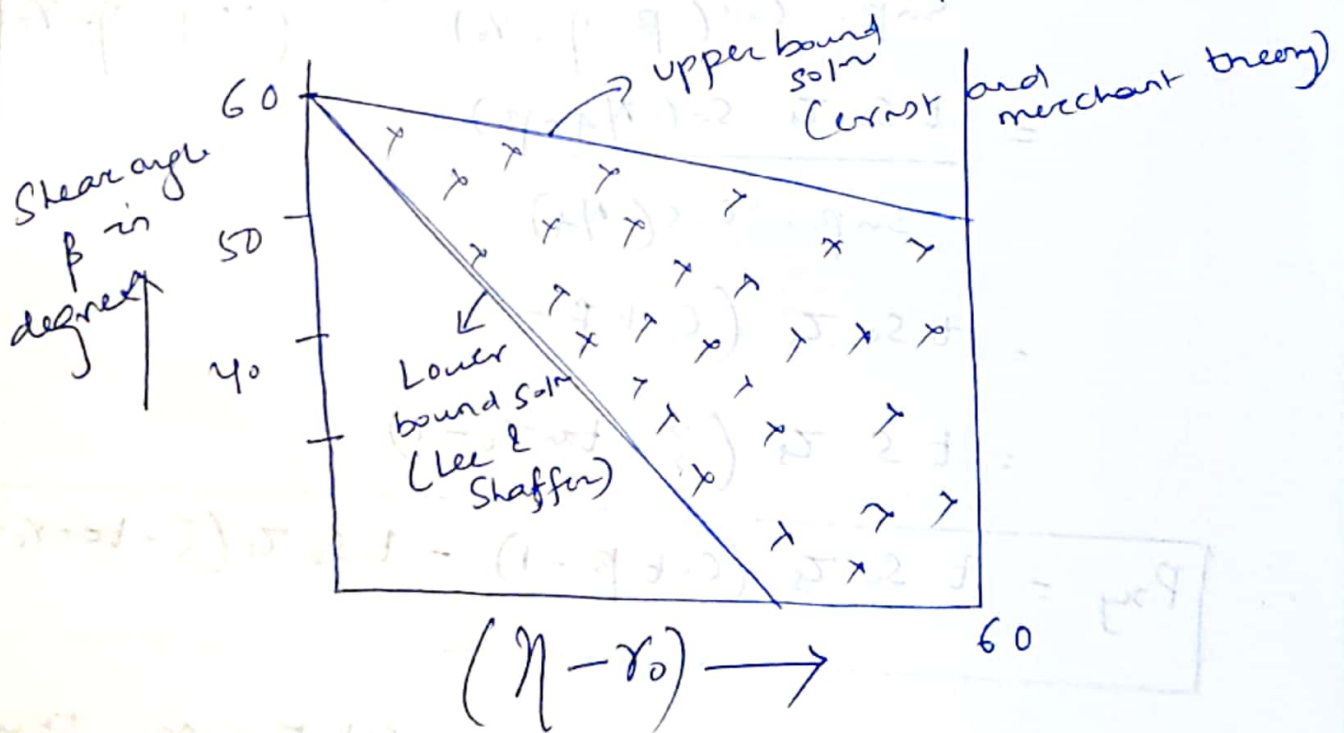
Lee and Shaffer has further extended this theory to consider the effect of built up edge and arrived at a soln

$$\beta = \frac{\pi}{4} - \eta + \gamma_0 + \Theta$$

Where  $\Theta$  is an additional angle depending on the size of the built up edge.

It is evident that the ideal soln of Ernst Merchant ( $2\beta + \eta - \gamma_0 = \pi/4$ ) is an upper bound equation.

Merchant's second soln ( $2\beta + \eta - \gamma_0 = C$ ) where  $C < \pi/4$  or Lee and Shaffer's ( $\beta + \eta - \gamma_0 = \pi/4 + \Theta$ ) are recognized approximations to fit experimental results.





What is BUE? Why it is produced?  
Indicate the cutting condn favourable for formation of BUE? What are the advantage & disadvantage of BUE?

When the frictional resistance at the chip tool - interface is considerably high, the contact layers subjected to all sided compression may shear from the bulk of the chip, such separated layers are called Built up edges. The edge rapidly grows in size and form as it is built up.

Built up edges are strongly deformed - strain-hardened material and form a zone of stagnation, reducing tool wear, increasing tool life and worsening surface finish.

It has been observed that the BUE form with increased friction but diminish with the increase in temperature.

Favourable cutting condn are :-

- (i) Low cutting speed.
- (ii) Low rake angle
- (iii) High speed.
- (iv) Lack of cutting fluid.
- (v) Large depth of cut.



Advantage ÷

It reduces the tool wear

It increases the tool life.

Disadvantage ÷

The BUE formed at the cutting edge of the tool is unstable, breaks down & some fragment leave with the chip as it passes off & the rest adheres to the work surface producing characteristics rough-surface. So it worsens the surface of the w/p.

Orthogonal and  
Oblique cutting →

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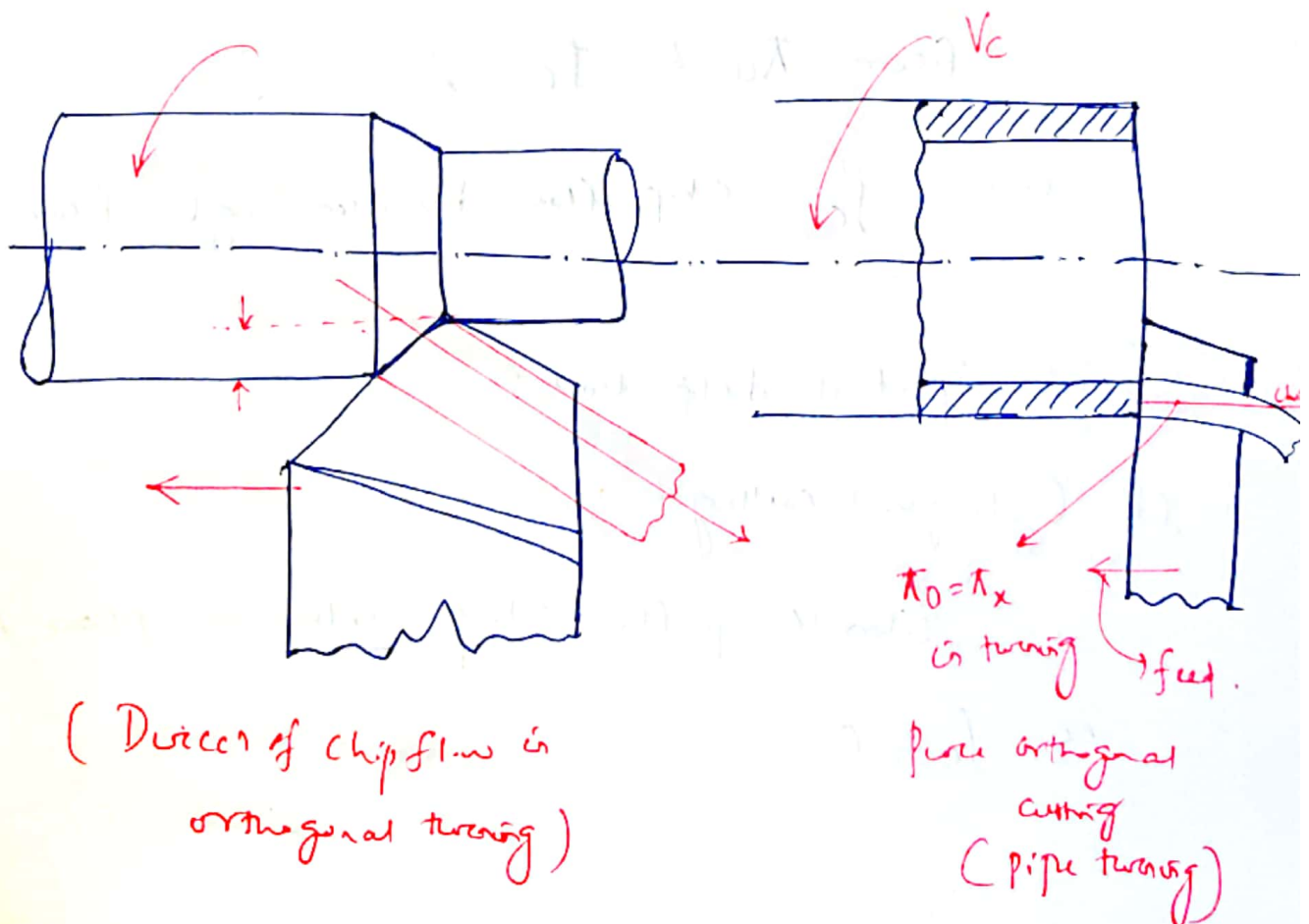
## Orthogonal & Oblique Cutting

Specific instructional objectives:

To enable the students, at the end of this lecture:

- (1) Define & distinguish, with illustrations, betn orthogonal cutting & oblique cutting.
- (2) Identify the causes of oblique cutting & chip flow deviation.
- (3) Determine the angle of chip flow deviation.
- (4) Illustrate & deduce effective rake angle.
- (5) State the effects of oblique cutting.

### Orthogonal Cutting



It is expected that, in turning ductile material by a sharp tool, the continuous chip flow over the tool rake surface, and in the direction apparently  $\perp$  to the principal cutting edge, i.e. along orthogonal plane (i.e. normal to the cutting plane containing principal cutting edge)

\* Practically, the chip may not flow along the orthogonal plane due to several factors such as presence of rake angle  $\lambda$  etc.

(1) when  $\lambda = 0$ , the chip flows along orthogonal plane, i.e.  $\phi_c = 0$

(2) when  $\lambda \neq 0$ , the chip flow is deviated from  $\pi_0$  &  $\phi_c = \lambda$

where  $\phi_c = \text{chip flow deviation angle from } \pi_0$

Some important definitions:

(1) Orthogonal cutting:

when chip flow along orthogonal plane  $\pi_0$ ,

or  $\phi_c = 0$ .



## Oblique cutting:

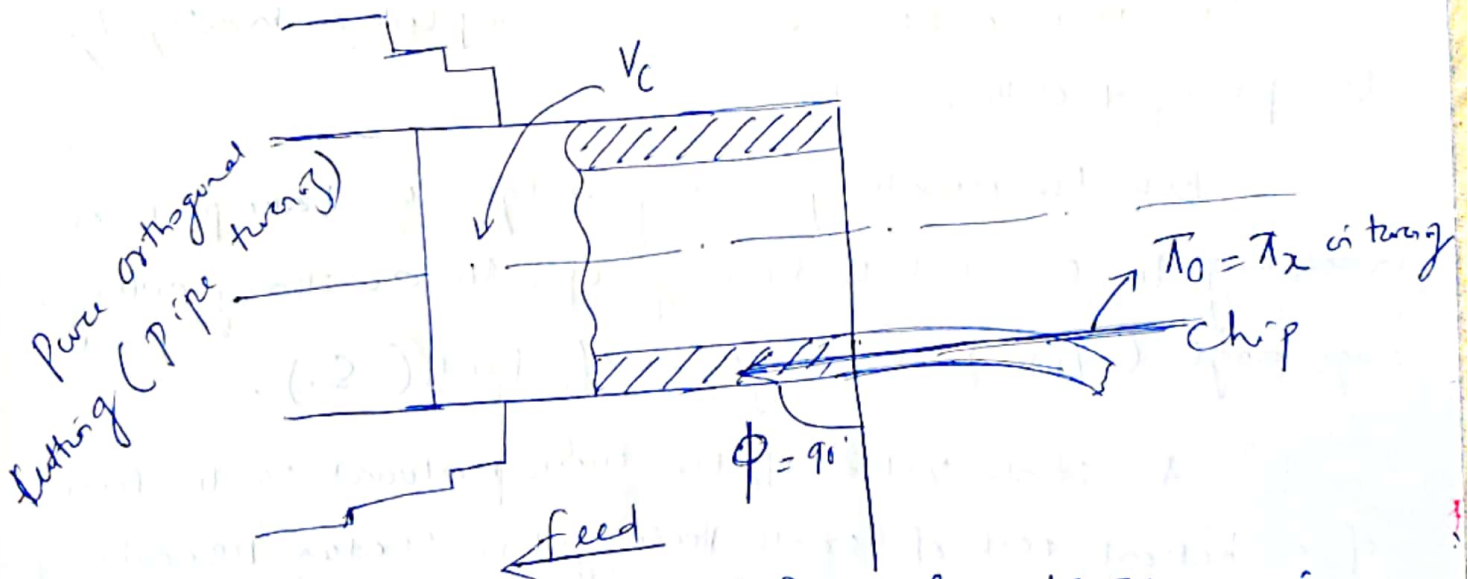
When chip flow deviates from orthogonal plane (i.e.  $\phi_c \neq 0$ ).

Practically  $\phi_c$  may not be zero even if  $\lambda = 0$  &  $\phi_c$  may not be exactly equal to  $\lambda$ .

This is because there are some other factors (than  $\lambda$ ) which also may cause chip flow deviation.

## Pure orthogonal cutting:

This refers to chip flow along  $\pi_0$  &  $\phi = 90^\circ$ .



where a tubular job of uniform thickness is turned (reduced in length) in a lathe by a turning-tool of geometry  $\lambda = 0$  &  $\phi = 90^\circ$  resulting in chip flow along  $\pi_0$  (which is ~~also~~  $\pi_x$  in this case).

## Causes of chip flow deviation & Angle of deviation

The direction of chip flow in machining like turning by single point tool may deviate from orthogonal plane due to the following 3 reasons:

- (1) Restricted cutting effect (RCE),
- (2) Presence of tool nose radius, ( $r$ )
- (3) Presence of inclination angle  $\lambda (\neq 0)$ .

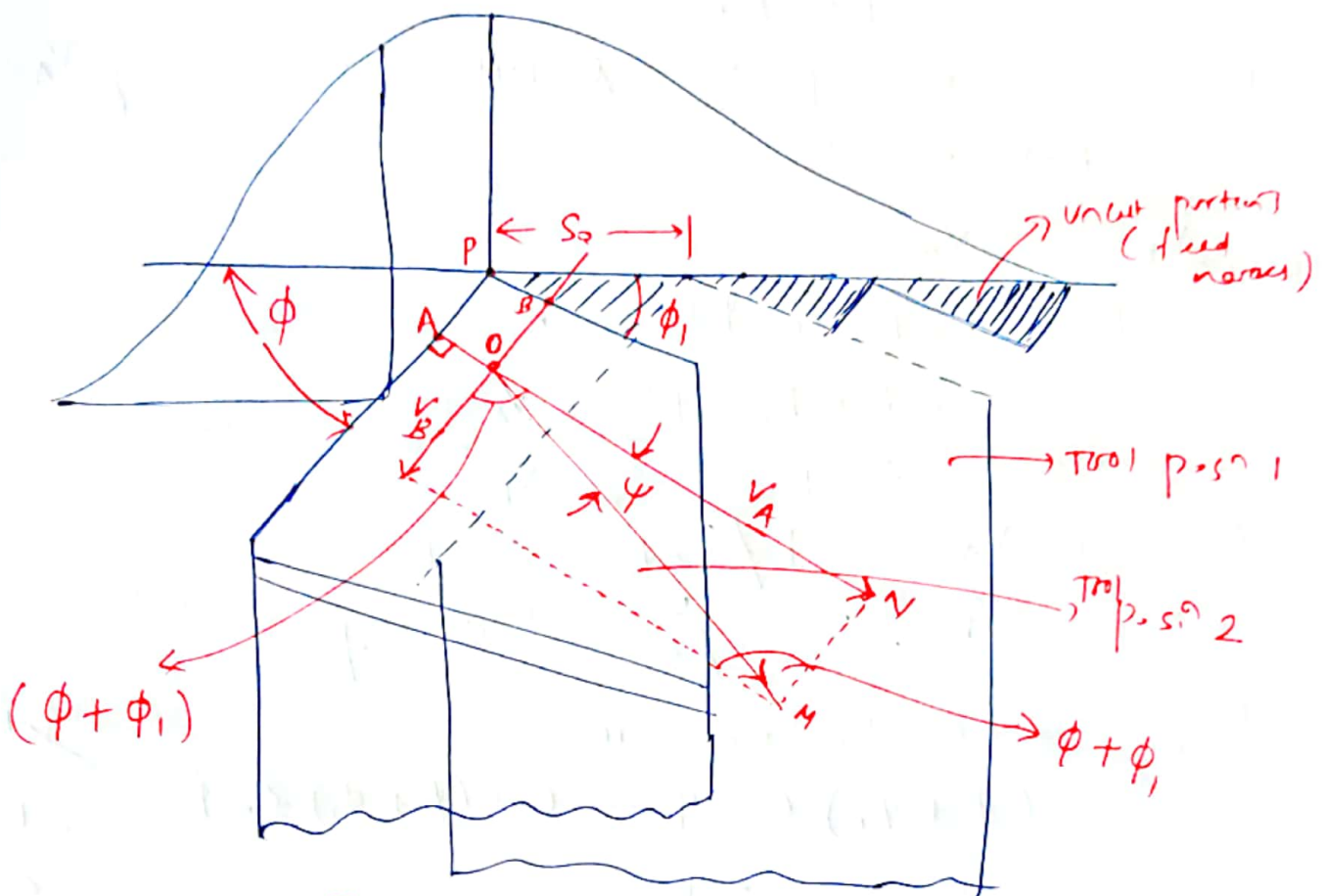
### i) Restricted cutting effect (RCE)

The material removal is accomplished mainly by the principal cutting edge.

But the auxiliary cutting edge also take part in machining to some extent depending upon the auxiliary cutting-edge angle ( $\phi_1$ ) & the magnitude of feed ( $S_0$ ).

A small volume of the work material in the form of a helical rib of small triangular section remains uncut. This causes surface roughness in the form of fine-threads called feed marks / scallop marks.

The work material flows <sup>out</sup> in the form of chip at velocity  $V_A$  when the auxiliary cutting edge plays a negligible role on chip formation.



(Chip flow deviation due to RCE)

$V_A \rightarrow$  plays important role when auxiliary cutting edge plays negligible role in chip formation.

However, when the auxiliary cutting edge comes into sizeable contact with the w/p then the material that comes out from that edge at velocity  $V_B$ , interferes with the main stream of the chip causing chip flow deviation from the direction of  $V_A$  by an angle say  $\psi$ .

This phenomenon is called Restricted cutting effect (RCE)



$$\angle APB = 180^\circ - (\phi + \phi_1)$$

$$\angle AOB = \phi + \phi_1$$

From properties of  $\Delta$ ,  $\Delta OMN$ , and assuming  $V_A = ON$  &  $V_B = MN$ , we get

$$\frac{V_B}{\sin \psi} = \frac{V_A}{\sin (\phi + \phi_1 - \psi)}$$

$$\text{or } \frac{\sin (\phi + \phi_1 - \psi)}{\sin \psi} = \frac{V_A}{V_B} \quad \text{--- (i)}$$

$$\text{Assuming } \frac{V_A}{V_B} = \frac{t / \sin \phi}{S_o / 2} = \frac{2t}{S_o \sin \phi} \quad \text{--- (ii)}$$

Eqn (i) can be rewritten as

$$\frac{\sin (\phi + \phi_1) \cos \psi - \cos (\phi + \phi_1) \sin \psi}{\sin \psi} = \frac{2t}{S_o \sin \phi} \quad \text{--- (iii)}$$

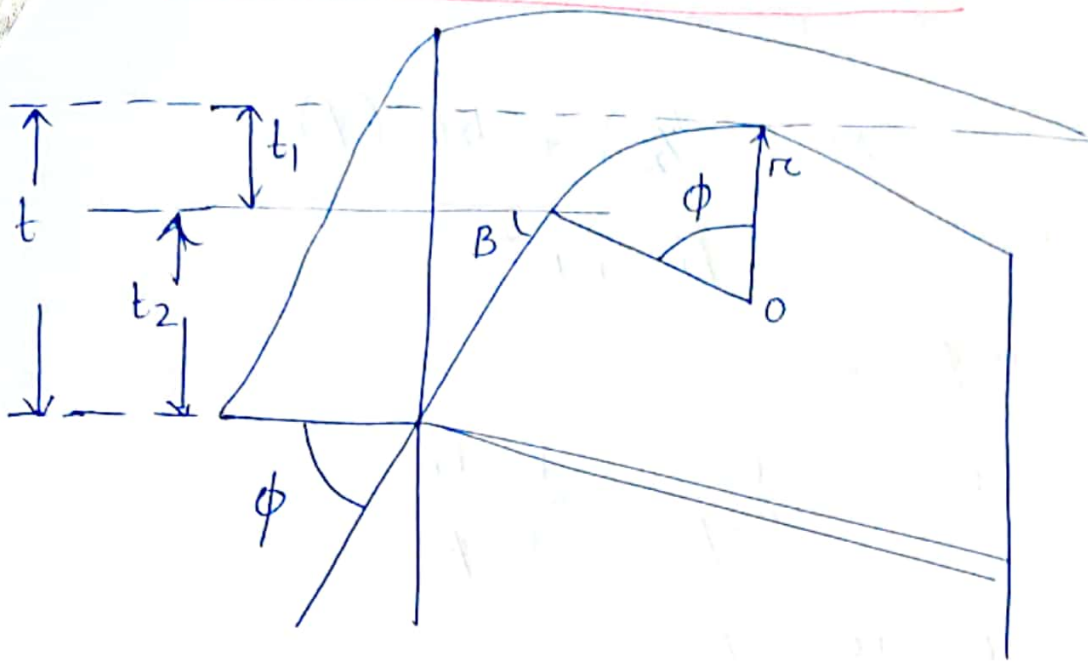
On simplification, eqn (iii),  $\psi$  can be expressed as:

$$\tan \psi = \frac{\sin (\phi + \phi_1)}{\left( \frac{2t}{S_o} \sin \phi \right) + \cos (\phi + \phi_1)} \quad \text{--- (iv)}$$

Eqn (iv) above reveals that even in absence of  $\lambda$  the chip flow may deviate, & the angle of deviation  $\psi$ , though small, depends upon the cutting angles & depth of cut to feed ratio ( $t/S_o$ ).



## Effect of Tool - nose radius



(Variation of principal cutting edge along the nose of tool)

From eqn 
$$\tan \psi = \frac{\sin(\phi + \phi_1)}{\left(\frac{2t}{s}\right) \sin \phi + \cos(\phi + \phi_1)} \quad \text{--- (A)}$$

It indicates that chip flow direction is significantly influenced by the principal cutting edge angle  $\phi$ .

In a tool having nose radius, the value of  $\phi$  continuously varies from zero, over the curved portion of the principal cutting edge.

Such variation reasonably influences the chip flow direction. Therefore, to incorporate the effect of tool-nose radius also,  $\phi$  in eqn (A) by average value of  $\phi$  (i.e.  $\phi_{avg}$ ) which can be determined with the help of

the diagram in fig above.

From figure, we get:

$$\phi_{arg} = \frac{\overline{AB} \cdot (\phi/2) + \overline{BC} (\phi)}{\overline{AB} + \overline{BC}}$$

where  $\overline{AB} = r\phi$  &

$$BC = \frac{t_2}{\sin \phi} = \frac{t - t_1}{\sin \phi}$$

Here  $t_1 = r - r \cos \phi$

$$\phi_{arg} = \frac{(\frac{\phi}{2}) + \left[ \left( \frac{t}{r} \right) + \cos \phi - 1 \right] (1/\sin \phi)}{1 + \left[ (t/r) + \cos \phi - 1 \right] / \phi \sin \phi} \quad \text{--- (B)}$$

It can be noted in eqn (B) that the difference between  $\phi$  &  $\phi_{arg}$  is governed mainly by the ratio of depth of cut to nose radius.

Hence to incorporate the effect of nose radius along with restricted cutting effects,  $\phi$  in eqn (A) has to be replaced by  $\phi_{arg}$  to be determined by eqn (B).

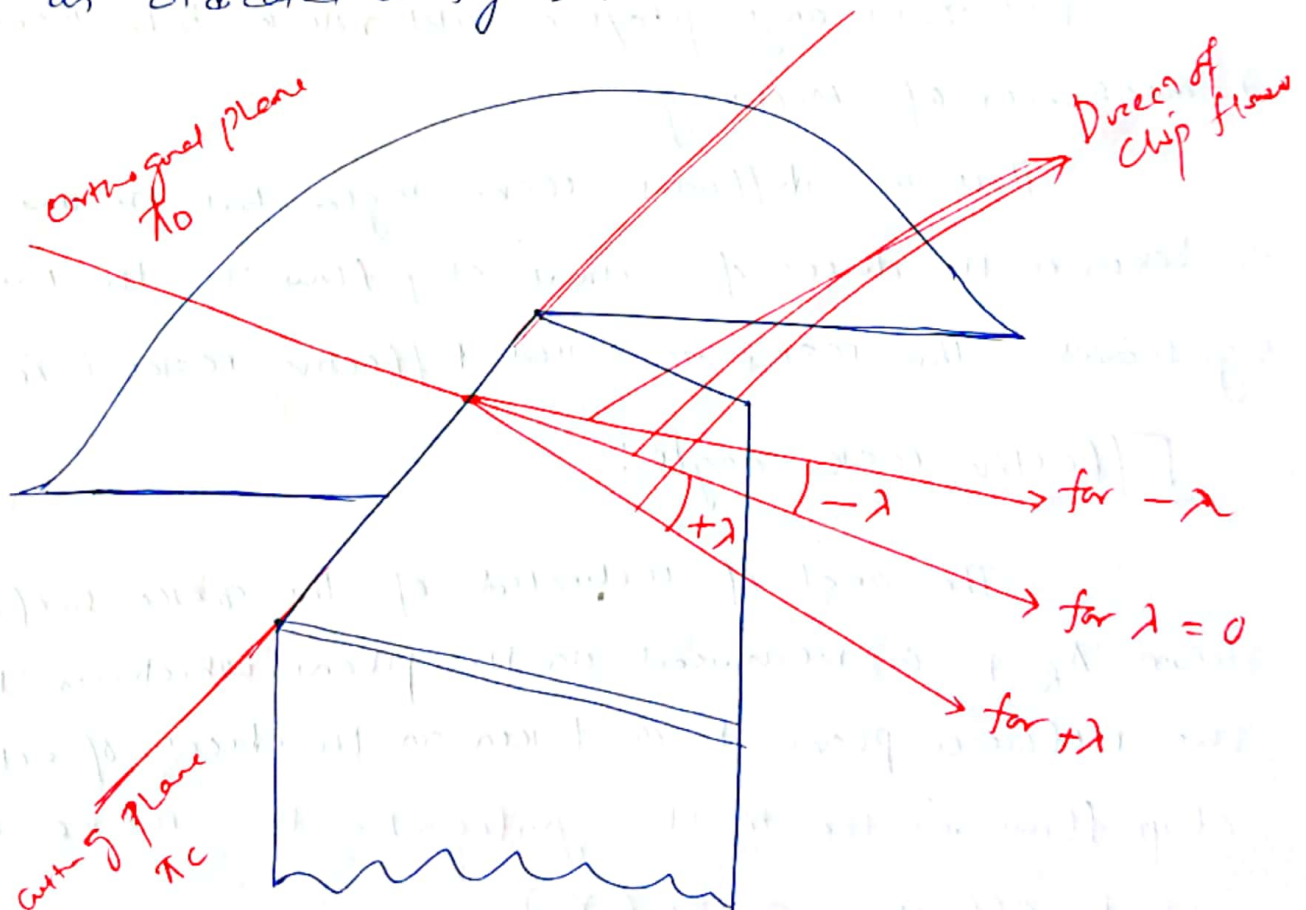
In this case

$$\tan \psi = \frac{\sin(\phi_{arg} + \phi_1)}{\left(\frac{2t}{S_0}\right) \sin \phi_{arg} + \cos(\phi_{arg} + \phi_1)}$$

⌋ ⊙

### Effect of inclination angle ( $\lambda$ )

In the absence of RCE & nose radius, the chip flow deviation will be governed only by the value of  $\lambda$  as indicated in fig below.



Therefore, the combined effect of RCE, tool nose radius & presence of  $\lambda$  will affect chip flow deviation angle  $\phi_c$  as

$$\phi_c = \psi + \lambda \quad (\pm)$$

Generally compared to  $\lambda$ ,  $\psi$  is very small ( $0^\circ$  to  $5^\circ$ )

So approximately  $\phi_c = \lambda$  where  $\lambda$  may be +ve or -ve.

### Effective rake angle ( $\gamma_e$ )

Tool rake angle plays a vital role in both mechanism & mechanics of machining.

There are different rake angles but the one which is taken in the direction of actual chip flow is the most significant. This rake is called Effective rake ( $\gamma_e$ ).

Effective rake angle:

The angle of inclination of the rake surface from  $\pi_R$  is measured on the plane which is  $\perp$  to the reference plane & is taken in the direction of actual chip flow is the most significant. This rake is called effective rake ( $\gamma_e$ ).



$$\sin \gamma_c = \cos \lambda \cdot \cos \phi_c \cdot \sin \gamma_n + \sin \lambda \cdot \sin \phi_c$$

Assuming  $\lambda = \phi_c$  we get

$$\sin \gamma_c = \cos^2 \lambda \cdot \sin \gamma_n + \sin^2 \lambda$$

where  $\tan \gamma_n = \tan \gamma_0 \cdot \cos \lambda$ .

It is again noted that if  $\lambda = 0$ , then  $\gamma_c = \gamma_n = \gamma_0$ .

In case of oblique cutting, which is practically more common, the actual chip flow direction & the corresponding rake angle (i.e. effective rake) should be used for a more accurate assessment of cutting forces, friction & tool wear.

### Effect of Oblique cutting

Contrary to simpler orthogonal cutting, oblique cutting leads to the following effects in chip formation and mechanics of machining.

- (1) Chip does not flow along the orthogonal plane.
- (2) positive  $\lambda$  causes
  - (a) Chip flow deviation away from the

finished surface, which may result lesser further damage to the finished surface.

(b) Reduction of mechanical strength of the tool tip.

(c) Increase in temperature at the tool tip.

(d) More vibration in turning slender rods due to increase in  $P_y$  (tangential force).

(3) Negative  $\lambda$  causes

(a) Increase in mechanical strength of the tool tip.

(b) Reduction in tool tip temperature.

(c) Change of chip c/s from rectangle (ideal) to skewed trapezium.

(d) The ductile metals to produce more compact helical chips if not broken by chip breaker.

(e) Analysis of cutting forces, chip-tool friction, etc more complex.